SAXON

HOMESCHOOL

Upper Grades Sampler

Algebra 1, Algebra 2, Geometry, Advanced Mathematics, and *Calculus*

Algebra 1, Algebra 2, Geometry, Advanced Mathematics, and *Calculus* each contain a series of lessons covering all areas of general math. *Advanced Mathematics* is a comprehensive precalculus course that includes advanced algebra, geometry, trigonometry, discrete mathematics, and mathematical analysis.

Each lesson in the Saxon math program presents a small portion of math content (called an increment) that builds on prior knowledge and understanding.

This sampler includes materials that are representative of the Saxon math program, including samples of lessons and other types of practice activities, such as Investigations and Labs.

We hope these materials will assist you in your evaluation of the Saxon program.

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LESSON 59 *Rearranging Before Substitution*

In every substitution problem encountered thus far, one of the equations has expressed x in terms of y, as in the bottom equation in (a), or y in terms of x, as in the top equation in (b).

$$\text{(a)}\ \begin{cases} 2x + 3y = 5 \\ x = 2y + 3 \end{cases} \qquad \text{(b)}\ \begin{cases} y = 2x + 4 \\ 2x - y = 7 \end{cases}$$

If neither of the equations is in one of these forms, we begin by rearranging one of the equations.

example 59.1 Use substitution to solve for x and y: $\begin{array}{l} \text{(a)} \\ \text{(b)} \end{array} \begin{cases} x - 2y = -1 \\ 2x - 3y = 4 \end{cases}$

solution To use substitution to solve this system of equations, it is necessary to rearrange one of the equations. We choose to solve for x in equation (a) because the x term in this equation has a coefficient of 1, and thus we can solve this equation for x in just one step.

$$\begin{array}{rcll} x - 2y &=& -1 & \text{equation (a)} \\ +\ 2y && +2y & \text{added } 2y \text{ to both sides} \\ \hline x &=& 2y - 1 & \end{array}$$

Now we can substitute the expression $2y - 1$ for x in equation (b) and complete the solution.

$$\begin{array}{rll} 2x - 3y = 4 && \text{equation (b)} \\ 2(2y - 1) - 3y = 4 && \text{substituted } 2y - 1 \text{ for } x \\ 4y - 2 - 3y = 4 && \text{multiplied} \\ y - 2 = 4 && \text{added like terms} \\ y = 6 && \text{added 2 to both sides} \end{array}$$

We can find the value of x by replacing the variable y with the number 6 in either of the original equations. We will use both of the original equations to demonstrate that either one can be used to find x.

Using Equation (a)	Using Equation (b)
$x - 2y = -1$	$2x - 3y = 4$
$x - 2(6) = -1$	$2x - 3(6) = 4$
$x - 12 = -1$	$2x - 18 = 4$
$x = 11$	$x = 11$

Thus the ordered pair of x and y that will satisfy both equations is **(11, 6).**

example 59.2 Use substitution to solve for x and y: $\begin{array}{l} \text{(a)} \\ \text{(b)} \end{array} \begin{cases} 2x - y = 10 \\ 4x - 3y = 16 \end{cases}$

solution We will first solve equation (a) for y and then substitute the resulting expression for y in equation (b).

$$\begin{array}{rcll} 2x - y &=& 10 & \text{equation (a)} \\ -2x && -\ 2x & \text{added } -2x \text{ to both sides} \\ \hline -y &=& 10 - 2x & \\ y &=& -10 + 2x & \text{multiplied both sides by } -1 \end{array}$$

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Now we substitute $-10 + 2x$ for y in equation (b).

$4x - 3y = 16$	equation (b)
$4x - 3(-10 + 2x) = 16$	substituted $-10 + 2x$ for y
$4x + 30 - 6x = 16$	multiplied
$-2x + 30 = 16$	added like terms
$-2x = -14$	added -30 to both sides
$x = 7$	divided both sides by -2

To finish the solution, we can use either of the original equations to solve for y.

Using Equation (a)	Using Equation (b)
$2x - y = 10$	$4x - 3y = 16$
$2(7) - y = 10$	$4(7) - 3y = 16$
$14 - y = 10$	$28 - 3y = 16$
$-y = -4$	$-3y = -12$
$y = 4$	$y = 4$

Thus the solution is the ordered pair **(7, 4).**

example 59.3 Use substitution to solve for x and y: $\begin{matrix}\text{(a)} \\ \text{(b)}\end{matrix} \begin{cases} 4x - 2y = 38 \\ 2x + y = 25 \end{cases}$

solution We will first solve equation (b) for y and then substitute the resulting expression for y in equation (a).

$$\begin{array}{rcll} 2x + y &=& 25 & \text{equation (b)} \\ \underline{-2x } && \underline{-\, 2x} & \text{added } -2x \text{ to both sides} \\ y &=& 25 - 2x & \end{array}$$

Now we substitute $25 - 2x$ for y in equation (a).

$4x - 2y = 38$	equation (a)
$4x - 2(25 - 2x) = 38$	substituted $25 - 2x$ for y
$4x - 50 + 4x = 38$	multiplied
$8x = 88$	simplified
$x = 11$	divided both sides by 8

To finish the solution, we can use either of the original equations to solve for y.

Using Equation (a)	Using Equation (b)
$4x - 2y = 38$	$2x + y = 25$
$4(11) - 2y = 38$	$2(11) + y = 25$
$44 - 2y = 38$	$22 + y = 25$
$-2y = -6$	$y = 3$
$y = 3$	

Thus the solution is the ordered pair **(11, 3).**

practice Use substitution to solve for x and y:

a. $\begin{cases} x - 3y = -7 \\ 2x - 3y = 4 \end{cases}$ **b.** $\begin{cases} 4x - y = 41 \\ 2x + y = 25 \end{cases}$

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problem set 59

1. (58) The store owner gave a 35 percent discount, yet Joe and Carol still had to pay \$247 for the camera. What was the original price of the camera? Draw a diagram as an aid in solving the problem.

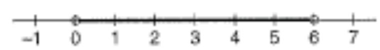
2. (58) The weight of the elephant was 1040 percent of the weight of the bear. If the elephant weighed 20,800 pounds, what did the bear weigh? Draw a diagram as an aid in solving the problem.

3. (39) The gallimaufry contained things large and small in the ratio of 7 to 2. If the total was 1098 items, how many were large?

4. (56) Given the sets $A = \{-3, -2, -1\}$, $B = \{1, 2, 3\}$, and $C = \{-1, 1, -2, 2, -3, 3\}$, are the following statements true or false?

   (a) $-3 \in A$ (b) $-2 \in B$ (c) $2 \notin C$ (d) $3 \notin C$

5. (46) Write a conjunction that describes this graph.



6. (50) 2.625 of what number is 8.00625?

7. (35) If $g(x) = -\sqrt{x}$, find $g(9)$.

8. (31) Solve: $1.591 + 0.003k - 0.002 + 0.002k = -(0.003 - k)$

Simplify:

9. (55) $\dfrac{\frac{1}{x}}{a}$

10. (55) $\dfrac{x + y}{\frac{1}{c}}$

Use substitution to solve for x and y:

11. (54) $\begin{cases} 2x - 3y = 5 \\ x = -2y - 8 \end{cases}$

12. (59) $\begin{cases} x + 2y = 5 \\ 3x - y = 1 \end{cases}$

Graph these equations on a rectangular coordinate system:

13. (51) $y = -3\frac{1}{2}$

14. (56) $4y - 4x = 8$

Add:

15. (44) $\dfrac{-x}{a^2b} + \dfrac{a - b}{b}$

16. (52) $\dfrac{m}{k(k + c)} + \dfrac{m}{k}$

Add. Write the answers with all exponents positive.

17. (57) $bx + cy^{-1}$

18. (57) $x^{-1}ay^{-2} - bz^{-1}$

19. (48) Add. Write the answer in descending order of the variable:

$$4(x^2 - 3x + 5) - 2(x^3 + 2x^2 - 4) - (2x^4 - 3x^3 + x^2 + 3)$$

20. (49) Multiply. Write the answer in descending order of the variable: $(-5x - 2)(-x + 4)$

21. (35) Factor the greatest common factor of $12x^4yp^3 - 4x^3y^2pz - 8x^2p^2y^2$.

