Houghton Mifflin Harcourt

Research-Based Approach

GO MATH

Grades 6–8

Houghton Mifflin Harcourt's *Go Math!* © 2014, Middle School, Grades 6-8: A Research-Based Approach

Houghton Mifflin Harcourt

Contents

Overview

Houghton Mifflin Harcourt's *Go Math!* © 2014 is a focused, middle school mathematics program designed to meet the objectives and intent of the Common Core State Standards for Mathematics. The program provides thorough coverage of the standards, with an emphasis on depth of instruction. The program is designed to provide coherent and focused progressions across the grades. Particular attention was given to providing support for teachers as they deliver this focused, rigorous curriculum. Students and teachers are supported by the program's unique, write-in, interactive *Student Editions* at every grade, which allow students to represent, solve, and explain in one place, and by the features of the fully integrated *Go Digital!* resources, which harness the power of technology to support students' deep mathematical learning.

The purpose of this document is to demonstrate clearly and explicitly the research upon which *Go Math!* is based. This research report is organized by the major instructional strands that underpin the program:

- Alignment with the Common Core State Standards;
- Effective approaches to mathematics instruction;
- Data-driven instruction and ongoing assessment;
- Instruction that meets the needs of all learners; and
- Use of technology to teach mathematics.

Each strand is supported by research in mathematics education, and by research on teaching and learning across the content areas. The content, activities, and strategies presented in *Go Math!* align with what we know about teaching for mathematical understanding and align to the Common Core State Standards for Mathematics.

To help readers make the connections between the research strands and the *Go Math!* program, the following sections are used within each strand:

- **Defining the Strand.** This section summarizes the terminology and provides an overview of the research related to the strand.
- **Research that Guided the Development of *Go Math!*** This section identifies subtopics within each strand and provides excerpts from and summaries of relevant research on each subtopic.
- **From Research to Practice.** This section explains how the research data is exemplified in *Go Math!*

The combination of the major research recommendations and related features of *Go Math!* will help readers better understand how the program incorporates research in its instructional design.

A list of references is provided at the end of this document.

Introduction to *Go Math!* © 2014

We live in a mathematical world. Never before has the workplace demanded such complex levels of mathematical thinking and problem solving (National Council of Teachers of Mathematics, 2009). Clearly, those who understand and can do mathematics will have opportunities that others do not—and building students' foundational skills is essential. An analysis of the results of the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) led researchers to conclude that "countries that want to improve their mathematics performance should start by building a strong mathematics foundation in the early grades" (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005, p. v).

The Common Core State Standards for Mathematics were written provide such a foundation for students. The standards were developed through a bipartisan, state-led process, which incorporated the input of mathematics educators and researchers across the country, as well as the knowledge gained from previous state and international efforts. The Common Core Standards for Mathematics were written with the goal of providing greater focus and coherence to Kindergarten through Grade 12 mathematics instruction in the United States. As the writers of the Common Core document point out in their introduction to the Standards for Mathematics, "For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is 'a mile wide and an inch deep.' These Standards are a substantial answer to that challenge" (NGA & CCSSO, 2010a, p. 3).

The Common Core State Standards (CCSS) were systematically developed to:

- Be research-based;
- Focus on the critical skills at each grade level;
- Encourage conceptual mastery of key ideas;
- Develop students' mathematical understanding and procedural skill and fluency;
- Build students' ability to apply math flexibly in context;
- Present a coherent progression from grade to grade; and
- Prepare students for the demands of the future—in school and work.

In addition, the Standards serve the purpose of helping to ensure equity for all American students. Inconsistent standards, curriculum, and assessments across states have raised equity issues in the past (Reed, 2009) and wide disparities in performance on the National Assessment of Educational Progress (NAEP) (Schneider, 2007).

While the standards detail the knowledge and skills—content and processes—students need at each grade level, they do not describe the instructional approaches needed to meet the standards. Thus, an effective instructional program is needed to bridge between the expectations set out by the standards—and the desired student learning and achievement.

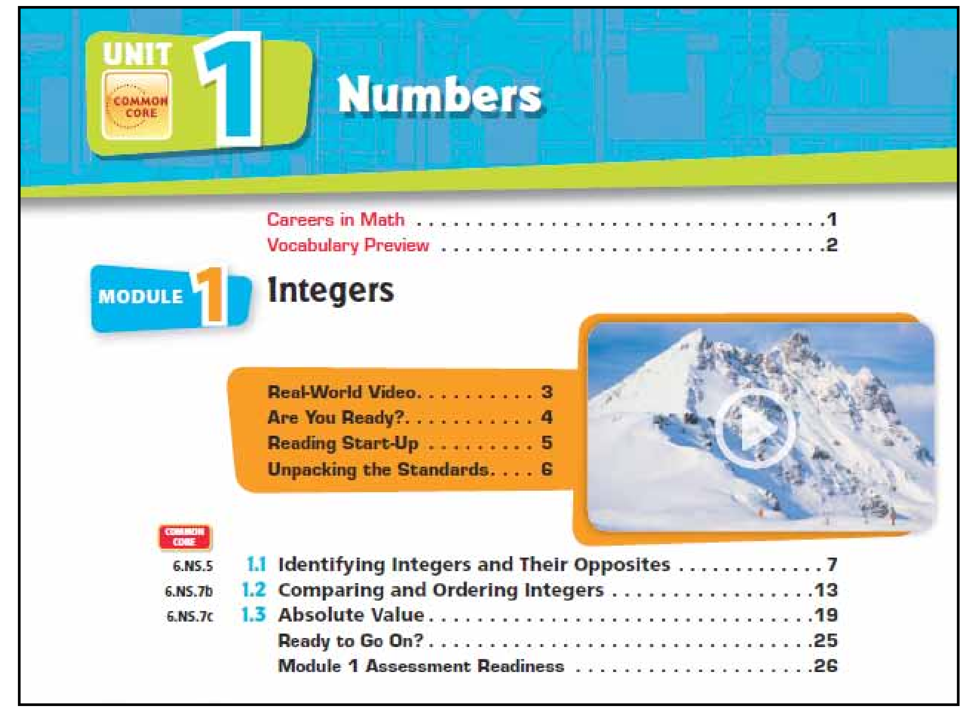
This alignment between standards, curriculum, instruction, and assessments is critical. Researchers looking at effective educational practices identified nine characteristics of high-performing schools—and reported that several of these relate to standards and standards alignment. High-performing schools have a clear, shared focus; high standards and expectations for all students; and curriculum, instruction, and assessments aligned to the standards (Shannon & Bylsma, 2003).

Houghton Mifflin Harcourt's *Go Math!* © 2014 was developed with the Common Core State Standards for Mathematics as a foundation, and uses research-tested approaches to address the rigors of the Common Core. *Go Math!* © 2014 is a program that is:

- **Coherent—**Content is organized into meaningful progressions that highlight the unity of the mathematics curriculum at each grade level. Concepts within each grade level in the *Go Math!* program are organized in units that align to the major domains of the Standards.

- **Focused—**Content is focused on essential learning so that students have time to master content at each grade level. Units in *Go Math!* focus on key big ideas, while modules align to clusters and build connections among the individual standards.

Throughout *Go Math!* alignment with the Common Core is made explicit, with standards references included alongside lesson content in the program's table of contents.



The Mathematical Practices are completely imbedded in the lessons. The program's Front Matter offers overviews of the standards. Each instructional section opens with a section on **Unpacking the Standards** where students can get a better understanding of the expectations for learning. Teachers and students who use *Go Math!* can be assured of meeting the expectations of the Common Core.

Beyond this alignment with the content and practices of the Common Core, *Go Math!* represents a comprehensive system of mathematics instruction that includes multiple instructional approaches, diagnostic and formative assessments linked to differentiated instructional resources and tiered interventions, and technology solutions designed to support and motivate students.

Strand 1: Teaching Mathematics to the Common Core State Standards

A standards-based curriculum combined with the creative use of classroom strategies can provide a learning environment that both honors the mathematical strengths of all learners and nurtures students where they are most challenged.

(McREL, 2010, p. 7)

The standards stress not only procedural skill but also conceptual understanding, to make sure students are learning and absorbing the critical information they need to succeed at higher levels—rather than the current practices by which many students learn enough to get by on the next test, but forget it shortly thereafter...

(National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010b, online)

Defining the Strand

Mathematical learning involves learning content *and* processes. Mathematical *content* relates to the subject of math—what students know and do; mathematical *practices* relate to the vehicles for doing math—how students acquire and use knowledge (NCTM, 2000). According to the National Research Council, linking content and practice—and reflecting both in the mathematics classroom—is essential to student understanding (2001). The Common Core State Standards for Mathematics address content and processes with a balanced approach in which “mathematical understanding and procedural skills are equally important” (NGA & CCSSO, 2010a, p. 4).

Rigor, too, is essential, but only if placed on a foundation of strong skills and fluency. While some have suggested that a solution to the problem of low student mathematical skills is to reduce the focus on computation and “simpler” math skills, research suggests that students’ performance on items of low- and high-difficulty correlate highly—suggesting that students’ “mathematical abilities to solve problems at different levels of mathematics rigor are complementary” (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005, p. v).

Finally, coherence, or the pattern by which topics are introduced and build across grades, may be “one of the most critical, if not the single most important, defining elements of high-quality standards” (Schmidt et al., 2005, p. 554). The Common Core reflect coherence; designers “drew on research on learning progressions” (Cobb & Jackson, 2011) during development.

The Common Core State Standards for Mathematics accomplish the goals of focus, balance, rigor, and coherence, articulating a rigorous, balanced progression that builds from grade to grade. *Go Math!* © 2014 meets these expectations of the Common Core through a comprehensive mathematics program designed to support teachers in effectively building students’ mathematical knowledge and skills, content and processes.

Research that Guided the Development of the *Go Math!* Program

Content Standards

The description of standards or instruction as “a mile wide and an inch deep” has become a common way to describe expectations and instruction that cover many topics—but none to mastery. Past comparisons of the U.S. with other countries have suggested that the U.S. K through 8 curriculum is “shallow, undemanding, and diffuse in content coverage” (National Research Council, 2001, p. 4).

In contrast, research suggests that a greater focus on fewer content areas leads to greater mastery. Reviews of the mathematics curriculum in top-performing countries find that they “present fewer topics at each grade level but in greater depth” (National Mathematics Advisory Panel, 2008, p. 20). The Common Core State Standards for Mathematics “promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible” (Achieve, 2010, p. 1). Cobb and Jackson (2011) reviewed the standards and came to the conclusion that “the developers make good on their intention to focus on a small number of core mathematical ideas at each grade” (p. 184).

Focusing on specific areas of content does not mean arbitrarily reducing focus on computation and skill building. An analysis of TIMSS and PISA results led researchers to conclude that “the evidence does not support proposals to reduce attention to learning computational and simpler mathematical skills in order to focus on strengthening students’ ability to handle more complicated mathematics reasoning” (Ginsburg, Cooke, Leinwand, Noell, & Pollack, 2005, p. v). Instead, students need to focus each year on developing the skills that will allow them to perform well in low- and high-level problem-solving situations.

International comparisons have shown that American students do not perform as well as students from other countries on assessments of math achievement (see TIMSS study by Gonzales, Williams, Jocelyn, Roey, Katsberg, & Brenwald, 2008, and PISA study by Baldi, Jin, Skemer, Green, & Herget, 2007). In an effort to unpack the specific factors that contribute to this relatively low performance across grade levels, Ginsburg and colleagues concluded that “the distribution of that [instructional] time across mathematics content areas differs in ways consistent with our findings about relative performance across content areas” (Ginsburg et al., 2005, p. v). For example, in comparing time spent on specific content areas, researchers found that “the United States devotes about half the time to its study of geometry—its weakest subject—that other countries spend” (Ginsburg et al., 2005, p. 22). In other words, if teachers want to improve students’ performance across mathematical content areas, they would benefit from focusing instruction accordingly.

Because math learning occurs sequentially, building on previous learning and developing in sophistication, part of a discussion of content in mathematics must address the idea of sequence or progression. As stated previously, the coherence of standards, as illustrated by the logical progression across grade levels, is an essential element of effective standards. Researchers Cobb and Jackson (2011) reviewed the Common Core State Standards for Mathematics and concluded that the standards represent “a major advance in this regard” (p. 184). The standards build on the foundations of earlier years, with new learning extending upon what has already been learned. Strong learning progressions build deep content knowledge and build the complexity of student skills over time.

In the Common Core, the content of the standards in grades 6 through 8 builds on students’ foundations, preparing them to move onto more demanding math concepts, procedures, and applications.