Simplify. Write the answers with all exponents positive.

22. (53) $(4x^0y^2m)^{-2}(2y^{-4}m^0x)^4$

23. (53) $\dfrac{(x^2)^{-3}(yx)^2x^0}{x^2y^{-2}(xy^{-2})^3}$

24. (40) Expand by using the distributive property. Write the answer with all exponents negative.

$$\left(\frac{4p^2}{m^2b^3} - \frac{4m^{-2}}{ab^2p}\right)\frac{p^{-3}m^2}{4a^{-1}b^{-3}}$$

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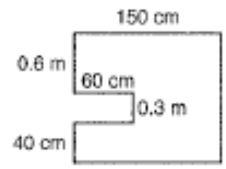
25. (41) Simplify by adding like terms. Write the answer with all exponents positive.

$$\frac{4(a^{-1}b^2)^2}{a^0b^2} + \frac{2aab^0}{(a^{-1}b)^{-2}} - \frac{5a^3a^{-1}b^0}{(ab^{-1})^2} - \frac{3(a^0a^3)^{-2}}{a^{-4}b^{-2}}$$

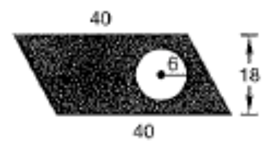
26. (41) Evaluate: $x^{-1}y^{-2} - x^{-2}y^{-3}$ if $1 - 2x = 3$ and $6 + 5y = -4$

27. (29) Simplify: $2^0|1 - 4| - (-3)^0|1 - 5| - 4^0|1 - 6| + \sqrt[3]{-27}$

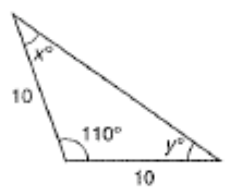
28. (8) Find the area of this figure in square meters. Corners that look square are square.



29. (8) Find the area of the shaded portion of this parallelogram. Dimensions are in inches.



30. (2) Find x and y.



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LESSON 75 *Writing the Equation of a Line • Slope-Intercept Method of Graphing*

75.A writing the equation of a line

We remember that the graph of a vertical line is everywhere equidistant from the y axis. In the figure on the left, every point on the line A is 3 units to the left of the y axis, and the equation of this vertical line is

$$x = -3$$

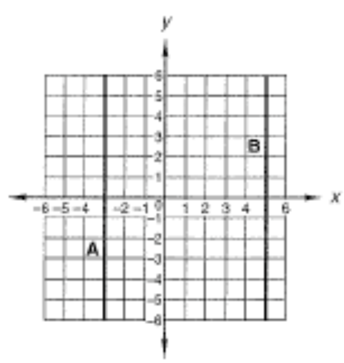
Every point on line A satisfies the equation $x = -3$ as every point on line A has -3 as its x coordinate. The y coordinate can be any real number since there are no restrictions on it. Every point on line B has 5 as its x coordinate, so the equation of line B is

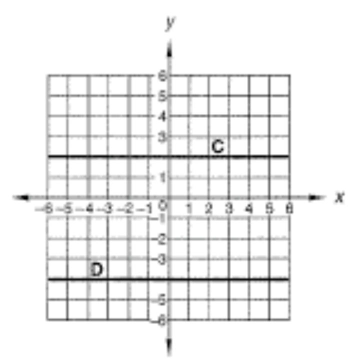
$$x = 5$$

The graph of every vertical line on the coordinate plane has the form

$$x = k$$

where k is the value of the x coordinate of any point on the vertical line. Note that k can be either positive or negative.





The graph of a horizontal line is everywhere equidistant from the x axis. Every point on line C is 2 units above the x axis, and the equation of this line is

$$y = 2$$

Every point on line C has a y coordinate of 2, and so the equation of line C is

$$y = 2$$

Note that there are no restrictions on the value of the x coordinate, so the points on the graph of the line $y = 2$ are those points whose x coordinates can be any real number and whose y coordinates must be 2. Applying similar reasoning, note that every point on line D has -4 as its y coordinate, so its equation is

$$y = -4$$

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If we use k to represent the value of the y coordinate of any point on a horizontal line, we can say that the equation of a horizontal line is

$$y = k$$

Thus, we see that the equations of vertical and horizontal lines can be determined by inspection. These equations contain an x and one number or a y and one number.

$$x = +5 \qquad x = -3 \qquad y = +2 \qquad y = -4$$

The equation of a line that is neither vertical nor horizontal cannot be so simply written. However, the equations of these lines can be written in what we call the **slope-intercept form.** The following equations are equations of three different lines written in slope-intercept form.

(a) $y = -6x + 2$ (b) $y = \frac{2}{3}x - 5$ (c) $y = 0.007x + 3$

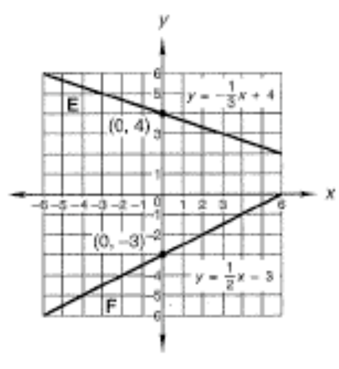
We note that each equation contains an equals sign, a y, an x, and two numbers. **The only difference in the equations is that the numbers are different.**

We use the letters m and b when we write this equation without specifying the two numbers.

$$y = mx + b$$

Since the equation of any line that is not a vertical line or a horizontal line can be written in this form, the problem of finding the equation of a given line is reduced to the problem of finding the two numbers that will be the values of m and b in the equation.

intercept In the slope-intercept form of the equation $y = mx + b$, we will call the constant b the **intercept** of the equation because b is the y coordinate of the line at the point where the line intercepts the y axis. Note that b is the value of y when x has a value of 0. The figure shows the graphs of two lines. Line E intercepts the y axis at +4, so the intercept b in the equation of this line has a value of 4. Line F intercepts the y axis at −3, so the intercept b in the equation of this line has a value of −3.



example 75.1 Find the y intercept of the line whose equation is $y = 3x - 5$.

solution The equation of the line $y = 3x - 5$ is written in the form $y = mx + b$. The constant b is the y intercept, so in this case, the y intercept is **−5.**

Another way to solve this problem is to remember that the y intercept is the y coordinate of the point of intersection of the line and the y axis. In other words, the y intercept is the value

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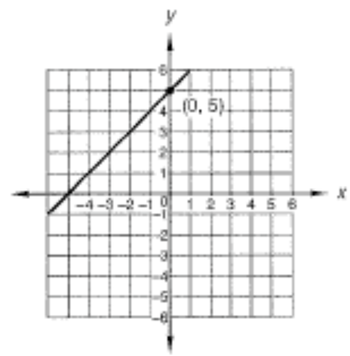
of y which satisfies the equation of the line when x is set equal to 0. Setting $x = 0$ in the equation of the line, we get

$$y = 3(0) - 5$$
$$= -5$$

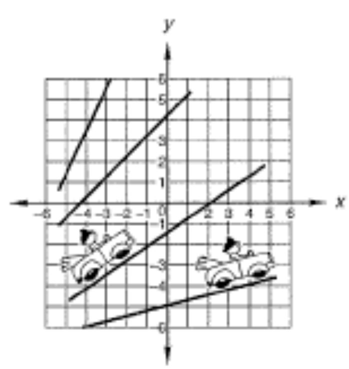
Therefore, the line intersects the y axis at -5 and the y intercept is -5.

example 75.2 Find the y intercept of the line shown.

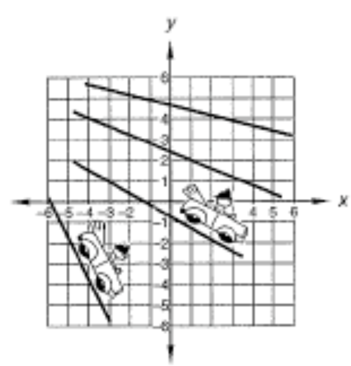


solution The line shown intersects the y axis at $y = 5$, so the y intercept is **5**.

slope **In the slope-intercept form, $y = mx + b$, we call the constant m the slope of the line.** Thus, in the equation $y = -2x + 6$, we say that the slope of this line is -2 because the coefficient of x is -2. **We note that the slope has both a sign and a magnitude** (absolute value). A line represented by a line segment that points toward the upper right-hand part of the coordinate plane has a positive slope. A line represented by a line segment that points toward the lower right-hand part of the coordinate plane has a negative slope. As a mnemonic to help us remember this, we will use the little man and his car. **He always comes from the left-hand side, as shown here.**



Positive slopes



Negative slopes

The little man sees the first set of lines as uphill with positive slopes and the second set of lines as downhill with negative slopes.