Critical Areas in the Common Core State Standards for Mathematics, Grades 6 through 8 and Algebra	
Grade	**Critical Areas / Overview**
6	1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; 2. Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; 3. Writing, interpreting, and using expressions and equations; and 4. Developing understanding of statistical thinking.
7	1. Developing understanding of and applying proportional relationships; 2. Developing understanding of operations with rational numbers and working with expressions and linear equations; 3. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and 4. Drawing inferences about populations based on samples.
8	1. Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; 2. Grasping the concept of a function and using functions to describe quantitative relationships; and 3. Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.
Algebra	• Seeing Structure in Expressions • Arithmetic with Polynomials and Rational Expressions • Creating Equations • Reasoning with Equations and Inequalities

Mathematical Practices

What is mathematics? By looking at the many interrelated skills and knowledge involved in learning and doing mathematics, it is clear that mathematics is not simply a body of content or topics to be learned.

Developing children's mathematical ways of thinking is an essential element of effective mathematics instruction. "[C]ompetence in a domain requires knowledge of both concepts and procedures. Developing children's procedural knowledge in a domain is an important avenue for improving children's conceptual knowledge in the domain, just as developing conceptual knowledge is essential for generation and selection of procedures" (Rittle-Johnson, Siegler, & Alibali, 2001, p. 359-360). Research by Franke, Kazemi, and Battey (2007) suggests that students need an environment to develop both concepts and skills in order to become flexible when engaging with mathematical ideas, and to develop as critical thinkers.

In attempting to define the many aspects of mathematics learning and understanding, the National Research Council (2001) identified five strands of mathematical proficiency:

> *Conceptual understanding*—comprehension of mathematical concepts, operations, and relations
>
> *Procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

> *Strategic competence*—ability to formulate, represent, and solve mathematical problems
>
> *Adaptive reasoning*—capacity for logical though, reflection, explanation, and justification
>
> *Productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (p. 5)

The group concluded that "The integrated and balanced development of all five strands of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) should guide the teaching and learning of school mathematics" (National Research Council, 2001, p. 11).

In developing the *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics identified expectations for content as well as for process. Under its Process Standards, NCTM includes Problem Solving, Reasoning and Proof, Communication, Connections, and Representation.

The Common Core State Standards for Mathematics are an extension of these earlier efforts, by NCTM and the NRC, to define the processes and proficiencies of mathematics. In the Common Core State Standards for Mathematics, the Standards for Mathematical Practice, "describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years" (NGA & CCSSO, 2010a). Students meet the Standards for Mathematical Practice by demonstrating the ability to:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

From Research to Practice

Content Standards in Go Math!

The Common Core State Standards are a balanced combination of procedure and understanding. The Standards reflect a focus on coherence and rigor in the expectations for students. This is reflected in Houghton Mifflin Harcourt's *Go Math!* © 2014.

Throughout *Go Math!* © 2014 a strong focus is maintained on the content of the Common Core State Standards for Mathematics. At every grade level, the program is organized into units that include the big ideas of the Common Core State Standards for Mathematics and align with the standards, as shown in the table that follows.

Alignment between *Go Math!* © 2014 and the Common Core State Standards for Mathematics	
Common Core State Standards for Mathematics – Grade 6	***Go Math!* © 2014 – Grade 6**
• Ratios and Proportional Relationships	Unit 3: Proportionality: Ratios and Rates
• The Number System	Unit 1: Numbers Unit 2: Number Operations Unit 5: Equations and Inequalities Unit 6: Relationships in Geometry
• Expressions and Equations	Unit 4: Equivalent Expressions Unit 5: Equations and Inequalities
• Geometry	Unit 6: Relationships in Geometry
• Statistics and Probability	Unit 7: Measurement and Data
Common Core State Standards for Mathematics – Grade 7	***Go Math!* © 2014 – Grade 7**
• Ratios and Proportional Relationships	Unit 2: Ratios and Proportional Relationships
• The Number System	Unit 1: The Number System
• Expressions and Equations	Unit 1: The Number System Unit 2: Ratios and Proportional Relationships Unit 3: Expressions, Equations, and Inequalities
• Geometry	Unit 4: Geometry
• Statistics and Probability	Unit 5: Statistics
Common Core State Standards for Mathematics – Grade 8	***Go Math!* © 2014 – Grade 8**
• The Number System	Unit 1: Real Numbers, Exponents, and Scientific Notation
• Expressions and Equations	Unit 1: Real Numbers, Exponents, and Scientific Notation Unit 2: Proportional and Nonproportional Relationships and Functions Unit 3: Solving Equations and Systems of Equations
• Functions	Unit 2: Proportional and Nonproportional Relationships and Functions
• Geometry	Unit 4: Transformational Geometry Unit 5: Measurement Geometry
• Statistics and Probability	Unit 2: Proportional and Nonproportional Relationships and Functions Unit 6: Statistics

For each grade level, correlations for the *Go Math!* program with the Common Core State Standards for Mathematics are provided in the front matter pages.

COMMON CORE

Common Core Standards for Mathematics

Correlations for *HMH Go Math* Grade 6

Standard	Descriptor	Taught	Reinforced
6.RP Ratios and Proportional Relationships			
Understand ratio concepts and use ratio reasoning to solve problems.			
CC.6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.	SE: 149–150, 152	SE: 153–154, 167–168, 197–198
CC.6.RP.2	Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a{:}b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.	SE: 155–156, 158	SE: 159–160, 167–168
CC.6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	SE: 151–152, 157–158, 162–164, 173, 176, 179, 182, 185–188, 193–194, 209–212, 215, 218, 220; *See also below.*	SE: 153–154, 159–160, 165–166, 167–168, 177–178, 183–184, 189–190, 195–196, 197–198, 213–214, 221–222, 223–224; *See also below.*
CC.6.RP.3a	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	SE: 151, 161, 164, 173–176	SE: 153–154, 165–166, 177–178
CC.6.RP.3b	Solve unit rate problems including those involving unit pricing and constant speed.	SE: 155, 157–158, 175, 180–182, 193–194	SE: 159–160, 167–168, 177–178, 183–184, 195–196
CC.6.RP.3c	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.	SE: 203–206, 216, 219–220	SE: 207–208, 221–222, 223–224
CC.6.RP.3d	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	SE: 185–188, 191–194	SE: 189–190, 195–196, 197–198

In *Go Math!* strong learning progressions help students develop a deep understanding of mathematical content. Through the program, students build to more complex skills and content knowledge over time.

	Grade 6	Grade 7	Grade 8
Ratios and Proportionality	• Understand ratio concepts and use ratio reasoning to solve problems.	• Analyze proportional relationships and use them to solve real-world and mathematical problems.	
The Number System	• Apply and extend previous understandings of multiplication and division to divide fractions. • Compute fluently with multi-digit numbers and find common factors and multiples. • Apply and extend previous understandings of numbers to the system of rational numbers	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	• Know that there are numbers that are not rational, and approximate them by rational numbers.
Expressions and Equations	• Apply and extend previous understandings of arithmetic to algebraic expressions. • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationships between dependent and independent variables.	• Use properties of operations to generate equivalent expressions. • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	• Work with radicals and integer exponents. • Understand the connections between proportional relationships, lines, and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations.
Geometry	• Solve real-world and mathematical problems involving area, surface area, and volume.	• Draw, construct and describe geometrical figures and describe the relationships between them. • Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	• Understand congruence and similarity using physical models, transparencies, or geometry software. • Understand and apply the Pythagorean theorem. • Solve real-world and mathematical problems involving volume of cylinders, cones and spheres
Statistics and Probablilty	• Develop understanding of statistical variability. • Summarize and describe distributions.	• Use random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Investigate chance processes and develop, use, and evaluate probability models.	• Investigate patterns of association in bivariate data.
Functions			• Define, evaluate, and compare functions. • Use functions to model relationships between quantities.

In *Go Math!* © 2014, all learning is shown in the context of a progression of skills—prerequisite skills, knowledge, and vocabulary is made explicit and students and teachers can clearly see the expected progression of learning. In the *Teacher Edition,* modules include graphic presentations of prerequisites and expected outcomes for learning.

Before	In this module	After
Students understand whole numbers, fractions, and decimals: • compare and order • relate fractions and decimals	Students recognize, order, and perform computations with integers: • identify a number and its opposite • compare and order integers using a number line • find the absolute value of a number	Students work with factors and multiples: • find the greatest common factor • use the distributive property

***Mathematical Practices in* Go Math!**

The *Go Math!* © 2014 program provides balanced instruction on mathematical content and practices. In *Go Math!* instructional time is devoted to developing both students' content skills as well as their mathematical practices. Numerous program features build students' mathematical practices, including:

- **Analyze Relationships**
- **Check for Reasonableness**
- **Communicate Mathematical Ideas**
- **Critical Thinking**
- **Critique Reasoning**
- **Draw Conclusions**
- **Error Analysis**
- **Explain the Error**
- **Justify Reasoning**
- **Look for a Pattern**
- **Make a Conjecture**
- **Math Talk**
- **Multiple Representations**
- **Persevere in Problem Solving**
- **Reflect**
- **Represent Real-World Problems**
- **What If?**

And so on.

For specific examples of how the *Go Math!* program supports the Standards for Mathematical Practice, see the pages referenced in the table that follows.

Common Core State Standards / Standards for Mathematical Practice in *Go Math!* © 2014	
Standards for Mathematical Practice	**Examples from *Go Math!* © 2014, Middle School, Grades 6-8**
1. Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	*The mathematical practices standards are integrated throughout the each grade level in the program.* **Problem-solving examples and exercises** throughout all grade levels lead students through problem-solving steps. For Grade 6 examples, see pages 36, 97-98, 190, 268, 302, 376, and 454. For Grade 7 examples, see pages 100, 155, 222, 282, 324, and 389. For Grade 8 examples, see pages 14, 120, 178, 202, 211, 219, 242, 254-255, 308, 3880, 392, 415, and 455-456.
2. Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	*The mathematical practices standards are integrated throughout the each grade level in the program.* **Focus on Higher Order Thinking** exercises in every lesson and **Performance Tasks** in every unit require students to use logical reasoning, represent situations symbolically, use mathematical models to solve problems, and state answers in terms of a problem context. For Grade 6 examples, see pages 64, 90, 193, 254, 320, 382, and 462. For Grade 7 examples, see pages 94, 149, 218, 273, 347-348, and 392. For Grade 8 examples, see pages 14, 38, 82, 103, 111, 153-154, 198-202, 214, 254-255, 354-355, and 375-376.

Common Core State Standards / Standards for Mathematical Practice in *Go Math!* © 2014	
Standards for Mathematical Practice	**Examples from *Go Math!* © 2014, Middle School, Grades 6-8**
3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	*The mathematical practices standards are integrated throughout the each grade level in the program.* The **Essential Question Check-In** and **Reflect** activities in every lesson ask students to evaluate statements, explain relationships, apply mathematical principles, make conjectures, construct arguments, and justify reasoning. For Grade 6 examples, see pages 24, 112, 208, 248, 318, 406, and 468. For Grade 7 examples, see pages 18, 134, 210, 258, 340, and 404. For Grade 8 examples, see pages 20, 100, 146, 208, 258, 302, 352, 410, and 444.
4. Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	*The mathematical practices standards are integrated throughout the each grade level in the program.* **Real-world examples** and **mathematical modeling** apply mathematics to other disciplines and real-world contexts such as science and business. For Grade 6 examples, see pages 17, 100, 215-216, 249, 324, 385, and 468. For Grade 7 examples, see pages 12, 122, 193, 270, 322, and 386. For Grade 8 examples, see pages 73, 129-130, 204-205, 254-255, 363-364, 413, 457 and 461.

Common Core State Standards / Standards for Mathematical Practice in *Go Math!* © 2014	
Standards for Mathematical Practice	**Examples from *Go Math!* © 2014, Middle School, Grades 6-8**
5. Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	*The mathematical practices standards are integrated throughout the each grade level in the program.* **Exploration Activities** in the lessons use concrete and technological tools, such as manipulatives or graphing calculators, to explore mathematical concepts. For Grade 6 examples, see pages 8, 91, 185, 276, 303, 371, and 458. For Grade 7 examples, see pages 100, 147, 191, 243-246, 323, and 422. For Grade 8 examples, see pages 22, 53, 197, 227-231, 285, 315, 347-348, 353, 375, and 399.
6. Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	*The mathematical practices standards are integrated throughout the each grade level in the program.* Precision refers not only to the correctness of calculations but also to the proper use of mathematical language and symbols. **Communicate Mathematical Ideas** exercises and **Key Vocabulary** highlighted for each module and unit help students to learn and use the language of math to communicate mathematics precisely. For Grade 6 examples, see pages 13, 93, 214, 242, 336, 424, and 452. For Grade 7 examples, see pages 62, 128, 190, 250, 346, and 380. For Grade 8 examples, see pages 56, 106, 143, 173-178, 214, 234, 250, 352, 404, and 437.