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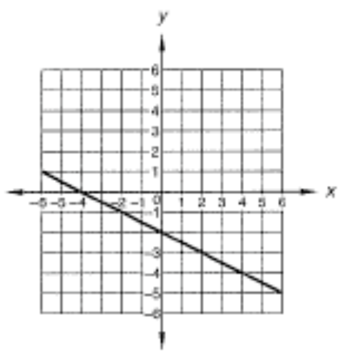
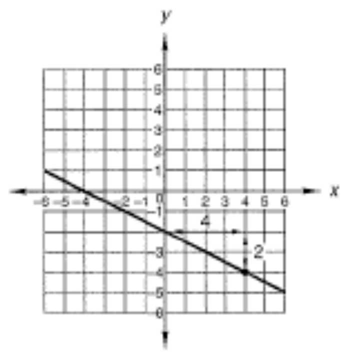
The magnitude, or absolute value, of the slope is defined to be the ratio of the absolute value of the change in the *y* coordinate to the absolute value of the change in the *x* coordinate as we move from one point on the line to another point on the line.

$$|m| = \frac{|\text{change in } y|}{|\text{change in } x|}$$

The figure on the left shows the graph of a line that has a negative slope. To find the magnitude of the slope of this line, we arbitrarily choose two points on the line, draw a right triangle, and label the lengths of the triangle. This has been done in the figure on the right.





The length of the horizontal leg of the triangle is 4 and is the difference of the *x* coordinates of the two points. The length of the vertical leg of the triangle is 2 and is the difference of the y coordinates of the two points. Since the magnitude of the slope is the ratio of the absolute value of the change in the *y* coordinate to the absolute value of the change in the *x* coordinate, we see that the magnitude, or absolute value, of the slope of this line is $\frac{1}{2}$.

$$|m| = \frac{|\text{change in } y|}{|\text{change in } x|} \longrightarrow |m| = \frac{2}{4} \longrightarrow |m| = \frac{1}{2}$$

We call the change in *x* the **run** and the change in y the **rise.** Using these words, the magnitude of the slope can be defined as the **absolute value of the rise over the absolute value of the run.**

$$|\text{Slope}| = \frac{|\text{rise}|}{|\text{run}|} \qquad \text{or} \qquad |m| = \frac{|\text{rise}|}{|\text{run}|}$$

The general form of the equation of a line is $y = mx + b$, and to write the equation of this line, we need to know (1) the value of the intercept b, (2) the sign of the slope, and (3) the magnitude, or absolute value, of the slope. We see that

1. The y coordinate of the point where the line intercepts the y axis is -2, so $b = -2$.
2. The line points to the lower right and thus the *sign* of the slope is negative.
3. The magnitude of the slope is $\frac{2}{4}$, which is equivalent to $\frac{1}{2}$.

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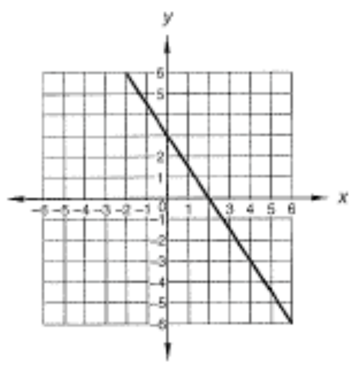
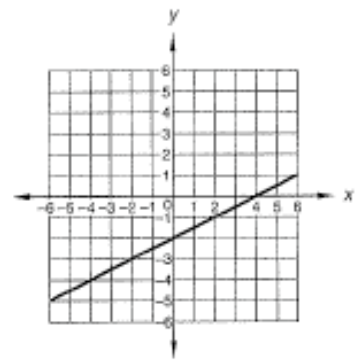
So the equation of this line is

$$y = -\frac{1}{2}x - 2$$

example 75.3 Find the equations of the lines graphed in the accompanying figures.





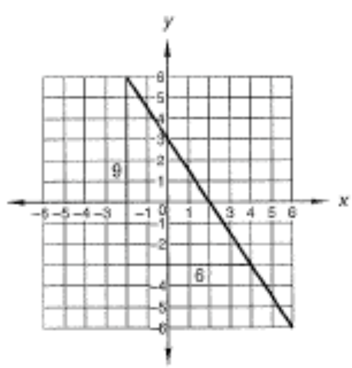
solution The desired equation is $y = mx + b$, and we need to find m and b.

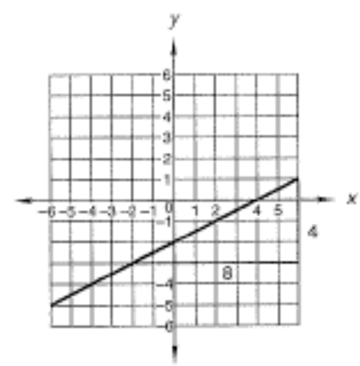
By inspection, $b = +3$.
By inspection, the sign of m is $-$.

The desired equation is $y = mx + b$, and we need to find m and b.

By inspection, $b = -2$.
By inspection, the sign of m is $+$.

Now we need to find the magnitudes, or absolute values, of the slopes. We will arbitrarily choose two points on each of the lines, draw the right triangles, and compute the slopes.





$$|m| = \frac{9}{6} = \frac{3}{2}$$

So $b = +3$ and $m = -\frac{3}{2}$.

Using these values in $y = mx + b$ yields

$$\mathbf{y = -\frac{3}{2}x + 3}$$

$$|m| = \frac{4}{8} = \frac{1}{2}$$

So $b = -2$ and $m = +\frac{1}{2}$.

Using these values in $y = mx + b$ yields

$$\mathbf{y = \frac{1}{2}x - 2}$$

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75.B slope-intercept method of graphing

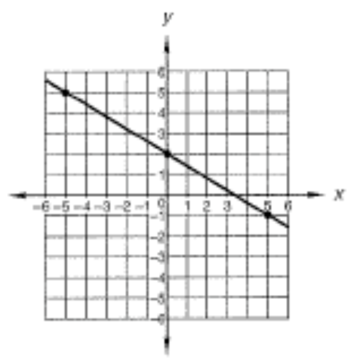
Thus far, we have graphed a line by finding ordered pairs of x and y that lie on the line. To graph $y = -\frac{3}{5}x + 2$, we choose values for x and write them in a table.

x	0	5	–5
y			

Then we use each of these numbers one at a time in the equation and find the corresponding values of y.

x	0	5	–5
y	2	–1	5

We finish by graphing the ordered pairs on the coordinate system below and drawing the line.



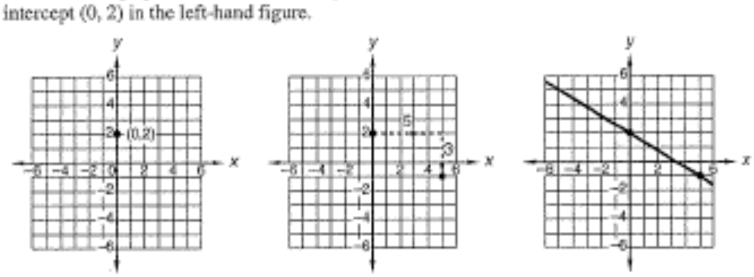
This method is dependable, but it is time-consuming. We can use the slope and the intercept of the line to get an accurate graph in less time. We will demonstrate this method by graphing the same line again.

example 75.4 Graph $y = -\dfrac{3}{5}x + 2$.

solution We begin by writing the slope in the form of a fraction that has a positive denominator. If we do this, the denominator will be +5 and the numerator will be –3.

$$y = \frac{-3}{+5}x + 2$$

Now we will graph the line in three steps, as shown below. As the first step, we graph the intercept (0, 2) in the left-hand figure.