Common Core State Standards / Standards for Mathematical Practice in *Go Math!* © 2014	
Standards for Mathematical Practice	**Examples from *Go Math!* © 2014, Middle School, Grades 6-8**
7. Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression *x*2 + 9*x* + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(*x* – *y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.	*The mathematical practices standards are integrated throughout the each grade level in the program.* Throughout the lessons, students will observe regularity in mathematical structures in order to make generalizations and make connections between related problems. For Grade 6 examples, see pages 18, 118, 214, 263-264, 311, 430, and 471. For Grade 7 examples, see pages 97, 146, 175, 283, 349, and 410. For Grade 8 examples, see pages 10-11, 33-35, 45, 133-135, 153-154, 208, 297-300, 381, 434, and 439-441.
8. Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (*y* – 2)/(*x* – 1) = 3. Noticing the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	*The mathematical practices standards are integrated throughout the each grade level in the program.* Throughout the program's examples and exercises, students will look for repeated calculations and mathematical patterns. Recognizing patterns can help students to make generalizations and obtain a better understanding of the underlying mathematics. For Grade 6 examples, see pages 19, 125, 149, 237, 242, 310, 378, and 462. For Grade 7 examples, see pages 61, 152, 203-204, 265, 290, and 407. For Grade 8 examples, see pages 8, 33-35, 45, 107, 197, 235, 243, 251, 297-300, 388, and 440.

Strand 2: Effective Instructional Approaches

...teacher and principal effectiveness has a greater impact on student learning than any other factor in a school system.

(Partnership for Learning, 2010, p. 2)

But what exactly do highly effective teachers do in their classrooms to help students learn at higher levels? Research tells us that one key trait of effective teachers is their use of instructional strategies that work.

(McREL, 2010)

Defining the Strand

Quality teaching matters. Extensive research has shown that teacher effectiveness has an enormous impact on student learning and achievement—more than any other in-school factor (Goldhaber, 2002; Partnership for Learning, 2010). Chetty, Friedman, and Rockoff (2012) looked at the long-term impacts of teachers and found that those who added value to their students' test scores, also added life-long value to their students' educational attainment and income earning.

What is it that effective teachers do to make gains in student learning and achievement? Quality teachers use effective classroom practices (Wenglinsky, 2002). Research—in cognitive science, on classroom practices of master teachers, and on specific supports that help students learn—points to specific principles and methods of effective instruction (Rosenshine, 2012). Teaching mathematics is not easy, but employing proven research techniques can help teachers to ensure that all students learn.

Go Math! © 2014 is a program designed to support teachers in effectively building students' mathematical skills and understandings. In its design, the program incorporates research-based strategies for effective teaching and learning. *Go Math!* supports teaching and learning by incorporating the following proven instructional approaches:

- Following an effective instructional process
- Addressing misconceptions and common errors
- Communicating about mathematics
- Developing students' content-area vocabulary
- Making connections
- Providing opportunities for practice
- Engaging in higher-order thinking
- Teaching multiple representations
- Motivating students
- Engaging students

Research that Guided the Development of the *Go Math!* Program

Instructional Process

Using a predictable, logical sequence in both the planning and delivery of instruction has been shown to positively impact student outcomes.

In their seminal work *Understanding by Design*, Wiggins and McTighe (1998) describe effective instructional design in the classroom occurring in three stages, each centered on a guiding question:

1. What is worthy and requiring of understanding?
2. What is evidence of understanding?
3. What learning experiences and teaching promote understanding, interest, and excellence?

In this model of instructional planning, teachers work backwards from the end goal. In a standards-based system particularly, this kind of planning and design makes sense—the standards show the expectations and what is needed is for educators and curriculum developers to think carefully about how to get there—and how to know when students meet the standard.

Once the larger design of instruction is in place, teachers must consider how to deliver instruction. Following a predictable, structured, research-based routine has been shown to be effective. Such a routine may include key elements of engagement, active, problem-based learning, and other research-based strategies. In considering research on how to organize instruction and study to improve student learning, Pashler and his colleagues (2007) found that teachers can use specific strategies in organizing their instruction to increase student learning and achievement, specifically by spacing learning over time, incorporating worked example solutions with problem-solving exercises, combining graphics and text, using quizzes to promote learning, and asking deep questions.

Making the organization of instruction clear to students has been shown to be effective in increasing learning. Research on effective organization of classroom instruction has found that the use of explicit organization behaviors—identifying the organizational structure of the lesson at the outset of instruction, offering outlines of lesson content, including explicit transitions and connections between parts of the lesson, and summarizing the content of the lesson—results in higher achievement by students (Kallison, 1986).

Addressing Misconceptions and Common Errors

One of the challenges to successful mathematical learning is students' existing misconceptions. A misconception is a mistaken understanding of a mathematical concept that may be the result of informal, everyday thinking or a lack of previous successful instruction on the concept. Addressing misconceptions is crucial to successful math learning (Ben-Hur, 2006). However, research in cognitive science more generally, and on student misconceptions specifically, suggests that simply repeating a lesson for students on a concept for which they hold a misconception is ineffective; students who hold misconceptions on a topic need to have these misconceptions explicitly addressed, and need to be re-educated on the concept or skill (Klopfer, Champagne, & Gunstone, 1983; Resnick, 1983). Identifying and resolving misconceptions leads to more effective math instruction and learning. Reviewing research on mathematical misconceptions led Welder (2012) to believe that student "learning of algebra may be supported through identifying and preventing student misconceptions of prerequisite algebra concepts" (p. 255).

Feedback is a powerful tool in addressing—and preventing—student misconceptions. In Bangert-Drowns, Kulik, Kulik, and Morgan's 1991 meta-analysis of research on feedback, they found that the primary benefit of feedback appeared to be error correction. Providing feedback "can signal an error to students and supply the correct response. The student then initiates some degree of mindful, metacognitively driven knowledge alteration" (p. 234). Key here is the idea that the feedback is not just an indication that the learner is correct or incorrect; rather, the feedback must inform the learner of the correct answer. Similarly, Moreno (2004) looked at feedback in a multimedia environment, in which an animated agent provided students with either explanatory feedback, which offered verbal explanations about student choices, or with corrective feedback, which simply indicated whether students' answers were correct or incorrect. She found that students receiving explanatory feedback reported higher motivation and engagement and out-performed those receiving only corrective feedback. Feedback has real implications on students' misconceptions. Additional research suggests that failing to provide feedback not only fails to challenge students' existing misconceptions, but may also enable students to create new misconceptions (Brown & Campione, 1994).

Regular assessment is crucial to identifying and re-directing students' misconceptions. Examining incorrect student responses to a set of items can "reveal specific student misunderstandings" (Popham, 2006, p. 86)—and teachers can respond instructionally. By analyzing the mistakes that students make, teachers can determine which specific concepts, algorithms, or procedures need additional instruction (Ketterlin-Geller & Yovanoff, 2009). Studying student responses as a group can also allow teachers to find evidence of any recurring misconceptions held by several students (NCTM, 2000). After a review of research on the principles of effective instruction, Rosenshine (2012) came to the conclusion that one of the most important elements of effective instruction is that the teacher continually checks for student understanding—in part to determine if students held previous misconceptions, or were developing new misconceptions. Scheuermann and van Garderen (2008), too, point to the importance of regular assessment—in their research, by looking at student use of graphic representations—to determine students' misconceptions and identify error patterns.

Finally, because the majority of student errors are consistent and systematic (Riccomini, 2005), curricular materials that help teachers understand and respond to errors and misconceptions can be especially helpful to supporting effective mathematics instruction.

Communication

A wide body of research attests to the effectiveness of writing and speaking in the content-area classroom. Communicating mathematically is a frequent and consistent thread throughout research on effective instructional strategies for teaching mathematics—and is one of the strategies highlighted in NCTM's *Principles and Standards for School Mathematics* and is a recurrent thread throughout the Common Core Standards for Mathematical Practice. When students write about and discuss math concepts, they have the chance to think through, defend, and support their ideas. A review of studies conducted by the National Council of Teachers of Mathematics revealed that "the process of encouraging students to verbalize their thinking—by talking, writing, or drawing the steps they used in solving a problem—was consistently effective...Results of these students were quite impressive, with an average effect size of 0.98..." (Gersten & Clarke, 2007, p. 2). Communicating about math increases learning; "encouraging students to verbalize their current understandings and providing feedback to the student increases learning" (Gersten & Chard, 2001, online). Because mathematics learning is so connected to the language of mathematics, communication in mathematics is particularly important. As Edwards, Esmonde, and Wagner (2011) put it: "...language and learning are inextricably intertwined, and ... understanding mathematical learning, assessing students' mathematical learning, and designing mathematical learning environments require examining the role of language in mathematical activity" (p. 62).

Numerous studies have emphasized the importance of writing in content-area learning, and in the mathematics classroom. Bosse and Faulconer (2008) report that writing in the mathematics classroom results in deeper student learning. Other researchers have found that students' conceptual understanding and problem-solving skills improve when they are encouraged to write about their mathematical thinking (Burns, 2004; Putnam, 2003; Russek, 1998; Williams, 2003). Most important, writing appears to benefit all students, with researchers finding benefits for low-achieving students (Baxter, Woodward, & Olson, 2005) and for high-achieving students (Brandenburg, 2002).

What makes writing to learn so effective? According to Vygotsky (1978), writing gives students the chance to explore their own thinking. According to Burns (2004) "Writing in math class supports learning because it requires all students to organize, clarify, and reflect on their ideas—all useful processes for making sense of mathematics" (p. 30). In addition, researchers theorize that verbalizing forces students to slow their thinking down so that they are more careful in their processes—and less likely to make careless errors (Gersten & Clarke, 2007). Other researchers have found that writing during math instruction gives students more confidence in their math abilities, creates more positive attitudes toward math, and makes it easier for students to understand complex math concepts (Furner & Duffy, 2002; Taylor & McDonald, 2007). Pugalee (2005) argues that writing builds students' mathematical reasoning and problem-solving skills.

Writing can be incorporated into the mathematics classroom in numerous ways, including free writing; biography; learning logs, blogs, and journals; summaries; word problems; and formal writing (Urquhart, 2009). Students can engage in more structured or more informal journaling or note-taking. In a study with grade 9 algebra students, Pugalee (2004) found that journal writing positively impacted students' problem solving. Albert and Antos (2000) examined the impact of journal writing, and found that using the journals "gives students practice in communicating their ideas clearly and allows for each student to make a personal connection that strengthens his or her learning and understanding of mathematical concepts and ideas" (p. 530-531). Note-taking has also been shown to be an effective instructional and learning strategy in the classroom (Marzano, Pickering, & Pollock, 2001).

Providing opportunities for students to talk about mathematics and mathematical concepts also enhances their understanding of mathematics. Instructional practices—such as restating, prompting students, and engaging in whole-class discussion, small-group discussion, and paired conversations—have been shown to be effective in improving student understanding (Chapin, O'Connor, & Canavan Anderson, 2003). Lovitt and Curtis (1968) found that encouraging a student to verbalize problems before giving a written response increased the rate of correct answers. Hatano and Inagaki (1991) found that students who discussed and justified their solutions with peers demonstrated greater mathematical understanding than students who did not engage in such discussions. Leinwand and Fleischman (2004) reviewed research on effective mathematics instruction and concluded that talking about math and explaining the rationale for solutions can help ensure that students have a conceptual understanding—rather than a rote knowledge of a rule.

Talking about math has also been found to benefit students at different levels of learning and in different contexts. In their study, Hufferd-Ackles, Fuson, and Sherin (2004) found a math-talk community to be beneficial with students who were English language learners in an urban setting. Similarly, working in a transitional language classroom led researchers Bray, Dixon, and Martinez (2006) to concluded that as students "communicate verbally and in writing about their mathematical ideas, they not only reflect on and clarify those ideas but also being to become a community of learners" (p. 138).

In addition to promoting greater learning, communication in the mathematics classroom can facilitate teachers in assessing students' performance—and students in engaging in self-assessment; "Classroom communication about students' mathematical thinking greatly facilitates both teacher and student assessment of learning" (Donovan & Bransford, 2005, p. 239).

Vocabulary Development

In order to successfully communicate, students must know content-area vocabulary. Understanding the language of mathematics is essential to understanding and doing mathematics.

In numerous studies, students' knowledge of mathematical vocabulary has been shown to correlate with their mathematics achievement (Earp, 1970; Stahl & Fairbanks, 1986; Usiskin, 1996). Research by Freeman and Crawford (2008) found that focused and explicit attention to vocabulary and language helps students develop a deeper understanding of content. In a study of students in grade 7, researchers Jackson and Phillips (1983) compared the performance of two groups—one completed vocabulary-oriented activities over the four weeks of instruction on ratio and proportion; the other control-group classes received similar instruction, but without the vocabulary activities. At the end of the instructional unit, students in the vocabulary condition outperformed the control-group students on both computational items and verbal items, suggesting increased comprehension and skill resulted from the explicit focus on mathematical vocabulary learning.