Now in the center figure, from the intercept we move to the right (the positive x direction) a distance of 5 (the denominator of the slope representing the "run"). Then we move up or down

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the distance indicated by the numerator of the slope. We move down 3 since our numerator is −3 and the numerator represents the "rise." We graph this new point, and in the figure on the right we draw the line through the two points.

example 75.5 Use the slope-intercept method to graph the equation $x - 2y = 4$.

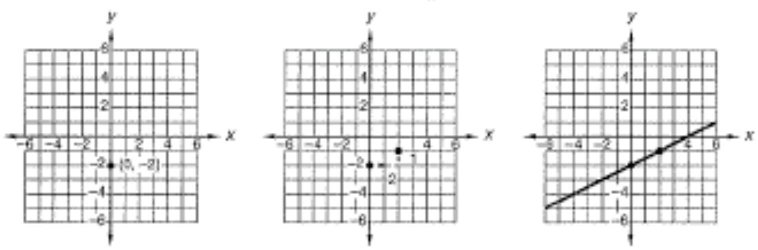
solution As the first step, we write the equation in slope-intercept form by solving for y.

$$x - 2y = 4 \quad \longrightarrow \quad -2y = -x + 4 \quad \longrightarrow \quad 2y = x - 4 \quad \longrightarrow \quad y = \frac{1}{2}x - 2$$

Now we write the slope as a fraction with a positive denominator.

$$y = \frac{+1}{+2}x - 2$$

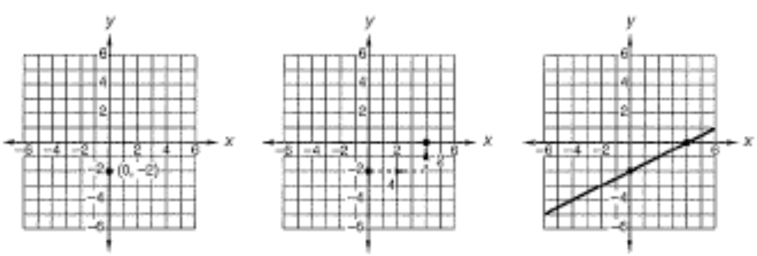
In the figure on the left we graph the intercept $(0, -2)$. In the figure in the middle we move from the intercept a horizontal distance of +2 (to the right) and a vertical distance of +1 (up). In the figure on the right we draw the line through the two points.



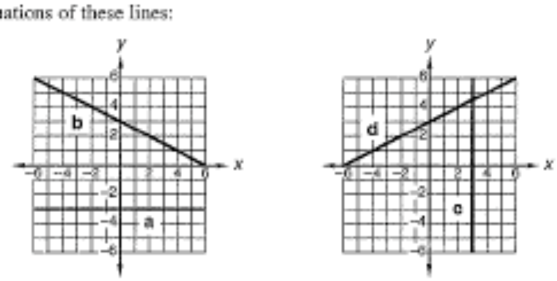
When the points are close together, as in this case, it is difficult to draw the line accurately. To get another point, we multiply the denominator and the numerator of the slope by a convenient integer and use the new form of the slope to get the second point. For the line under discussion, we will multiply the slope by 2 and get

$$\frac{+1}{+2} \cdot \frac{(2)}{(2)} \quad \longrightarrow \quad \frac{+2}{+4}$$

In the figures below, we use the same intercept but move an x distance of +4 and a y distance of +2 to find the new point.



practice Find the equations of these lines:



Algebra 1, Lesson 75

Sample taken from Algebra 1 (Third Edition), page 312

Use the slope-intercept method to graph the equations:

e. $y = -\frac{2}{3}x + 2$ f. $5 + 3y = x$

problem set 75

1. (70) Richardson has a bag with 5 white marbles and 7 red marbles. A marble is drawn at random and replaced. Then another marble is randomly drawn. What is the probability that the first marble will be red and the second marble will be white?

2. (70) The spinner shown is spun 4 times. What is the probability that the spinner stops on 4, 2, 3, and 1, in that order?

3. (73) David has a bucket with 5 yellow golf balls and 9 white golf balls. A ball is drawn at random and not replaced. Then another ball is randomly drawn. What is the probability that both are white golf balls?

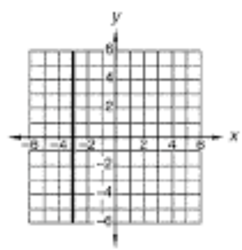
4. (58) Three percent of the caterpillars metamorphosed into butterflies. If Ramona could count 120 butterflies, how many caterpillars had there been? Draw a diagram as an aid in solving the problem.

5. (58) Sakahara socked it to them. If 4800 were present and Sakahara socked 34 percent of them, how many did he sock? Draw a diagram as an aid in solving the problem.

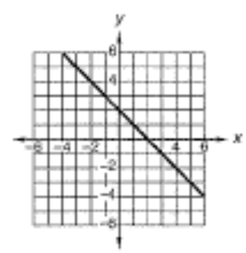
6. (58) Muhammad counted the tents and found that 784 were patched. If there were 1400 tents in all, what percent was patched? Draw a diagram as an aid in solving the problem.

Find the equations of these lines:

7. (75)



8. (75)



Use the slope-intercept method to graph these equations on a rectangular coordinate system:

9. (75) $y = -\frac{3}{2}x + 3$

10. (75) $2y = -x + 2$

Write these numbers in scientific notation:

11. (74) 0.00123×10^{-5}

12. (74) 0.00123×10^{8}

Factor these binomials. Always factor the common factor first.

13. (73) $b^3x^2 - 4b^3$

14. (73) $16x^2 - a^2$

15. (73) $-m^2 + 9p^2$

Factor the trinomials. Always begin by writing the trinomials in descending order of the variables and by factoring out the greatest common factor.

16. (69) $x^2 + 3x - 10$

17. (69) $4x + x^2 - 21$

18. (71) $5x^2 - 15x - 50$

19. (71) $x^3 - 3x^2 + 2x$

20. (72) $18(x + y) + 9z(x + y) + z^2(x + y)$

21. (72) $(m + a)x^2 + 3(m + a)x - 18(m + a)$

Algebra 1, Lesson 75

Sample taken from Algebra 1 (Third Edition), page 313

22. (67) Use elimination to solve:

$$\begin{cases} 4x - 5y = -1 \\ 2x + 3y = 5 \end{cases}$$

23. (71) Use substitution to solve:

$$\begin{cases} N_Q + N_D = 25 \\ N_Q = N_D + 3 \end{cases}$$

Simplify:

24. (66) $15\sqrt{12} - 30\sqrt{18} + 2\sqrt{300}$

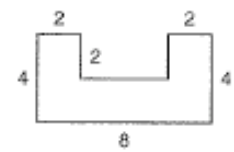
25. (59) $\dfrac{5}{-3^{-2}} - \sqrt[5]{-32}$

26. (53) $\dfrac{x^{-2}y^{0}(x^{-2})^{-2}y^{2}}{(y^{2}x^{-4})^{2}(y^{3}x^{2})^{-1}}$

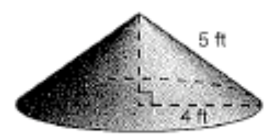
27. (48) $\dfrac{a^{-1} + b^{-1}}{a^{-1}b^{-1}}$

28. (27) Solve: $2.2x - 0.1x + 0.02x = -2 - 0.12$

29. (8) Find the area of this figure. All angles are right angles. The dimensions are in inches.



30. (72) A right circular cone has a base of radius 4 ft and a slant height of 5 ft, as shown. Find the surface area of the right circular cone.

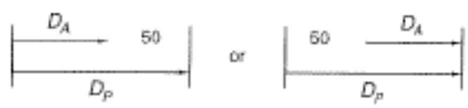


Algebra 1, Lesson 99

Sample taken from Algebra 1 (Third Edition), page 418

LESSON 99 *Uniform Motion—Unequal Distances*

Some uniform motion problems tell us that one person or object traveled a distance that is greater by a specified amount than the distance traveled by another person or object. The distance diagram for these problems usually takes one of the following forms:

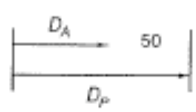


In the picture on the left, both started from the same place and P went 50 farther than A. In the picture on the right, A started out 50 in front of P, and they both ended at the same place. In either case the distance that A traveled plus 50 equals the distance that P traveled. The distance equation for both diagrams is the same.

$$D_A + 50 = D_P \qquad \text{so} \qquad R_AT_A + 50 = R_PT_P$$

example 99.1 At 8 p.m. Achilles left camp and headed south at 20 kilometers per hour. At 10 p.m. Patroclos headed south from the same camp. If Patroclos was 50 kilometers ahead by 3 a.m., what was his speed?

solution Since they had the same starting point, both arrows begin at the same point. Patroclos went farther, so his arrow is longer.



Patroclos went 50 kilometers farther, so we write the distance equation as

$$D_A + 50 = D_P$$

and we substitute R_AT_A for D_A and R_PT_P for D_P to get

$$R_AT_A + 50 = R_PT_P$$

We reread the problem to get the rate and time equations.

$$R_A = 20 \qquad T_A = 7 \qquad T_P = 5$$

Now we solve.

$$\begin{aligned} (20)(7) + 50 &= R_P(5) && \text{substituted} \\ 140 + 50 &= 5R_P && \text{simplified} \\ 190 &= 5R_P && \text{simplified} \\ R_P &= \mathbf{38}\ \frac{\textbf{km}}{\textbf{hr}} && \text{divided} \end{aligned}$$

example 99.2 Rachel has a 15-kilometer head start on Charlene. How long will it take Charlene to catch Rachel if Rachel travels at 70 kilometers per hour and Charlene travels at 100 kilometers per hour?

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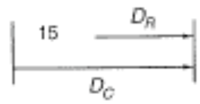
Sample taken from Algebra 1 (Third Edition), page 419

solution Rachel began 15 kilometers ahead and they ended up in the same place, so the distance diagram is



We get the distance equation from the diagram as

$$15 + D_R = D_C$$

and we replace D_R with $R_R T_R$ and D_C with $R_C T_C$ to get

$$15 + R_R T_R = R_C T_C$$

Then we reread the problem to get the other three equations.

$$R_R = 70 \qquad R_C = 100 \qquad T_R = T_C$$

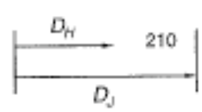
Now we solve.

$$\begin{aligned} 15 + 70T_R &= 100T_C && \text{substituted} \\ 15 + 70T_C &= 100T_C && \text{used fact } T_R = T_C \\ 15 &= 30T_C && \text{simplified} \\ \frac{1}{2} &= T_C && \text{divided} \end{aligned}$$

So Charlene will catch Rachel in $\frac{1}{2}$ **hour.**

example 99.3 Harry and Jennet jog around a circular track that is 210 meters long. Jennet's rate is 230 meters per minute, while Harry's rate is only 200 meters per minute. In how many minutes will Jennet be a full lap ahead?

solution This problem is simpler if we straighten it out and get the following distance diagram.



We get the distance equation from this diagram as

$$D_H + 210 = D_J \qquad \text{so} \qquad R_H T_H + 210 = R_J T_J$$

The time equation is $T_H = T_J$, and the rate equations are $R_J = 230$, $R_H = 200$. Thus the four equations are

$$R_H T_H + 210 = R_J T_J \qquad T_H = T_J \qquad R_J = 230 \qquad R_H = 200$$

We use substitution to solve.

$$\begin{aligned} 200T_H + 210 &= 230T_H && \text{substituted} \\ 210 &= 30T_H && \text{simplified} \\ 7 \text{ minutes} &= T_H && \text{divided} \end{aligned}$$

Thus $T_J = 7$ minutes because $T_J = T_H$. Therefore, Jennet will be a full lap ahead of Harry in **7 minutes.**

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Sample taken from Algebra 1 (Third Edition), page 420

practice

a. At 5 a.m. Napoleon headed south from Waterloo at 4 kilometers per hour. At 7 a.m. Wellington headed south from Waterloo. If Wellington passed Napoleon and was 20 kilometers ahead of Napoleon at 2 p.m., how fast was Wellington traveling?

b. Helen has a 4-kilometer head start on Paris. How long will it take Paris to catch Helen if Helen travels at 6 kilometers per hour and Paris travels at 8 kilometers per hour?

problem set 99

1. *(99)* Ferris and Julia jog around a circular track that is 500 meters long. Julia's rate is 250 meters per minute, while Ferris's rate is only 230 meters per minute. In how many minutes will Julia be a full lap ahead? Begin by drawing a diagram of distances traveled and writing the distance equation.

2. *(95)* Eleanor started out at 60 miles per hour at 9 a.m., two hours before Alexi started out to catch her. If she was still 60 miles ahead at 3 p.m., how fast was Alexi driving? Begin by drawing a diagram of distances traveled and writing the distance equation.

3. *(33)* The product of 5 and the sum of a number and -8 is 9 greater than the product of 2 and the opposite of the number. Find the number.

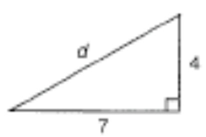
4. *(83)* When the car overturned, the jar broke and spilled 450 nickels and quarters all over the freeway. If their value was \$62.50, how many coins of each type were there?

5. *(56)* When the nurse gave the shots, she noticed that 34 percent of the people winced and the rest were stolid. If 3300 people were stolid, how many shots did she give?

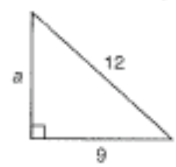
6. *(76)* Bobby and Joan found four consecutive integers such that 5 times the sum of the second and third was 6 less than 7 times the first. What were their integers?

Use the Pythagorean theorem to find the unknown lengths in the following right triangles:

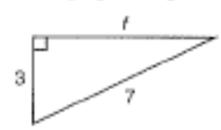
7. *(97)*



8. *(97)*



9. *(97)*



10. *(98)* Given the points (4, 3) and (7, -2):

(a) Find the slope of the line that passes through these two points.

(b) Find the distance between these two points.

11. *(98)* Given the points (4, -2) and (-2, 3):

(a) Find the slope of the line that passes through these two points.

(b) Find the distance between these two points.

12. *(85)* Given the following five functions:

$$f(x) = x^3 \quad g(x) = x^2 \quad h(x) = x \quad k(x) = -x^2 \quad p(x) = -x^3$$

Identify the function whose graph most resembles the shape shown.

(a) 

(b) 

(c) 

(d) 

(e) 

13. *(96)* Use the difference of two squares theorem to find all the solutions to the following equations:

(a) $x^2 = 64$ (b) $x^2 = 32$ (c) $x^2 = 11$

14. *(93)* Simplify: $\dfrac{x^2 + 11x + 28}{-x^2 + 5x} + \dfrac{x^2 + x - 12}{x^3 - 3x^2 - 10x}$

Algebra 1, Lesson 99

Sample taken from Algebra 1 (Third Edition), page 421

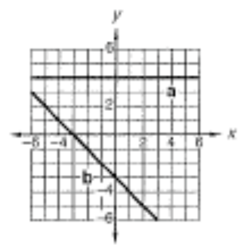
15. (88) Solve by factoring: $81 = 4x^2$

16. (81,83) Indicate whether the following numbers are rational numbers or irrational numbers:

(a) $0.\overline{3}$ (b) $\sqrt{12 + 4}$ (c) $\sqrt{9 - 4}$ (d) $\frac{25}{7}$

17. (82) What is the domain of $f(x) = \sqrt{3x - 4}$?

18. (75) Find the equations of lines (a) and (b).