Why is vocabulary learning so tied to mathematics achievement? This may be because the language of mathematics is so closely tied to the content of mathematics; mathematics vocabulary terms offer the means to communicate mathematical ideas that are by nature abstract and complex (Kouba, 1989). In addition, mathematics involves many different kinds of terms, including *technical* terms (that represent concepts, such as *trapezoid*), *subtechnical* terms (that have more than one meaning, different in math than in everyday life, such as *volume*), *general* words (that again have multiple meanings, such as *negative*), and *symbolic* words (that represent highly abstract numbers) (Monroe & Panchyshyn, 1995). As Edwards, Esmonde, and Wagner (2011) put it:

> ...mathematics is a language—a semiotic system with particular syntactic and semantic entailments that differ from those of "everyday" language...The mathematics register is highly technical, including particular uses of everyday words (e.g., *set, order*), specialist vocabulary that is specifically mathematical (e.g., *sine, equation*), composite words and expressions with mathematical meanings distinct from everyday usage (e.g., *square root, differential operator*), precise and specific meanings of linguistic features (e.g., conjunctions such as *if, therefore*; directives such as *assume;* modifiers such as *clearly, obvious*), highly dense nominal structures (e.g., the sum of the squares of two sides of a right triangle), and grammatical constructions that imply logical relationships (e.g., *if...then...*)..." (p. 62-63)

To support students' proficiency, teachers should incorporate vocabulary instruction in the math classroom so that students can understand and use this "language with its own symbols, syntax, and grammar" (Leiva, 2007, online). Certain strategies have been shown to be particularly effective. Research suggests that instructional practices for promoting vocabulary learning must include *integrating* new vocabulary, connecting it to previously learned words and concepts; *repeating* vocabulary words and offering opportunities for practice with new words; and *using words with meaning,* or offering opportunities for students to use the words in meaningful ways (Harmon, Hedrick, & Wood, 2005). In a study with fourth grade students comparing an integrated model of vocabulary instruction, which incorporated graphic organizers, with a definition-only instructional model, Monroe and Pendergrass (1997) found that students in the integrated condition outperformed students in the definition-only condition.

While instruction in vocabulary in mathematics benefits all learners, explicit instruction in the language of mathematics is particularly important for English language learners and for struggling readers (Bay-Williams & Livers, 2009).

Connections

Connections—among mathematical ideas, with other content areas, and in real-world contexts—are an essential part of successful mathematics learning. Making connections between new information and students' existing knowledge—knowledge of other content areas and of the real world—has proved to be more effective than learning facts in isolation (Beane, 1997; Bransford, Brown, & Cocking, 1999; Caine & Caine, 1994; Kovalik, 1994). Students learn best when they can make connections between ideas.

By its nature, mathematics lends itself to interdisciplinary learning; mathematics can be applied to situations in social studies, technology, engineering, science, and real-world contexts, and knowledge of mathematics is essential to study in many disciplines. Connecting mathematics to science, social science, and business topics can increase students' understanding of and ability with mathematics (Russo, Hecht, Burghardt, Hacker, & Saxman, 2011). These interdisciplinary connections build students' knowledge and increase their perceptions of mathematics as useful and interesting—thereby increasing their motivation to learn (Czerniak, Weber, Sandmann, & Ahem, 1999). Research shows that making inter-disciplinary connections can increase students' achievement (Russo, Hecht, Burghardt, Hacker, & Saxman, 2011).

Making connections makes learning more relevant to students. Students see the purpose of learning when they can apply it to real-world contexts; "When instruction is anchored in the context of each learner's world, students are more likely to take ownership for...their own learning" (McREL, 2010, p. 7). These connections build students' knowledge and increase their perception of the content as interesting and useful —thereby increasing their motivation to learn (Czerniak, Weber, Sandmann, & Ahem, 1999). In his examination of U.S. and Korean students' performance on the 2003 TIMSS, House (2009) found a strong correlation between connections linking mathematical study with students' daily lives and students' enjoyment of mathematics. In its review of studies on using real-world problems in the mathematics classroom, the National Mathematics Advisory Panel (2008) found that for "certain populations (upper elementary and middle grade students, and remedial ninth-graders) and for specific domains of mathematics (fraction computation, basic equation solving, and function representation), instruction that features the use of 'real-world' contexts has a positive impact on certain types of problem solving" (p. 50). Thus, the use of real-world problems can be recommended for this population of students.

Opportunities for Practice

Most learning requires repetition and practice. To learn something, we must do it repeatedly. Rosenshine's 2012 review of effective instructional practices led him to conclude that the importance of practice was one of ten research-based principles all teachers should know; "the best way to become an expert is through practice—thousands of hours of practice. The more the practice, the better the performance" (p. 19). Ericsson, Krampe, and Tesch-Romer (1993) similarly found that deliberate practice is essential to attaining expert performance. Mathematics is no different; practice is essential.

Learning mathematics is a sequential process, in which students at each stage must build on and expand on their previous understandings. At each level, students learn to think about mathematics in increasingly sophisticated ways. Initially, though, many mathematical concepts post challenges for students because of the complexity of abstract concepts and interactions among different elements involved in problem solving. Sweller's research (1988; see also Paas, Renkl, & Sweller, 2004) on cognitive load theory suggests that students cease to learn when the cognitive load is too high. Students' working memory can only hold so much at a given time, so when students are integrating different types of information to solve a problem, they may become overloaded cognitively. This problem can be alleviated by practice. When students practice, their schemas, or mental frameworks of pre-existing knowledge, become automated, allowing them to focus their mental energies on the new problem at hand. Similar to the way fluent readers can devote more attention to reading, students who have practiced enough in mathematics to gain automaticity can advance to more complex problem-solving situations. For example, basic multiplication—such as 5 x 3 = 15—is involved in many other mathematical procedures, including estimation, multidigit multiplication or division, multiplication of fractions and so on (National Research Council, 2001). When students can perform basic multiplication automatically, they can more easily perform more complex procedures that involve basic multiplication. Practice for review can also serve as a scaffold; review helps "...to refocus [students'] attention and give them further opportunity to develop their own understanding rather than relying on that of the teacher" (Anghileri, 2006, p. 41).

Rosenshine (2012) concluded that practice is essential both after each new concept is learned, and over time, for retention, recall, and fluency. To be effective, practice should address the concepts and procedures that have been learned and be repeated and frequent (National Research Council, 2001). Teacher feedback also increases the effectiveness of practice (Marzano, 2000). Brabeck and Jeffrey, in their module on *Practice for Knowledge Acquisition,* review research and conclude that effective practice is deliberate—not rote repetition—and is most effective when it is extensive and deliberate and distributed over time.

Research has shown practice to be effective in increasing student learning and understanding. Gonzalez and Birch (2000) found using the computer for additional practice exercises supported students' understanding of the material. Practice can take the form of working examples. Sweller (2006) found that studying worked examples or explaining them to other students were particularly powerful forms of practice. Quizzes and tests, too, offer an opportunity for practice (Brabeck & Jeffrey, n.d.). Homework can also be an effective form of practice (Marzano, 2000; Walberg, Paschal, & Weinstein, 1985).

Higher-Order Thinking

Higher-order thinking can be defined as the ability to combine information in memory, prior knowledge, with new information, and use these new integrated skills and knowledge to solve problems, analyze arguments, debate issues, or make predictions (Underbakke, Borg, & Peterson, 1993). Lewis and Smith (1993) suggest that we can see the term *higher-order thinking* encompassing problem solving, critical thinking, creative thinking, and decision making—and as such, we can see that it is an important skill for all; "Any time an individual is faced with a perplexing situation or a situation where it is necessary to decide what to believe or do, higher order thinking is necessary" (p. 136).

Instruction in higher-order thinking skills has been shown repeatedly to have a positive impact on student learning. Wenglinsky (2002) conducted an analysis of effective teacher practices and student academic performance and concluded that "As the qualitative literature leads on to expect, a focus on higher-order thinking skills is associated with improved student performance."

In a study that compared students exposed to teaching strategies that promoted higher-order thinking with those taught more traditionally, researchers found that experimental group students outperformed control group students, showing significant improvement in their critical thinking skills; "Our findings suggest that if teachers purposefully and persistently practice higher order thinking strategies for example, dealing in class with real-world problems, encouraging open-ended class discussions, and fostering inquiry-oriented experiments, there is a good chance for a consequent development of critical thinking capabilities" (Miri, David, & Uri, 2007, p. 353).

Adey and Shayer (1993) designed a study in which they looked at the impact of special higher-order thinking lessons on student performance over time. Students who received the higher-order thinking instruction showed immediate gains in cognitive development—and gains in science, mathematics, and English language arts two and three years after the intervention program.

How best can teachers develop students' higher-order thinking skills? Employing effective questioning strategies—asking questions that require students to make connections, allowing time for responses, offering feedback—is one way to encourage higher-order thinking. In a study in which researchers observed hundreds of classrooms to try to identify the elements of the most effective lessons, Weiss and Pasley (2004) concluded that questioning was a strategy employed in the most effective lessons. Problem-based learning, in which problems are used to drive learning in the mathematics classroom, is another way for students to develop higher-order thinking skills. Problem-based learning offers students the opportunity to engage in critical thinking, communicate about math, and develop strategies to solve problems and evaluate solutions (Roh, 2003). As Miri, David, and Uri (2007) suggest, using real-world problems, engaging students in discussions, and using inquiry-based methods can also foster higher-order thinking opportunities in the classroom.

Multiple Representations

Representations and models include a wide variety of images or likenesses that can be used to support understanding in the mathematics classroom. Representations and models might include those that students create themselves to better understand concepts, or those that teachers provide to better illustrate concepts for students. They might include pictures, manipulatives, symbols, or diagrams. Visual representations include graphic organizers, illustrations used organize and highlight content. Technology also provides new means of illustrating concepts with models and representations.

In *Principles and Standards for School Mathematics* (2000) the importance of representations in mathematics instruction is highlighted; "Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling" (p. 67).

Students' use of mathematical models and representations—images, likenesses, or depictions—can help to make mathematical concepts more concrete for students. Representations help students to understand and communicate mathematical ideas and to model and interpret concepts. Research by Clarke and Clarke (2004) suggests that the effective use of models and representations is an important element in successful mathematics instruction. Marzano included non-linguistic representations as one of the nine most effective instructional strategies teachers can use in the classroom (Marzano, Pickering, & Pollock, 2001). A review of 13 randomized controlled supported the use of visual representations and found that they improved student performance in general mathematics, prealgebra, word problems, and operations (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). Graphic organizers have been show to be effective in helping students organize and remember content-area information (Horton, Lovitt, & Bergerud, 1990). In mathematics, graphic organizers can offer students guidance and assistance in approaching new problems (Maccini & Gagnon, 2005).

Scheuermann and van Garderen (2008) suggest that students' ability to generate and use representations can serve as a valuable source of information for teachers trying to identify exactly how a student is struggling in mathematics. Looking at representations, their findings suggest, can help teachers direct instruction effectively.

Motivation

Generally speaking, motivation is the desire to engage in some activity—and persist to completion. Researchers generally refer to two types of motivation—intrinsic, in which one does something for its own value, or extrinsic, in which one does something in order to produce a certain result. Research suggests that there are four dimensions or factors influencing motivation. These include:

1. Competence—A student's belief that he or she can successfully complete the task;
2. Control—A student's sense of autonomy, that he or she can control both how to approach a task and what the outcome will be.
3. Interest and Value—A student's perception of the worth of an activity and his or her level of interest in the activity.
4. Relatedness—The level to which completing the activity will bring the student a sense of social connectedness (Usher & Kober, 2012).

Motivation plays a powerful role in student learning. To prepare the research report *Adding It Up: Helping Children Learn Mathematics*, researchers reviewed existing research in an attempt to identify how students best learn mathematics and what instructional processes and strategies can support that learning. For students to learn, the researchers found, they must be motivated to learn. To be motivated to learn, they must have the expectation that they can learn and a belief that their learning has a value (National Research Council, 2001).

Research has long documented the connection between a student's sense of confidence and self-efficacy for learning and his or her learning and achievement. In their study, in which they investigated the relationship between achievement and learning in mathematics and motivational and affective variables, Seegers and Boekaerts (1993) found that differences in mathematical performance were not explained by cognitive levels alone. Rather, students' self-efficacy, their perceptions of the relevance and interest of tasks, and their willingness to invest effort all contribute to their performance. Students who believe they can learn persist in learning, are engaged in learning, and subsequently learn more than peers who are less confident in their abilities. As a result, building students' confidence in learning is an important element of effective instruction.