19. (86) Divide: $(x^3 - 4) \div (x - 4)$

Simplify:

20. (84) $3\sqrt{2} \cdot 4\sqrt{3} \cdot 5\sqrt{12} + 2\sqrt{8}$

21. (84) $3\sqrt{2}(5\sqrt{2} - 4\sqrt{42})$

22. (81) Solve by graphing and check: $\begin{cases} y = x - 4 \\ y = -x + 2 \end{cases}$

23. (91) Graph on a number line: $-x - 3 < 2$; $D = \{\text{Reals}\}$

24. (80) Simplify: $\dfrac{(22{,}000 \times 10^{-7})(500)}{(0.0011)(0.002 \times 10^{14})}$

25. (52) Add: $\dfrac{x}{x + 4} + \dfrac{3}{x} - \dfrac{x + 2}{x^2}$

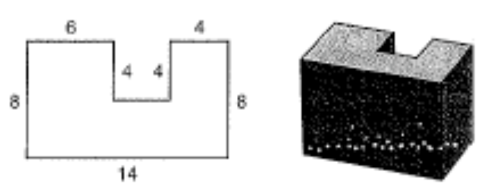
26. (78) Solve: $\dfrac{p}{6} - \dfrac{p + 2}{4} = \dfrac{1}{3}$

27. (43,62) Evaluate: $-x^0y(y - x^0) - x^2y^{-1}$ if $2x = -4$ and $y = \sqrt[3]{-64}$

28. (35,36) Simplify: (a) $\dfrac{-2p^2a^2 - p^2a}{-p^2a}$ (b) $\dfrac{-3^2}{-(-3)^{-2}}$

29. (93) Find the surface area of the sphere shown whose radius is 4 cm.

30. (20,65) A base of the right prism 10 meters high is shown. Find the volume of the right prism. All angles are right angles. Dimensions are in meters.



Algebra 2
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Sample taken from Algebra 2 (Third Edition), page 217

LESSON 50 *Quadratic Equations • Completing the Square*

50.A quadratic equations

A quadratic equation in one unknown has the number 2 as the highest power of the variable. The highest power of x in each of the three equations shown here is 2,

$$x^2 = -4x + 2 \qquad x - 3 = x^2 \qquad x^2 + 4x - 2 = 0$$

so these equations are all quadratic equations. We have solved quadratic equations thus far by two methods. To use the first method, we factor and then use the zero factor theorem. To review this method, we begin with the equation

$$x^2 + x - 2 = 0$$

First we factor and get

$$(x + 2)(x - 1) = 0$$

and then we set each of the factors equal to zero and solve.

$$\begin{array}{llll} \text{If } & x + 2 = 0 & \qquad \text{If } & x - 1 = 0 \\ & x = -2 & & x = 1 \end{array}$$

The other method can be used when the equation is in the form

$$(x - 2)^2 = 3$$

We can solve equations that are in this form by taking the square root of both sides and remembering the ± sign that appears on the right.

$$\begin{array}{rll} (x - 2)^2 = 3 & & \text{equation} \\ x - 2 = \pm\sqrt{3} & & \text{square root of both sides} \\ x = 2 \pm \sqrt{3} & & \text{added +2 to both sides} \end{array}$$

50.B completing the square

Unfortunately, the factoring method cannot always be used, because some equations cannot be factored into binomials whose constants are all integers. For example, the equation

$$x^2 + 4x - 2 = 0$$

cannot be factored. **The inability to factor some quadratic equations is offset by the fact that any quadratic equation can be rearranged into the form**

$$(x + a)^2 = k$$

and then the equation can be solved by taking the square root of both sides, as we have just demonstrated. The method used to accomplish this rearrangement is called **completing the square.** If we rearrange the equation

$$x^2 + 4x - 2 = 0$$

by completing the square, we can change the equation into this form

$$(x + 2)^2 = 6$$

which we can solve by taking the square root of both sides.

$$\begin{array}{rll} x + 2 = \pm\sqrt{6} & & \text{square root of both sides} \\ x = -2 \pm \sqrt{6} & & \text{added -2 to both sides} \end{array}$$

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Sample taken from Algebra 2 (Third Edition), page 218

To complete the square, it is necessary to remember the form of a trinomial that is the square of some binomial. On the left we show several binomials, and on the right we show the trinomials that result when the binomials are squared.

(a)	$x + 2$	(a′)	$(x + 2)^2 = x^2 + 4x + 4$
(b)	$x - 5$	(b′)	$(x - 5)^2 = x^2 - 10x + 25$
(c)	$x - 6$	(c′)	$(x - 6)^2 = x^2 - 12x + 36$
(d)	$x + 8$	(d′)	$(x + 8)^2 = x^2 + 16x + 64$
(e)	$x - 3$	(e′)	$(x - 3)^2 = x^2 - 6x + 9$

In each of these examples we note that the last term of the trinomial is the square of one half of the middle term of the trinomial. Thus,

	Last Term		$\left(\frac{1}{2}\text{ Middle Term}\right)^2$
In (a′)	4	is	$\left(\frac{4}{2}\right)^2$
In (b′)	25	is	$\left(-\frac{10}{2}\right)^2$
In (c′)	36	is	$\left(-\frac{12}{2}\right)^2$
In (d′)	64	is	$\left(\frac{16}{2}\right)^2$
In (e′)	9	is	$\left(-\frac{6}{2}\right)^2$

This pattern occurs every time we square a binomial. There is nothing to understand. It happens, so we will remember it and use it. This pattern is the key to completing the square.

Thus, if we have the expression

$$x^2 + 10x + ?$$

and ask what number should replace the question mark if the trinomial is to be the square of some binomial, the answer is 25 because

$$\left(\frac{10}{2}\right)^2 = 25$$

So the completed expression is

$$x^2 + 10x + 25$$

Thus, the binomial that was squared to get this result had to be $x + 5$ because

$$(x + 5)^2 = x^2 + 10x + 25$$

example 50.1 Solve $x^2 + 6x - 4 = 0$ by completing the square.

solution We want to rearrange the equation into the form

$$(x + a)^2 = k$$

We begin by enclosing the first two terms in parentheses and moving the -4 to the right-hand side as $+4$.

$$(x^2 + 6x \quad) = 4$$

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Sample taken from Algebra 2 (Third Edition), page 219

Note that we left a space inside the parentheses. Now we want to change the expression inside the parentheses so that it is a perfect square. To do this, we first divide the coefficient of x by 2 and square the result.

$$\left(\frac{6}{2}\right)^2 = 9$$

Then we add this number to both sides of the equation.

$$(x^2 + 6x + 9) = 4 + 9$$

Now the left-hand side can be written as $(x + 3)^2$ and the right-hand side as 13.

$$(x + 3)^2 = 13$$

We finish the solution by taking the square root of both sides and remembering the $\pm$ sign that will appear on the right.

$$x + 3 = \pm\sqrt{13} \qquad \text{square root of both sides}$$
$$\mathbf{x = -3 \pm \sqrt{13}} \qquad \text{added } -3 \text{ to both sides}$$

example 50.2 Solve $2x + x^2 - 5 = 0$ by completing the square.

solution We want to change the form of the equation to

$$(x + a)^2 = k$$

We begin by moving the constant term to the right-hand side and enclosing the other two terms in parentheses. We leave a space inside the parentheses.

$$(x^2 + 2x \quad) = 5$$

Now we square $\frac{1}{2}$ of the coefficient of x

$$\left(\frac{2}{2}\right)^2 = 1$$

and add it to both sides.

$$(x^2 + 2x + 1) = 5 + 1$$

The left-hand side is a perfect square, and the right-hand side is 6.

$$(x + 1)^2 = 6 \qquad \text{simplified}$$
$$x + 1 = \pm\sqrt{6} \qquad \text{square root of both sides}$$
$$\mathbf{x = -1 \pm \sqrt{6}} \qquad \text{added } -1 \text{ to both sides}$$

example 50.3 Solve $x^2 = 5x - 5$ by completing the square.

solution We begin by placing the constant term on the right and enclosing the other two terms in parentheses, remembering to leave a space in the parentheses.

$$(x^2 - 5x \quad) = -5$$

Next we divide -5 by 2 and square the result.

$$\left(\frac{-5}{2}\right)^2 = \frac{25}{4}$$

Now we add $\frac{25}{4}$ to both sides.

$$\left(x^2 - 5x + \frac{25}{4}\right) = -5 + \frac{25}{4}$$

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Sample taken from Algebra 2 (Third Edition), page 220

Next we write the left-hand side as a perfect square and simplify the right-hand side.

$$\left(x - \frac{5}{2}\right)^2 = \frac{5}{4} \qquad \text{simplified}$$

$$x - \frac{5}{2} = \pm\sqrt{\frac{5}{4}} \qquad \text{square root of both sides}$$

$$x = \frac{5}{2} \pm \sqrt{\frac{5}{4}} \qquad \text{added } \frac{5}{2} \text{ to both sides}$$

$$\mathbf{x = \frac{5}{2} \pm \frac{\sqrt{5}}{2}} \qquad \text{simplified}$$

practice Solve by completing the square: $x^2 = 9x - 7$

problem set 50

1. (21) The Two-Steppers were good recruiters, as they numbered 10 more than 5 times the number of the Waltzers. Also, there were 10 times as many Two-Steppers as Waltzers. How many of each were there?