How best can teachers create a motivating learning environment for students? To create the expectation that students can learn, teachers can provide activities at which students can be successful, and scaffolding for those activities beyond students' current levels. Matching tasks and instructional supports to students' current levels of skill will create an expectation among students that they can meet the challenge (National Research Council, 2001). Scaffolding can also help students to believe in their own abilities to succeed (Baker, Schirner, & Hoffman, 2006). As Hyde (2006) states, "Scaffolding does not necessarily make the problem easier, and the teacher does not do the work for students or show them how to do it. It enables the person to do it" (p. 28). This empowerment gives students confidence in their ability and allows them to take on increasingly more challenging material and assignments as they demonstrate success completing previous tasks. Williams (2008) found that "scaffolding tasks allowed students to work independently at appropriately challenging levels,…and develop a sense of self-confidence in their mathematics knowledge and skills" (p. 329).

To make the content of learning seem worth learning, teachers can communicate their own enthusiasm for the subject and can involve students in real-world applications of the course content (National Research Council, 2001).

Other instructional strategies that have been shown to foster student motivation in mathematics include showing the relevance of tasks; providing feedback and clarification; involving cooperation and social connections in the classroom; balancing challenge and skill; and supporting self-efficacy (Schweinle, Turner, & Meyer, 2006). Technology, too, shows promise as a motivational tool in the classroom (Usher & Kober, 2012).

Engagement

When taught well, mathematics can engage students with the "intriguing, surprising, fascinating, and beautiful" ideas that are at the heart of mathematics (Burger, 2005). In addition, the experience of teachers and the findings of research suggest that when students are engaged they are motivated to persist in learning and are better able to learn in the classroom. Research links engagement with student achievement and development (Finn, 1993; Newmann, 1992). In a study conducted by Park (2005) student engagement had positive effects on student academic growth—even after taking into account variables such as gender, minority status, SES, and interaction effects.

As discussed earlier in this report, making interdisciplinary connections engages students in the mathematics classroom. Also discussed previously, communicating about mathematics engages students. Research has shown that learners become more engaged in the learning process when they are asked to explain and reflect on their thinking processes (Good & Whang, 1999; Hettich, 1976; Surbeck, 1994). Continually requiring that students explain how they solved problems is another research-based strategy for maintaining student engagement (National Research Council, 2001). Teachers who press students to explain their answers help to keep their students engaged in a dynamic way. Choosing tasks that use students' prior knowledge as a foundation is another way that teachers can keep student engagement high (National Research Council, 2001). Thus, the sequence of instruction, and the reinforcement of prerequisite learning, is important to engagement. Multimedia learning, too, engages students. Students who often used computers in their mathematics classrooms reported greater enjoyment of mathematics than did students who did not frequently use computers for mathematics study (House & Telese, 2011). Reinking (2001) attributes the greater student engagement in multimedia learning environments to the interactive nature of the technology, the availability of scaffolds, the game-like nature of computer-based instructional materials, and the social learning environment that can be created with technological tools.

When students are motivated, they are more engaged. They are motivated when they expect that they will be able to perform mathematical tasks successfully. For this reason, teachers can keep students engaged by supporting students' expectations that they can succeed in solving problems (National Research Council, 2001).

From Research to Practice

Instructional Process in Go Math!

Go Math! © 2014 follows the principles of an effective instructional process and of Understanding by Design that supports all students in learning. The entire program is organized around the Common Core State Standards. With these expected outcomes, or end goals, in place, the program focuses then on how best to support teachers and students in meeting these high expectations.

In *Go Math!* each unit, module, and lesson of instruction follows a regular, predictable, five-step pattern of:

1. **Engage—**the **Essential Question** gets students thinking about the lesson content. The **Motivate the Lesson** suggestions encourage motivation to learn.
2. **Explore—**activities and exercises that encourage students to investigate lesson content and practice skills are provided.
3. **Explain—**students engage in activities like the following to explain their understandings—**Connect Vocabulary, Questioning Strategies, Connect to Daily Life,** and **Talk About It.**
4. **Elaborate—**students elaborate on their understandings of content and new skills through discussion (as with the **Talk About It** feature) and through practice (using print and digital activities and tools, such as **Engage with the Whiteboard** and **Focus on Technology**).
5. **Evaluate**—teachers assess student knowledge and skills using **Guided and Independent Practice** examples, as well as **Personal Math Trainer** assessments, **Lesson Quizzes, Leveled Practice** worksheets, and so on.

The program supports teachers through every stage of the instructional planning process helping them with planning tools that include the following:

- The **Online Teacher Edition** offers access to a full suite of teaching resources online, where teachers can plan, present, and manage classes, assignments, and activities.
- The **ePlanner** helps teachers to plan classes, create and view assignments, and access all program resources with a customizable planning tool.
- The **Teacher's Edition** help teachers support students with point-of-use questioning strategies, teaching tips, resources for differentiated instruction, additional activities, and more.

For teachers, the program resources and tools are designed to support teachers in designing instruction which will meet the needs of all students through a process of:

- **Engagement and Exploration:** Features like the **Real-World Videos,** the **Animated Math** online interactive simulations, and the **Explore Activities** engage student interest and invite them to explore math concepts.
- **Instruction:** The **Math on the Spot** video tutorials give students step-by-step instructions and explanations of key math concepts for every example in the textbook. The integrated formative assessment feature **Math Talk** invites students to discuss math concepts—and allows teachers to monitor and assess their progress. Specific exercises that build connections among Common Core standards are noted with the **Cluster Connection** icon.

- **Assessment and Intervention:** A wide variety of resources and tools for assessment and intervention support teachers and students throughout. The **Personal Math Trainer** offers a place for practice, assessment, homework, and intervention. Higher-order thinking exercises are offered throughout every lesson. Assessment resources, including **Leveled Module Quizzes, Leveled Unit Tests, Unit Performance Tasks,** and **Placement, Diagnostic, and Quarterly Benchmark Tests** allow for varied and ongoing assessment of student learning and progress.

Addressing Misconceptions and Common Errors in Go Math!

Throughout the *Go Math!* © 2014 program, students are challenged to address their misconceptions and recognize and avoid common errors.

At different points of instruction, teachers are given suggestions for how to help students **Avoid Common Errors**, such as in these examples from grade 8.

- "Be sure that students read Exercise 5 carefully before answering. The number given in the problem, 10, is the area, not the side length." (p. 15)
- "**Exercises 9-10** Remind students that it takes only one counterexample to show that a statement is false." (p. 17)
- "**Exercise 10** Remind students that the calculations have units." (p. 23)

Students are asked to analyze errors in the **Error Analysis** and **Explain the Error** features that appears throughout the program.

Throughout, students have the chance to engage in activities in which they analyze the thinking and reasoning in examples, such as with the **Critique Thinking** and **Critique Reasoning** features.

Addressing Misconceptions and Common Errors in *Go Math!* © 2014
Grade 6
Critique Thinking: For examples, see pages 52, 57, 64, 107, 214, 242, 247, 302, 310, and so on. **Critique Reasoning:** For examples, see pages 18, 24, 36, 52, 64, 112, 118, 160, 184, and so on.
Grade 7
Critique Thinking: For examples, see pages 9, 18, 30, 94, 133, 146, 216, 222, 246, and so on. **Critique Reasoning:** For examples, see pages 12, 41, 54, 87, 100, 127, 142, 152, 189, and so on.
Grade 8
Critique Thinking: For examples, see pages 20, 46, 88, 103, 112, 213, 234, 284, 290, and so on. **Critique Reasoning:** For examples, see pages 20, 26, 37, 38, 50, 82, 100, 106, 146, and so on.

Finally, the strong formative assessment program throughout *Go Math!* ensures that teachers are able to constantly check for student understanding at each point of learning—so that teachers can identify any previously held or newly "learned" misconceptions.

Communication in Go Math!

Opportunities that encourage writing and talking about math to learn, and to reflect on and refine mathematical ideas, are incorporated throughout *Go Math!* The program's write-in *Student Edition* allows students to explore concepts, take notes, answer questions, and complete homework right in the textbook. Continued opportunities for collaboration throughout *Go Math!* ensure that students use the power of communication to deepen their mathematical understandings. Students learn by engaging in conversations about mathematics in the *Go Math!* program.

Specific program features that encourage communication include:

- The program's **Communicate Mathematical Ideas** exercises help students to use the language of math to communicate mathematics ideas precisely and clearly.
- **Math Talk** activities offer a chance for students to talk about math—and an opportunity for teachers to monitor and asses student progress.

Communication in *Go Math!* © 2014
Grade 6
Communicate Mathematical Ideas: For examples, see pages 12, 20, 24, 36, 40, 48, and so on. **Math Talk:** Appears in most lessons; for examples, see pages 10, 14, 15, 20, 33, 48, and so on.
Grade 7
Communicate Mathematical Ideas: For examples, see pages 7, 8, 9, 12, 18, 20, 21, and so on. **Math Talk:** Appears in most lessons; for examples, see pages 61, 70, 77, 85, 91, 97, and so on.
Grade 8
Communicate Mathematical Ideas: For examples, see pages 20, 26, 37, 44, 49, 56, and so on. **Math Talk:** Appears in most lessons; for examples, see pages 16, 22, 34, 40, 47, 52, and so on.

Vocabulary Development in Go Math!

In *Go Math!* students learn the language of mathematics.

Each unit in the program opens with a **Vocabulary Preview,** in which students work alone, in pairs, or in small groups to complete a vocabulary activity that introduces them to mathematical language essential to learning in the unit. Regular program features such as the **Vocabulary Preview** offer students the chance to build their content-area vocabulary knowledge with specific mathematical terms.

Each module in the program opens with the **Reading Start-Up** feature, in which students review words, preview words, and complete activities to use the relevant mathematical terms in different ways. Each **Reading Start-Up** has students complete activities to **Understand Vocabulary** and **Visualize Vocabulary.**

Vocabulary in *Go Math!* © 2014
Grade 6
Vocabulary Preview: For examples, see pages 2, 74, 144, 232, 292, 366, and 444. **Reading Start-Up:** For examples, see pages 5, 29, 45, 77, 105, 147, 171, 201, 235, and so on.
Grade 7
Vocabulary Preview: For examples, see pages 2, 112, 168, 232, 306, and 362. **Reading Start-Up:** For examples, see pages 5, 35, 59, 115, 139, 171, 201, 235, 263, and so on.
Grade 8
Vocabulary Preview: For examples, see pages 2, 66, 192, 274, 342, and 428. **Reading Start-Up:** For examples, see pages 5, 31, 69, 93, 125, 151, 195, 225, 277, and so on.

Connections in Go Math!

The creators of *Go Math!* followed the research on the benefits of making interdisciplinary connections and real-world connections to enhance and add meaning to students' learning. Throughout the *Go Math!* program, the relevance of learning and the usefulness of mathematics is highlighted. Students are supported in making connections to other disciplines and to the world around them throughout.

Throughout the program, **real-world examples** and **mathematical modeling** apply mathematics to other disciplines and real-world contexts such as science and business. Real-world situations are a focus of *Go Math!* and are found throughout the program.

Each unit opens with an overview of a profession which involves math in the program feature, **Careers in Math.** Features like the **Careers in Math** help students to make connections between what they are learning—and how it applies in real-world contexts.

Explore Activities help students develop a deeper understanding of math concepts through real-world activities.

Each module opens with a link to a **Real-World Video** that helps to make the concepts in the module come alive for students. Through program features like the **Real-World Video** students see how what they learn applies to their lives outside of school.

The **Represent Real-World Problems** items in the **Independent Practice** sections ask students to take the mathematical concepts they have learned and apply them to problem solve in real-world situations.

Connections in *Go Math!* © 2014
Grade 6
Careers in Math: For examples, see pages 1, 73, 143, 231, 291, 365, and 443. **Real-World Video:** For examples, see **my.hrw.com** and pages 3, 27, 43, 75, 103, and so on. **Represent Real-World Problems:** For examples, see 17, 20, 21, 84, 90, 100, 124, and so on.
Grade 7
Careers in Math: For examples, see pages 1, 111, 167, 231, 305, and 361. **Real-World Video:** For examples, see **my.hrw.com** and pages 3, 33, 57, 113, 137, and so on. **Represent Real-World Problems:** For examples, see pages 11, 12, 18, 24, 41, 48, and so on.
Grade 8
Careers in Math: For examples, see pages 1, 65, 191, 273, 341, and 427. **Real-World Video:** For examples, see **my.hrw.com** and pages 3, 29, 67, 91, 123, and so on. **Represent Real-World Problems:** For examples, see pages 37, 55, 75, 120, 132, and so on.

Opportunities for Practice in Go Math!

In *Go Math!* students learn to mastery, learning deeply about key concepts to build a long-term understanding that they can access when they move ahead in mathematics. One of the ways that the program fosters this mastery is through opportunities for practice.

Placed strategically throughout the lessons, the **Your Turn** exercises allow students to immediately check understanding of new concepts.

The online **Personal Math Trainer** allows students to practice skills and complete homework in one place. The **Personal Math Trainer** offers a variety of learning aids—including videos, guided examples, and step-by-step solutions—to help develop and improve students' understanding of math concepts.

Assessment Readiness items provide the opportunity for students to check mastery of concepts and practice for high-stakes standardized tests.

Animated Math activities let students practice key math concepts and skills.

Higher Order Thinking (H.O.T.) activities encourage practice of higher-order thinking and mathematical processes in every lesson.

Guided Practice sections offer problems for students and instructional suggestions and activities for teachers to encourage practice of newly learned concepts and skills. **Guided and Independent Practice** at the end of each lesson gives students more opportunities to practice concepts and skills.

Higher-Order Thinking in **Go Math!**

In addition to building students' foundational understandings, the middle school years are the time for students to extend those understanding to higher-level applications. The *Go Math!* program offers students regular and ongoing opportunities for exploration and investigation .

The **Focus on Higher Order Thinking** exercises (marked with the H.O.T. icon) in every lesson and the **Performance Tasks** in every unit require students to use logical reasoning, represent situations symbolically, use mathematical models to solve problems, and state answers in terms of a problem context.

Higher-Order Thinking in *Go Math!* © 2014
Grade 6
H.O.T. (Higher Order Thinking) problems are found in every lesson. See, for example, pages 12, 18, 24, 52, 58, 64, 84, and so on.
Grade 7
H.O.T. (Higher Order Thinking) problems are found in every lesson. See, for example, pages 66, 74, 88, 100, 122, 134, 146, and so on.
Grade 8
H.O.T. (Higher Order Thinking) problems are found in every lesson. See, for example, pages 14, 20, 26, 38, 44, 50, 76, 82, 88, and so on.

Multiple Representations in **Go Math!**

The way that learners access and communicate the ideas of mathematics are through representations. Representations are essential to mathematics and the *Go Math!* program offers students opportunities to work with multiple representations throughout.

Multiple Representations in *Go Math!* © 2014
Grade 6
For examples, see pages 8, 12, 17, 47, 58, 91, 100, 113, 117, 125, 134, 154, 156, 184, and so on.
Grade 7
For examples, see pages 11, 42, 133, 134, 145, 186, 189, 196, 211, 241, 242, and 281.
Grade 8
For examples, see pages 49, 81, 120, 159, 308, and 438.

Motivation in **Go Math!**

In *Go Math!* students are motivated to learn mathematics, and teachers are provided with specific tools to motivate students for learning.

Each lesson opens with **Motivate the Lesson,** in which teachers can engage students with an idea or question that motivates them to want to learn the mathematical skills and concepts in the lesson.

For example, see this motivate the lesson from Grade 6, Lesson 4.3:

Motivate the Lesson
Ask: You have $2\frac{1}{4}$ lb of trail mix and need four $\frac{1}{8}$-lb servings. How can you know if you have enough trail mix? Begin the Explore Activity to find out more about dividing mixed numbers.

For example, see this motivate the lesson from Grade 7, Lesson 2.1:

Motivate the Lesson
Ask: Have you ever heard the expression "Two wrongs don't make a right"? Well, in math a surprising thing happens when you multiply two negative integers. Begin the Explore Activity to find out.

Or see this motivate the lesson from Grade 8, Lesson 6.1:

Motivate the Lesson
Ask: Suppose students are buying pencils and all of the pencils cost the same amount. Can you create a rule to describe the relationship between the number of pencils a student buys and their cost? Begin the Explore Activity to find out.

In addition, throughout *Go Math!* the program motivates students through such features and instructional strategies as:

- Showing the relevance of tasks, such as through the **Real-World Videos** and problems;
- Providing feedback for students, such as in the **Guided and Independent Practice** items;
- Encouraging social connectedness in the classroom, such as in the **Talk About It** suggestions;
- Offering scaffolds for students so that all can feel successful with learning, such as through the **Personal Math Tutor** and suggestions for differentiation.
- Employing technology effectively with a variety of tools, as discussed in a later strand of this report.

Engagement in Go Math!

In *Go Math!* students are the focus of the program and student engagement is promoted at every level—from the design of the program to the specific activities offered.

The program provides students with new ways to interact through the write-in **Student Edition**, which engages students in recording their strategies, explanations, solutions, and practice in their books.

Throughout the program, activities in the *Student Edition* and suggestions in the *Teacher Edition* engage students in learning. Specific program features designed for maximum engagement include:

- Every lesson opens with the **Engage** section, which provides the **Essential Question** for students, to immediately involve them in thinking about the concepts and skills in the lesson.
- The program's digital features engage students in multimedia learning opportunities. Every lesson opens with suggestions for the teacher for how to **Engage with the Whiteboard.**
- **Real-World Videos** engage students with interesting and relevant applications of the mathematical content of each module.
- The online simulations, tools, and games in **Animated Math** help students actively learn and practice key concepts.
- The **Explore Activities** invite students to interactively explore new concepts using a variety of tools and approaches.
- Engaging content is offered on a multitude of devices, including tables and interactive whiteboards—engaging digital natives using a medium that is a preferable form of learning for them.

Strand 3: Assessment

> *Assessment…refers to all those activities undertaken by teachers—and by their students in assessing themselves—that provide information to be used as feedback to modify teaching and learning activities…*
>
> (Black & Wiliam, 1998a, p. 140)

Defining the Strand

Assessments help teachers to use every minute of instructional time wisely. In a strong assessment system, diagnostic tools help teachers assess readiness and ensure proper placement and formative tools help them to make ongoing adjustments to instruction to meet diverse needs.

Effective assessment tools allow teachers to collect data about what is working—and what is not—so that they can take precise, swift, and effective action in meeting the specific needs of students. Formative assessment has a positive effect on learning (Black & Wiliam, 1998b; Cotton, 1995; Jerald, 2001). As noted by numerous research studies, the regular use of assessment to monitor student progress can mitigate and prevent mathematical weaknesses and improve student learning (Clarke & Shinn, 2004; Fuchs, 2004; Lembke & Foegen, 2005; Skiba, Magnusson, Marston, & Erickson, 1986). In their research, Baker, Gersten, and Lee (2002) concluded that "providing teachers and students with information on how each student is performing seems to enhance…achievement consistently" (p. 67). There is agreement that "assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions" (National Council of Teachers of Mathematics, 2000, p. 1).

Research also points to the importance of using varied item types and tasks in order to get the best reflection of student understanding. As noted by McREL (2010) "Using multiple types of assessments provides more insight into students' learning because students have more than one way to demonstrate their knowledge and skills" (p. 44).

Response-to-Intervention (RtI) is a framework with multiple layers, in which students are assessed to determine students' needs and the intensity of supports required, and then offered instructional interventions at different tiers, or levels (Fuchs & Fuchs, 2006). An effective diagnostic and ongoing assessment system is essential for the success of this kind of tiered intervention.

Go Math! supports data-driven instruction. Throughout the program, varied assessments provide valuable information about student learning that can help teachers plan and modify instruction. Specific examples of how *Go Math!* integrates effective assessment practices are provided on the following pages.

Research that Guided the Development of the *Go Math!* Program

Diagnostic Assessment

Effective instruction depends upon teachers who make good decisions about how best to meet their students' needs. To make these kinds of decisions, teachers need information that they can trust about students' strengths and weaknesses, knowledge and understandings. In an instructional context, a *diagnostic assessment* is one in which "assessment results provide information about students' mastery of relevant prior knowledge and skills within the domain as well as preconceptions or misconceptions about the material." (Ketterlin-Geller & Yovanoff, 2009, p. 1) A screening tool given to every student in a given grade at the opening of the school year can help to identify those who are at-risk or need additional support (Fuchs & Fuchs, 2006).

Studies attest to the benefits of using valid diagnostic measures—and tailoring instruction and supplemental practice according to the results of the diagnostics (for example, see Mayes, Chase, & Walker, 2008). Today's classrooms have disparity in students' prerequisite skills and knowledge and preparation and diagnostic assessment can help to identify the best instructional approach for each student at the outset so that instructional time is not wasted.

Formative Assessment

"Effective instruction depends on sound instructional decision-making, which in turn, depends on reliable data regarding students' strengths, weaknesses, and progress in learning content..." (National Institute for Literacy, 2007, p. 27) The phrase *formative assessment* encompasses the wide variety of activities—formal and informal—that teachers employ throughout the learning process to gather this kind of instructional data to assess student understanding and make and adapt instructional decisions. Its purpose is not an end in itself—such as the assignment of a grade—but rather, the purpose is to guide instruction. Formative assessment moves testing from the end into the middle of instruction, to guide teaching and learning as it occurs (Shepard, 2000; Heritage, 2007). Formative assessment shifts the way that students view assessments—"Assessment should not merely be done *to* students; rather, it should also be done *for* students, to guide and enhance their learning" (NCTM, 2000, p. 22).

Educators agree on the benefits of ongoing assessment in the classroom. "Well-designed assessment can have tremendous impact on students' learning ... if conducted regularly and used by teachers to alter and improve instruction" (National Research Council, 2007, p. 344). In its review of high-quality studies on formative assessment, the National Mathematics Advisory Panel (2008) found that "use of formative assessments benefited students at all ability levels" (p. 46). Several reviews of instructional practices used by effective teachers have revealed that effective teachers use formal (such as quizzes or homework assignments) and informal tools (such as discussion and observation) to regularly monitor student learning and check student progress (Cotton, 1995; Christenson, Ysseldyke, & Thurlow, 1989). A meta-analytic study by Baker, Gersten, and Lee (2002) found that achievement increased as a result of regular assessment use: "One consistent finding is that providing teachers and students with specific information on how each student is performing seems to enhance mathematics achievement consistently...The effect of such practice is substantial" (p. 67). In a study of student learning in a multimedia environment, Johnson and Mayer (2009) found that students who took a practice test after studying multimedia material outperformed students who studied the material again (without the assessment). Stecker, Fuchs, and Fuchs (2005) examined research on curriculum-based measurement, in which teachers used outcomes-based assessments regularly to monitor student progress, and found that the use of these assessments produced significant gains—when teachers used the data to make appropriate adjustments to instruction.

Research shows that regularly assessing and providing feedback to students on their performance is a highly effective tool for teachers to produce significant—and often substantial—gains in student learning and performance (Black & Wiliam, 1998a, 1998b; Hattie, 1992). Feedback is essential so that students know how to monitor their own performance and know which steps to take to improve (National Research Council, 2001).

An additional benefit of formative assessment is that it has been shown to be particularly helpful to lower-performing students. Gersten and Clarke (2007) conveyed similar findings for lower-achieving math students, concluding that "the use of ongoing formative assessment data invariably improved mathematics achievement of students with mathematics disability" (p. 2). In this way, use of formative assessments minimizes achievement gaps while raising overall achievement (Black & Wiliam, 1998b).

Varied Assessment Types and Options

One single assessment or type of assessment cannot serve all of the purposes of assessment. Research supports that looking at multiple means of assessment is the best way to capture a whole picture of student learning. As noted by Krebs' (2005) research, using one data point, such as written responses, to evaluate and assess students' learning can be "incomplete and incorrect conclusions might be drawn..." (p. 411). In addition, "using multiple types of assessments provides more insight into students' learning because students have more than one way to demonstrate their knowledge and skills" (McREL, 2010, p. 44). Therefore, variety in assessment item types is an integral part of an effective mathematics program.

Using performance-based assessments or problem-solving tasks in the classroom is one effective way to assess student understanding—and encourage critical thinking. Research indicates that high-quality tasks foster students' abilities to reason, solve problems, and conjecture (Matsumura, Slater, Peterson, Boston, Steele et al., 2006). Students can gain a deeper understanding of mathematics by exploring and reasoning through performance-based tasks.

Items in which students were asked to construct a response—rather than choose among options for answer choices—were shown to involve greater cognitive effort in a study by O'Neil and Brown (1998).

Asking students to respond to open-ended questions—in writing or through classroom discussion—is another useful way to assess what students are learning. As discussed by Moskal (2000) in her guidelines for teachers for analyzing student responses, students' responses to open-ended questions afford them the opportunity to show their approaches in solving problems and expressing mathematically what they know, which in turn allows the teacher to see the students' mathematical knowledge. Research by Aspinwall and Aspinwall (2003) on using open-writing prompts supports the use of open-ended questions in assessment in the mathematics classroom: "Students' responses to open-ended questions offer opportunities for understanding how students view mathematical topics...this type of writing allows teachers to explore the nature of students' understanding and to use this information in planning instruction" (p. 352-353). Similarly, by asking students to respond to open-ended questions verbally, researchers Gersten and Chard (2001) found that "encouraging students to verbalize their current understandings and providing feedback to the student increases learning."

Multiple-choice items can play an important role in an assessment system as well. The National Mathematics Advisory Panel (2008) found that formative assessments based on items sampled from important state standards objectives resulted in "consistently positive and significant" effects on student achievement (p. 47). In addition, the Panel found multiple-choice items to be equally valuable in assessing students' knowledge of mathematics (National Mathematics Advisory Panel, 2008).