2. (29) The first part of the trip was in a surrey at 8 mph, and the last part was in a buckboard at 12 mph. If the total trip was 104 miles and took 10 hours, how much of the trip was made in each type of carriage?

3. (27) What is the weight of the sodium (Na) in 348 grams of NaCl (Na, 23; Cl, 35)?

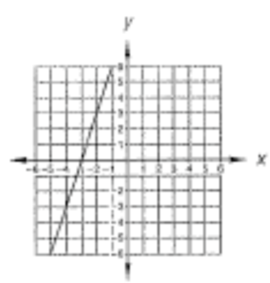
4. (6) Richard and Lynn found three consecutive even integers such that 7 times the sum of the first and third was 48 less than 10 times the second. What were the integers?

5. (9) Hadrian's soldiers increased their wall-building speed by 140 percent. If their new speed was 432 inches per day, what was their old wall-building speed?

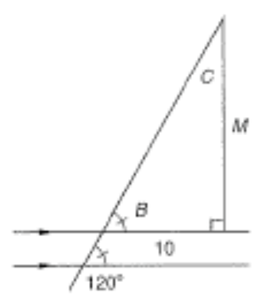
Solve by completing the square:

6. (50) $x^2 + 8x - 4 = 0$ **7.** (50) $12x + x^2 - 5 = 0$ **8.** (50) $x^2 = 7x - 3$

9. (45) Find the equation of this line.



10. (45) Find angle C and side M.



Solve:

11. (48) $\sqrt{x^2 - 4x + 20} = x + 2$ **12.** (48) $-5 = -\sqrt{x + 5} + 1$

13. (47) Use unit multipliers to convert 400 yards per second to miles per hour.

Simplify:

14. (46) $\sqrt[5]{2\sqrt[3]{2}}$ **15.** (47) $\sqrt{9\sqrt{3}}$

16. (46) $\sqrt{m^3y^5}\sqrt[3]{m^2y^2}$ **17.** (46) $5\sqrt{\frac{3}{11}} + 2\sqrt{\frac{11}{3}} - \sqrt{297}$

18. (42) Estimate: $\dfrac{(746{,}800 \times 10^{14})(703{,}916 \times 10^{4})}{500{,}000}$

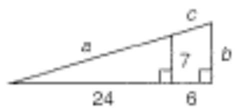
Algebra 2, Lesson 50

Sample taken from Algebra 2 (Third Edition), page 221

19. (40) Find c: $\frac{mxc}{p} - k = \frac{2}{r}$

20. (40) Find p: $\frac{4}{x} - \frac{3x}{p} = \frac{c}{m}$

21. (20) Find c.



22. (36) Divide $4x^3 + 3x + 5$ by $2x - 3$.

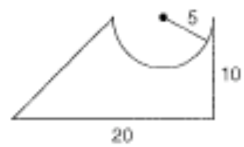
23. (38) Solve $x^3 - 28x = 3x^2$ by factoring.

Simplify:

24. (36) $\frac{x^2 + 49 - 14x}{x^3 - 13x^2 + 42x} \cdot \frac{x^3 - 4x^2 - 12x}{-35 - 2x + x^2}$

25. (35) $-49^{3/2}$

26. (9) The volume of a prism whose base is shown is $(600 - 50\pi)$ cm^3. Find the height of the prism. Dimensions are in centimeters.



27. (31) Find the equation of the line that passes through $(-2, 5)$ and is perpendicular to the line $3x - 5y = 2$.

28. (10) Find the distance between the points $(-3, 5)$ and $(-3, 7)$.

29. (4) Solve: $-2(x^0 - x - 3) - |-2^0 - 3| = x - 2(-2 - 4)$

30. (4) Multiply: $\frac{x^2y^{-2}}{z^2p^0}\left(\frac{4p^0x^{-5}z^2}{y^{-2}} - \frac{3x^{-4}y^2}{z^{-2}p}\right)$

Algebra 2, Lesson 81

Sample taken from Algebra 2 (Third Edition), page 335

LESSON 81 Complex Numbers and Real Numbers • Products of Complex Conjugates • Division of Complex Numbers

81.A complex numbers and real numbers

A complex number is a number that has a real part and an imaginary part. When the real part is written first, we say that we have written the complex number in **standard form.** Thus, the general expression for a complex number in standard form is

$$a + bi$$

The letter a can be any real number, and the letter b can be any real number. All of these numbers

$$\text{(a)}\ -\sqrt{2} + 3i \qquad \text{(b)}\ -\frac{4\sqrt{3}}{5} + 2\sqrt{3}i \qquad \text{(c)}\ 3 - \frac{23}{\sqrt{2}}i$$

are complex numbers in standard form, because all the replacements for a and b are real numbers. If a equals zero, the number does not have a real part. Thus, the following numbers are complex numbers whose real parts equal zero.

$$\text{(d)}\ +3i \qquad \text{(e)}\ +2\sqrt{3}i \qquad \text{(f)}\ -\frac{23}{\sqrt{2}}i$$

If the coefficient of the imaginary part of a complex number is zero, we get a complex number that has only a real part, such as the following:

$$\text{(g)}\ -\sqrt{2} \qquad \text{(h)}\ -\frac{4\sqrt{3}}{2} \qquad \text{(i)}\ 3$$

Thus, we see that every real number is a complex number whose imaginary part is zero, and every imaginary number is a complex number whose real part is zero. **Thus, the set of real numbers is a subset of the set of complex numbers, and the set of imaginary numbers is also a subset of the set of complex numbers.**

The complex number

$$\frac{4 - 3i}{5}$$

is not in standard form, because it is not in the form $a + bi$. However, it takes only a slight change to write it in standard form as

$$\frac{4}{5} - \frac{3}{5}i$$

81.B products of complex conjugates

We have noted that the product of a two-part number and its conjugate has the form $a^2 - b^2$.

$$\begin{array}{lcl} a + b & \qquad & 3 + 5\sqrt{2} \\ \underline{a - b} & & \underline{3 - 5\sqrt{2}} \\ a^2 + ab & & 9 + 15\sqrt{2} \\ \underline{\quad\ - ab - b^2} & & \underline{\quad - 15\sqrt{2} - 50} \\ a^2 \qquad\ - b^2 & & 9 \qquad\quad - 50 \end{array}$$

Algebra 2, Lesson 81

Sample taken from Algebra 2 (Third Edition), page 336

If we multiply a complex number in standard form by its conjugate, we get an answer in the form $a^2 + b^2$. The sign change between a^2 and b^2 is caused by the presence of an i^2 factor in the second part of the product.

$$\begin{array}{l} a + bi \\ \underline{a - bi} \\ a^2 + abi \\ \underline{\quad - abi - b^2i^2} \\ a^2 \qquad\quad + b^2 \end{array} \qquad\qquad \begin{array}{l} 3 + 4i \\ \underline{3 - 4i} \\ 9 + 12i \\ \underline{\quad - 12i - 16i^2} \\ 9 \qquad\quad + 16 \end{array}$$

We note that neither product has an imaginary part. We will find that we can use this fact to eliminate the i factor in the denominator of a fraction of complex numbers.

81.C division of complex numbers

The notation

$$\frac{x^2 - 2x + 7}{x + 3}$$

indicates that $x^2 - 2x + 7$ is to be divided by $x + 3$. There is a format and a procedure we can use to perform this division.

$$\begin{array}{r} x - 5 \\ x + 3\overline{)x^2 - 2x + 7} \\ \underline{x^2 + 3x} \quad\;\; \\ - 5x + 7 \\ \underline{- 5x - 15} \\ 22 \end{array}$$

By using this procedure, we find that $\frac{x^2 - 2x + 7}{x + 3}$ equals $\mathbf{x - 5 + \frac{22}{x + 3}}$.