Response to Intervention

Response to Intervention (RtI) is a model that integrates instruction, intervention, and assessment to create a more cohesive program of instruction that can result in higher student achievement (Mellard & Johnson, 2008). While the term RtI may be new to some educators, the practices are not new; "the process of RtI is the combination of well-established educational practices that together form a systematic and effective approach to improving instructional programs for all students as well as a possibly more complete manner in which to diagnose students with learning disabilities" (Riccomini & Smith, 2013, p. 346).

RtI is most commonly depicted as a three-tier model where Tier 1 represents general instruction and constitutes primary prevention. Students at this level respond well to the general curriculum and learn reasonably well without additional support. Tier 2 represents a level of intervention for students who are at-risk. Students at Tier 2 receive some supplementary support, in the form of instruction or assessment. Tier 3 typically represents students who need more extensive and specialized intervention or special education services (Smith & Johnson, 2011).

"At the heart of the RtI model is personalized instruction, during which each student's unique needs are evaluated and appropriate instruction is provided, so that students will succeed" (McREL, 2010, p. 15). While differentiation was conceived as a way to respond to the needs of diverse learners in the classroom, RtI was envisioned as a prevention system with multiple layers—a structured way to help students who were struggling before they fell behind their peers—and so it focuses on early, and ongoing, identification of needs and tiers of responses.

According to Griffiths, VanDerHeyden, Parson, and Burns (2006), an effective RtI model should include three elements:

1. Systematic assessment and collection of data to identify students' needs;
2. The use of effective interventions in response to the data; and
3. Continued assessment of students to determine the effectiveness of interventions—and the need for any additional intervention.

While research on RtI is still evolving (Riccomini & Smith, 2013), emerging evidence from field studies of RtI programs suggests that mathematics performance can be improved through RtI (Hughes & Dexter, 2011a, 2011b).

Research suggests that successful integration of RtI into instruction in the mathematics classroom involves a number of elements (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). At the Tier 1 level, all students should be screened to identify those at risk—and interventions for those at risk should be provided. At the Tiers 2 and 3 levels, the following are important and proven effective by research:

- The focus of instructional materials in grades K through 5 should be on whole numbers; in 4 through 8, rational numbers.
- Instruction during intervention should be explicit and systematic, and should include providing models of problem solving, communication about math, practice, feedback, and review.
- Interventions should require students to solve word problems.

- Interventions should provide opportunities for students to work with graphic organizers and visual representations.
- Interventions should devote regular time to arithmetic fact fluency.
- Students' progress should be monitored.
- Interventions should be designed to motivate students.

From Research to Practice

***Diagnostic Assessment in* Go Math!**

Specific features and tools in *Go Math!* support teachers in using diagnostic assessment effectively to assess students' need for instruction.

The program's assessment system includes these diagnostics measures:

- **Review Tests—**test students' knowledge from the previous year;
- **Benchmark Tests—**at the opening of each book help to ensure that students are properly placed;
- **Are You Ready?**—section assessments assess student readiness so that teachers and students do not have to wait until the end of the unit to find out that they are not ready to move ahead. Each module opens with **Are You Ready?** in the *Student Edition,* and connected suggestions for intervention or enrichment in the *Teacher Edition.*

Diagnostic Assessment in *Go Math!* © 2014
Grade 6
Review Test (Grade 5): See pages CC15-CC18. **Benchmark Test (Grade 6):** See pages CC19-CC22. **Are You Ready?** For examples, see pages 4, 28, 44, 76, 104, 146, 170, 200, 234, 258, and so on.
Grade 7
Review Test (Grade 6): See pages CC17-CC20. **Benchmark Test (Grade 7):** See pages CC21-CC24. **Are You Ready?** For examples, see pages 4, 34, 58, 114, 138, 170, 200, 234, 262, and so on.
Grade 8
Review Test (Grade 7): See pages CC15-CC18. **Benchmark Test (Grade 8):** See pages CC19-CC22. **Are You Ready?** For examples, see pages 4, 30, 68, 92, 124, 150, 194, 224, 276, 312, and so on.

***Formative Assessment in* Go Math!**

Opportunities for assessing what students know and can do are incorporated throughout *Go Math!* Assessments identify students' strengths and their weaknesses so that teachers can focus instruction accordingly.

Each section closes with a **Ready to Go On?** assessment which gives teachers key information about students' understanding and mastery of new topics.

Formative Assessment in *Go Math!* © 2014
Grade 6
Ready to Go On? For examples, see pages 25, 41, 65, 101, 135, 167, 197, 223, 255, and so on.
Grade 7
Ready to Go On? For examples, see pages 31, 55, 101, 135, 159, 197, 223, 259, 295, and so on.
Grade 8
Ready to Go On? For examples, see pages 27, 57, 89, 121, 147, 179, 221, 265, 309, and so on.

The *Go Math!* **Personal Math Trainer** is a tool that allows students to take an assessment—and then immediately move ahead, or get the reteaching help they need to master concepts. Using an algorithmic formula, the program monitors students' understanding so that they continue to be assessed on the same type of question until they achieve mastery or until the teacher decides to step in and provide targeted, intervention instruction. In the **Personal Math Trainer** students can get help, see additional examples, or watch an instructional video

***Varied Assessment Types and Options in* Go Math!**

Throughout the *Go Math!* program, multiple effective types of assessment appear in an effort to best allow students to demonstrate their knowledge and skills. Assessment resources include:

- **Leveled Module Quizzes**
- **Leveled Unit Tests**
- **Unit Performance Tasks**
- **Placement, Diagnostic, and Quarterly Benchmark Tests**

Go Math! features strong performance task assessments, as well as multiple-choice items, constructed-response tasks, and other problem-solving prompts. Varied assessment types include the following.

Varied Assessment Types and Options in *Go Math!*	
Assessment Type	**Description and Examples**
Multiple-Choice Assessments	Multiple-choice items allow for teachers to quickly get a sense of what students know and do not know, and can help to prepare students for on-demand, statewide assessments. In *Go Math!* the **Assessment Readiness** tests help students to ensure readiness for large-scale assessment formats including multiple-choice items.
Mixed Response Formats	Mixed response format items—such as constructed response items—allow for a deeper look at students' thinking and understanding of concepts and practices. The *Go Math!* **Lesson Quizzes** (available online), **Guided Practice, Independent Practice,** and **Assessment Readiness** features all employ varied formats, including mini-tasks. **Independent Practice** items at the end of each module include short-answer items that require students to communicate mathematical ideas and focus on higher order thinking.
Performance Assessments	Performance assessments can reveal thinking strategies that students use to work through problems. *Go Math!* includes a **Performance Assessment** for each unit, in which students must use logical reasoning, represent situations symbolically, use mathematical models to solve problems, and state answers in terms of a problem context.
Online Assessments	The *Go Math!* **Personal Math Trainer** allows for immediate diagnosis of students' strengths and weaknesses and online intervention instruction, where students can see additional examples or watch instructional videos. Users can choose from course assessments or customize assessments based on course content, standards, and difficulty levels. Student progress is easily monitored through online reports and alerts.

***Response to Intervention in* Go Math!**

Through print and digital resources, *Go Math!* © 2014 supports a Response to Intervention (RtI) instructional model.

Key to success with Response to Intervention (RtI) is the use of effective assessment resources. In *Go Math!* assessment resources include:

- **Leveled Module Quizzes**
- **Leveled Unit Tests**
- **Unit Performance Tasks**
- **Placement, Diagnostics, and Quarterly Benchmark Tests**

In *Go Math!* teachers gain valuable insight into student performance through tools and resources including:

- **Are You Ready?**
- **Mini-Tasks**
- **Assessment Readiness**
- **Ready to Go On?**

Because of the many varied options and resources in the program, teachers can select instructional strategies and resources that specifically align with each student's level of understanding and preferred learning style.

The program offers resources for each level of RtI:

- Tier 1: On-Level Intervention: The *Go Math!* program uses research-based instructional strategies to ensure quality instruction for all students.
- Tier 2: Strategic Intervention: For students who are not responding to on-level instruction, Tier 2 is met with increased time and focus to support struggling learners and reinforce skills that might not have been previously mastered.
- Tier 3: Intensive Intervention: For those students who need more specialized intervention, *Go Math!* offers additional support.

The program's **Personal Math Trainer** offers **Online Assessment and Intervention** at **my.hrw.com.** The program allows teachers to monitor student progress through reports and alerts, and to create and customize assignments aligned to specific lessons or standards. With the **Personal Math Trainer** teachers and students have an online site for:

- **Practice**—Students get practice on key concepts supported by guided examples, step-by-step solutions, and video tutorials.
- **Assessments**—Teachers can choose from course assignments or customize their own based on course content, standards, and difficulty levels.
- **Homework**—Students can complete online homework with a wide variety of problem types—and the system can automatically grade the homework to give students and teachers immediate feedback.
- **Intervention—**The **Personal Math Trainer** can automatically prescribe targeted, personalized intervention for each student.

Each module opens with the **Are You Ready?** feature, which in the *Teacher Edition*, is linked to specific suggestions for what to do if students need intervention to meet the module's prerequisite skills—or if they are ready for additional enrichment.

Strand 4: Meeting the Needs of All Students

All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.

(NCTM Principles & Standards for School Mathematics, 2000, p. 12)

Defining the Strand

Students in today's classrooms come from increasingly diverse backgrounds, in regard to culture and language as well as in their background knowledge, abilities, motivations, interests and modes of learning (Tomlinson, 2005). Mathematical learning is important to each of these different students; "All young Americans must learn to think mathematically, and they must think mathematically to learn" (National Research Council, 2001, p. 1). To effectively teach mathematics skills and concepts, teachers of mathematics must be knowledgeable of, and sensitive to, the needs of all learners in the mathematics classroom.

In the classroom, teachers encounter students who are on-grade, above grade, below grade—as well as English language learners, students with special needs, and students with varying learning styles and cultural backgrounds. As Vygotsky (1978) noted in his seminal research on learning, "Optimal learning takes place within students' 'zones of proximal development'—when teachers assess students' current understanding and teach new concepts, skills, and strategies at an according level." Research continues to support the notion that for learning to take place, activities must be at the right level for the learner (Tomlinson & Allan, 2000; Valencia, 2007). Therefore, teachers must correctly identify each child's needs for instruction and additional support. Differentiation offers teachers a means to provide instruction to a range of students in today's classroom (Hall, Strangman, & Meyer, 2009).

Most importantly, instruction needs to meet the needs of all students. To achieve this universal access, teachers must employ effective assessment practices, to diagnose student needs, and formative assessment, to assess progress regularly. Teachers must teach intentionally, scaffolding instruction so that all students can access the information.

Go Math! centers on helping all students gain a deeper understanding of mathematics concepts and practices. *Go Math!* supports the achievement of diverse learners by incorporating tactics to meet the varying needs of students through effective differentiation and by providing universal access.

Research that Guided the Development of the *Go Math!* Program

Differentiation and Universal Access

As every teacher finds upon entering his or her classroom, students differ in many important ways. As Tomlinson (1997) emphasizes in her discussion of differentiation, "Students are not all alike. They differ in readiness, interest, and learning profile...Shoot-to-the-middle teaching ignores essential learning needs of significant numbers of struggling and advanced learners" (p. 1). In the discussion of how to meet the needs of *every* learner in the classroom, educators will hear the terms differentiation, universal access, and universal design. Universal access refers to the idea of providing an equal opportunity for high-quality curriculum and instruction for all students. But how best to achieve this universal access? Differentiation and universal design are terms used to describe two related and complimentary approaches to meeting the needs of all students in the classroom.

In universal design, the needs of all students are considered at the point of instructional design; methods, materials, and assessments (diagnostic and formative) are created to recognize and address the wide range of student needs. In differentiation, modifications take place at the point of instruction; in differentiating instruction, teachers are responsive to what happens in the classroom, and are "flexible in their approach to teaching and adjust the curriculum and presentation of information to learners rather than expecting students to modify themselves for the curriculum" (Hall, Strangman, & Meyer, 2009).

To create an environment in which the barriers that limit student access to learning are removed, and there is universal access to learning, educators can apply the approaches of universal design. The movement to universal design in education was inspired by the same movement in architecture, to allow access for all (Shaw, 2011). In education, various researchers have employed different terms for related ideas—Universal Design for Learning (UDL), Universal Design for Instruction (UDI), and so on—but one essential characteristic that allows for universal access is that curricular materials be designed to be flexible, allowing for flexible methods of presentation, expression, and engagement, such as by offering multiple examples, employing multiple media and formats, engaging in supported practices, and allowing flexible opportunities for demonstrating skill (Hall, Strangman, & Meyer, 2009; Shaw, 2011). Technology can be particularly beneficial to allow for such flexibility (Hitchcock, Meyer, Rose, & Jackson, 2002).

Differentiating instruction is an organized, but flexible way to alter teaching and learning to help all students maximize their learning (Tomlinson, 1999), and is necessary in order to meet the needs of all learners in today's diverse classrooms (Tomlinson, 2000). To differentiate instruction, teachers can adjust the content of what is being learned, adjust the process of learning (by providing additional supportive strategies, for example, or adjusting pacing); and tailor the expected outcomes (assessments, products, or tasks) of how learning is assessed (Tomlinson, 2001). In the mathematics classroom, "mathematics instructors must respond to the diverse needs of individual students...using differentiated instruction, a process of proactively modifying instruction based on students' needs" (Chamberlin & Powers, 2010, p. 113).

Research points to the benefits of differentiation. In a study of numerous teachers using differentiated instruction, researchers found these benefits: students felt learning was more relevant; students were motivated to stay engaged in learning; students experienced greater success; students felt greater ownership of content, products and performances; and teachers gained new insights (Stetson, Stetson, & Anderson, 2007).

Research also shows that differentiation benefits all students—low-achievers in mathematics as well as high-achievers. In its review of studies on teaching mathematically gifted students, the National Mathematics Advisory Panel (2008) found that:

- The studies reviewed provided some support for the value of differentiating the mathematics curriculum for students with sufficient motivation, especially when acceleration is a component (i.e., the pace and level of instruction are adjusted).
- A small number of studies indicated that individualized instruction, in which pace of learning is increased and often managed via computer instruction, produces gains in learning.

The Panel concluded that gifted students, too, benefit from a differentiated curriculum (National Mathematics Advisory Panel, 2008).

From Research to Practice

***Differentiation and Universal Access in* Go Math!**

Go Math! supports teachers in implementing effective differentiation so that they meet the varied needs of students in their classrooms. With *Go Math!* practical, point-of-use support is built into each lesson so all learners can achieve success. The program's write-in *Student Edition* allows students to explore concepts, take notes, answer questions, and complete homework right in the textbook, encouraging active learning. Additional videos, activities, and learning aids support students at the point-of-use in the print and online textbook, supporting students who learn best from different formats, presentations, and/or activities.

Multiple program features employing varied and flexible multimedia formats allow for universal access and support differentiated instruction throughout *Go Math!* These include:

- **Differentiate Instruction** suggestions provided in the **Teacher's Edition** offer ways for teachers to meet the needs of specific populations.
- **Leveled Practice Worksheets** and **Problem Solving** exercises provide ongoing practice for students at different levels of understanding.
- **Reteach** suggestions target ways for teachers to develop the skills of students who may not master concepts the first time around.
- **Challenge** activity suggestions and **Extend the Math** Pre-AP activities in text and online offer challenging extensions for students ready to move ahead.
- **Math on the Spot** video tutorials provide step-by-step instruction of the math concepts covered in each example.
- The **Personalized Math Trainer** provides an online place to practice skills and complete homework, with a variety of learning aids—including videos, guided examples, and step-by-step solutions—designed to develop and improve student understanding of math concepts.
- **Animated Math Activities** let students interactively explore and practice key math concepts and skills.
- ELL and ELD students are supported specifically with **Success for English Learners** and vocabulary development activities that help them get up to speed on the language of mathematics.

Strand 5: Technology

Research on instructional software has generally shown positive effects on students' achievement in mathematics as compared with instruction that does not incorporate such technologies.

(National Mathematics Advisory Panel, 2008, p. 50)

Defining the Strand

At its most basic, technology can refer to any tools, inventions, or techniques that help us solve problems or perform activities. Technology has always played a role in mathematical learning and study, and can serve as a valuable tool in teaching and learning. Technology can support students' development of skills, exploration and communication of concepts, and ability to reason and problem-solve. With advances in technology, specifically in graphics technology and in information technology, new opportunities for mathematical teaching and learning are constantly emerging. Wenglinsky (1998) found benefits particularly for teaching and learning higher-order skills; "computers may serve as important tools for improving student proficiency in mathematics, as well as the overall learning environment in the school" (p. 4).

According to the findings of the National Research Council's 2001 review, "research has shown that instruction that makes productive use of computer and calculator technology has beneficial effects on understanding and learning algebraic representation" (p. 420). Numerous studies and meta-analyses support the use of computers in the classroom to improve student learning (see Britt & Aglinskas, 2002; Li & Ma, 2010; Means, Toyama, Murphy, Bakia, & Jones, 2009; North Central Regional Educational Laboratory, 2003; Teh & Fraser, 1995). Studies point to the effectiveness of computer-based instruction in mathematics particularly. In their meta-analysis, Cheung and Slavin (2011) found that educational technology applications had a positive effect on student assessment performance in mathematics. Sosa, Berger, Saw, and Mary (2011) found that computer-assisted instruction yielded larger student learning effects in statistics. The effective use of computers in the mathematics classroom also correlates with higher levels of motivation for and interest in learning mathematics, a finding supported by House's 2009 study of U.S. and Korean students' computer use and TIMSS assessment results.

Today's "digital natives" (Prensky, 2001) use technology daily (Rideout, Foehr, & Roberts, Kaiser Family Foundation, 2010), and while 94% of students believe technology will improve their school and workplace opportunities, only 39% believe that their school meets their expectations for technology (CDW, 2011).

Clearly, technology is an effective tool to reach today's students. *Go Math!* was developed to take advantage of the instructional benefits of technology and engage and support student learning through multimedia and varied digital tools. Specific program resources are described on the following pages.

Research that Guided the Development of the *Go Math!* Program

Multimedia Learning

In Mayer's second edition (2009) of *Multimedia Learning,* he again lays out the case for multimedia learning, presenting a cognitive theory of multimedia learning and citing the results of numerous, systematically designed studies which demonstrate the ways in which people learn more deeply from words and visuals rather than from verbal messages alone. According to Mayer, "the case for multimedia learning is based on the idea that instructional messages should be designed in light of how the human brain works" (Mayer, 2001, p. 4). Mayer (2001, 2005), a leading researcher in the field of multimedia learning, argues that student learning is increased in multimedia environments because information can be presented in multiple formats—including words, audio, and pictures. Students are able to learn more and retain information when they can access information using these different pathways.

Studies in mathematics suggest that digital instruction has specific benefits to the mathematics learner. Meta-analyses looking at the benefits of digital instruction in the mathematics classroom have found positive effects on learning and achievement (Cheung & Slavin, 2011; Li & Ma, 2010). In investigating the efficacy of a computer program to teach geometrical concepts of reflection and rotation, Dixon (1997) found that students in the "dynamic instructional environment outperformed students experiencing a traditional instructional environment on content measures involving reflections and rotations" (p. 356). Taconis (2013) concluded that computers are an excellent tool for teaching problem solving, and are effective for "delivering a variety of learning tasks, worked problems, and exercise problems that focus on strengthening the knowledge base and thinking skills..." (p. 381). Weiss, Kramarski, and Talis (2006) examined the impact of multimedia activities on the mathematics learning of young children and found that students who engaged in multimedia learning either individually or in cooperative learning groups significantly outperformed control group students. In a study which compared users of classroom computer games with a control group, Kebritchi, Hirumi, and Bai (2010) found that the games had a "significant positive effect on students' mathematics achievement" (p. 435).

Multimedia learning opportunities can help to close achievement gaps between groups of students and can be particularly effective with average and with lower-achieving students (see Huppert, Lomask, & Lazarowitz, 2002; Mayer, 2001; White & Frederiksen, 1998). Means, Toyama, Murphy, Bakia, and Jones (2009) found that online learning approaches were effective across types of learners. Computer technologies offer powerful tools for teachers seeking to differentiate and provide for universal access to learning in the classroom; "With the power of digital technologies, it is possible to provide a malleable curriculum in which content and activities can be presented in multiple ways and transformed to suit different learners" (Hitchcock, Meyer, Rose, & Jackson, 2002, p. 9). Multimedia learning environments are able to reach students who learn in different ways—visual learners, auditory learners, kinesthetic learners. With technology, too, scaffolds can be embedded at the point of use.

Technology offers tools that are particularly supportive to teachers. Interactive whiteboards—touch-sensitive boards that connect to a computer and projector—are helpful to teachers, enabling them to "demonstrate mathematical processes and illustrate concepts" (Schweder & Wissick, 2008, p. 55).

Studies indicate that whiteboards positively impact student motivation (Glover & Miller, 2001; Levy, 2002). Whiteboard technology can help students meet content standards, increase computer skills, and increase student motivation (Starkman, 2006). Marzano and Haystead (2009) compared students using interactive whiteboards in learning with those who did not use the technology, finding a 16 percentile point performance gain among the technology group. López (2010) found that interactive whiteboard technology helped to increase ELL student achievement and close achievement gaps. Wood and Ashfield (2008) explored the use of interactive whiteboards for creative literacy and mathematics instruction and found that students "felt that the IWB had enhanced whole-class teaching and learning...Aspects of direct teaching such as explaining, modeling, directing and instructing are all facilitated by the IWB..." (p. 93-94).

Whiteboards may be so effective because they align with what we know about how students learn (López, 2010). The National Research Council commissioned research on the human brain, cognition and educational psychology to identify fundamentals of effective instruction (see Bransford, Brown, & Cocking, 1999). Interactive whiteboards support the implementation of each in the classroom.

- Learning must build on previous experiences. Interactive whiteboards can help to connect prior experiences with new learning.
- Learning must take place in a social setting. Interactive whiteboards lend themselves to interactive group settings for learning.
- Students learn best in different ways. Interactive whiteboards allow for multimedia and multi-sensory delivery of information.
- Learning is enhanced when information presented is connected, organized, and relevant. Interactive whiteboards can facilitate these kinds of connections and organization.
- Regular evaluation and feedback are essential. Interactive whiteboards facilitate the assessment and feedback processes.

From Research to Practice

***Multimedia Learning in* Go Math!**

Go Math! through its digital program *Go Digital* employs technology to support instruction and enhance student learning.

In *Go Digital* digital natives are engaged with the fully interactive electronic **Student Edition,** a completely enhanced student experience with additional videos, activities, tools, direct links, and learning aids. *Go Digital* offers these resources for students and tools for teachers:

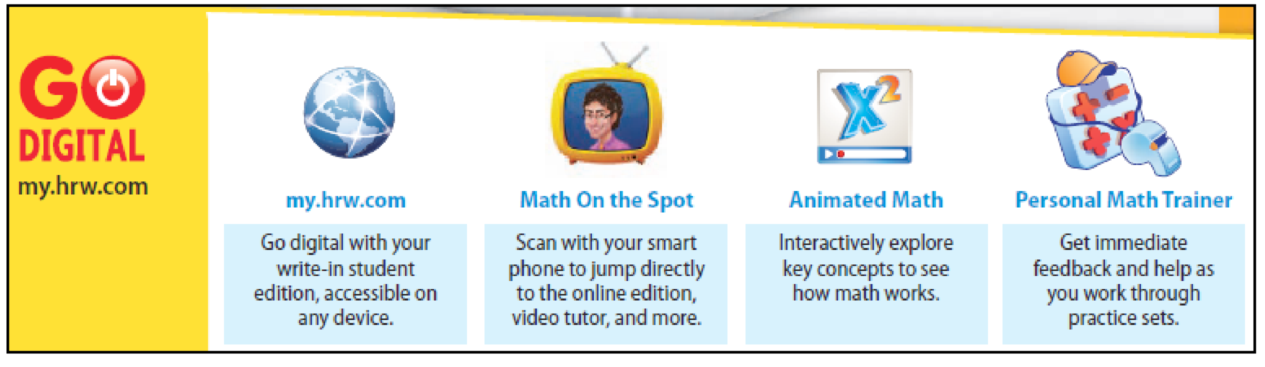
- **Math on the Spot** video tutorials provide step-by-step instruction of the math concepts covered in each example.
- The **Personal Math Trainer** allows students to practice skills and complete homework online. In addition, the **Personal Math Trainer** provides a variety of learning aids that develop and improve students' understanding of math concepts including videos, guided examples, and step-by-step solutions.
- **Animated Math** activities let students interactive explore and practice key math concepts and skills.

Additional *Go Digital* features include:

- **Online Teacher Edition**
- **ePlanner**
- **Interactive Answers and Solutions**
- **Online Assessment and Intervention**
- **Animated Math**

Teachers can teach content and reach students using **Interactive Whiteboards** and taking advantage of the program's numerous suggestions for whiteboard instruction.

Each program module opens in the *Teacher Edition* with references to the *Go Digital* features that can support learning.



GO DIGITAL
my.hrw.com

Online Teacher Edition
Access a full suite of teaching resources online—plan, present, and manage classes and assignments.

ePlanner
Easily plan your classes and access all your resources online.

Interactive Answers and Solutions
Customize answer keys to print or display in the classroom. Choose to include answers only or full solutions to all lesson exercises.

Interactive Whiteboards
Engage students with interactive whiteboard-ready lessons and activities.

Personal Math Trainer: Online Assessment and Intervention
Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated practice tests aligned with Common Core.

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