If we encounter the expression

$$\frac{2 + 3i}{4 - 2i}$$

we see that this notation indicates that $2 + 3i$ is to be divided by $4 - 2i$. **Unfortunately, there is no simple format or procedure that we can use to perform this division. However, we can change the form of the expression by multiplying above and below by the conjugate of the denominator. The resulting expression will have a denominator that is a real number.**

example 81.1 Simplify: $\frac{2 + 3i}{4 - 2i}$

solution We can change the denominator to a rational number if we multiply above and below by $4 + 2i$, which is the **conjugate of the denominator.**

$$\frac{2 + 3i}{4 - 2i} \cdot \frac{4 + 2i}{4 + 2i}$$

We have two multiplications indicated, one above and one below. We will use the vertical format for each multiplication.

$$\begin{array}{l} \text{Above} \\ 2 + 3i \\ \underline{4 + 2i} \\ 8 + 12i \\ \underline{\quad + 4i + 6i^2} \\ 8 + 16i - 6 \quad = 2 + 16i \end{array} \qquad\qquad \begin{array}{l} \text{Below} \\ 4 - 2i \\ \underline{4 + 2i} \\ 16 - 8i \\ \underline{\quad + 8i - 4i^2} \\ 16 \qquad\; + 4 \quad = 20 \end{array}$$

Algebra 2, Lesson 81

Sample taken from Algebra 2 (Third Edition), page 337

Thus, we can write our answer as

$$\frac{2 + 16i}{20} = \frac{1 + 8i}{10}$$

This answer is not in the preferred form of $a + bi$. We can write this complex number in standard form if we write

$$\mathbf{\frac{1}{10} + \frac{4}{5}i}$$

example 81.2 Simplify: $\dfrac{4 - 2i}{2i - 3}$

solution Although it is not necessary, we will begin by writing the denominator in standard form as

$$\frac{4 - 2i}{-3 + 2i}$$

We can change the denominator to a real number if we multiply above and below by $-3 - 2i$.

$$\frac{4 - 2i}{-3 + 2i} \cdot \frac{-3 - 2i}{-3 - 2i}$$

We have two multiplications to perform.

ABOVE	BELOW
$4 - 2i$	$-3 + 2i$
$\underline{-3 - 2i}$	$\underline{-3 - 2i}$
$-12 + 6i$	$9 - 6i$
$\underline{\quad - 8i + 4i^2}$	$\underline{\quad + 6i - 4i^2}$
$-12 - 2i - 4 = -16 - 2i$	$9 \quad + 4 = 13$

Thus, the new form of the expression is

$$\frac{-16 - 2i}{13}$$

which can be written in standard form as follows:

$$\mathbf{-\frac{16}{13} - \frac{2}{13}i}$$

practice Simplify:

a. $\dfrac{2 + 3i}{3 - 3i}$ **b.** $\dfrac{2 - 2i}{2i - 2}$

problem set 81

1. (80) Monkeys varied directly as turtles squared. When there were 2 turtles, there were 100 monkeys. How many monkeys were there when there were 5 turtles? Work the problem once using the direct variation method and again using the equal ratio method.

2. (80) The number of macaws varied inversely as the number of apes squared. When there were 4 macaws, there were 10 apes. How many macaws were there when there were only 2 apes?

3. (74) Roger made the 375-mile trip in 10 hours less than it took Judy. This was because he traveled 3 times as fast as Judy traveled. How fast did each travel, and for how long did each travel?

4. (52) Wittlocoodee has one solution that is 10% glycol and another that is 40% glycol. How much of each should she use to get 200 liters of solution that is 19% glycol?

5. (5) The curmudgeon chortled with glee when the results were announced, because only 60% had made it. If 1120 had not made it, how many had tried?

Algebra 2, Lesson 81

Sample taken from Algebra 2 (Third Edition), page 338

Simplify:

6. $\dfrac{3 - i}{2 + 5i}$

7. $\dfrac{3 - 2i}{2i - 4}$

8. Find an angle whose supplement is 30° greater than 4 times its complement.

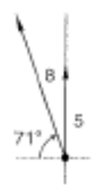
Solve:

9. $\sqrt{x^2 - x + 30} - 3 = x$

10. $\sqrt{p - 48} = 12 - \sqrt{p}$

11. $\begin{cases} x + 2y - 3z = 5 \\ 2x - y - z = 0 \\ y - 3z = 0 \end{cases}$

12. The two forces act on a point as shown. Find the resultant force.



13. Add: $\dfrac{7x - 2}{x^2 - 9} + \dfrac{3x}{3 - x}$

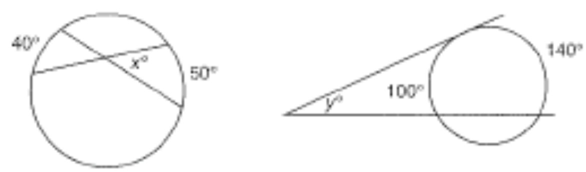
Simplify:

14. $\dfrac{\sqrt{2} - 5}{\sqrt{2} - 2}$

15. $\dfrac{2\sqrt{3} - 1}{1 - 3\sqrt{3}}$

16. $\dfrac{1 + \sqrt{2}}{3 - \sqrt{2}}$

17. Find x and y.



18. Begin with $ax^2 + bx + c$ and derive the quadratic formula by completing the square.

Simplify:

19. $2a^2 - \dfrac{3a}{a + \dfrac{1}{a}}$

20. $\sqrt{-9} - \sqrt{-2}\sqrt{-2} + \sqrt{-2}\sqrt{2} - 3i^3 - 2i^2$

21. Solve: $\begin{cases} \dfrac{2}{3}x - \dfrac{1}{3}y = 6 \\ 0.15x + 0.01y = 0.84 \end{cases}$

22. Solve $2 = -2x^2 - 3x$ by using the quadratic formula.

23. Divide $4x^3 - x + 2$ by $x - 4$.

Algebra 2, Lesson 81

Sample taken from Algebra 2 (Third Edition), page 339

24. (79) Find x and y.

Simplify:

25. (47) $3\sqrt{9\sqrt[4]{3}}$

26. (40) $2\sqrt{\frac{1}{5}} - 3\sqrt{5} + 3\sqrt{20}$

27. (27) Add: $\frac{x}{x + y} + \frac{3}{x^2y} + \frac{2}{xy}$

28. (9) Solve: $-4^2 - 3^0 - 2^0(x - x^0) - 3^0(-2x - 5) = 7$

29. (70) Find the distance between $(-2, 8)$ and $(5, -3)$.

30. (65) Use a calculator to simplify. Estimate first.

(a) $\frac{0.5061 \times 10^5}{0.0071643 \times 10^{-18}}$ (b) $\sqrt[6.2]{594}$

Algebra 2, Lesson 103

Sample taken from Algebra 2 (Third Edition), page 426

LESSON 103 Advanced Polynomial Division

We can divide polynomials that have more than one variable by using the same method that we use when only one variable is present.

example 103.1 Divide $x^3 + y^3$ by $x + y$.

solution We use the format for long division.

$$x + y\,\overline{)\,x^3 + y^3}$$

x^3 divided by x is x^2, so we record an x^2 above

$$\begin{array}{rl} & x^2 \\ x + y & \overline{)\,x^3 + y^3} \end{array}$$

and multiply x^2 by $x + y$ and record.

$$\begin{array}{rl} & x^2 \\ x + y & \overline{)\,x^3 \qquad + y^3} \\ & \underline{\;x^3 + x^2y\;} \end{array}$$

Now we mentally change the signs and add.

$$\begin{array}{rl} & x^2 \\ x + y & \overline{)\,x^3 \qquad + y^3} \\ & \underline{\;x^3 + x^2y\;} \\ & \qquad - x^2y \end{array}$$

Now $-x^2y$ divided by x equals $-xy$, so we record $-xy$ above and then multiply and add.

$$\begin{array}{rl} & x^2 - xy \\ x + y & \overline{)\,x^3 \qquad\qquad\quad + y^3} \\ & \underline{\;x^3 + x^2y\;} \\ & \qquad - x^2y \\ & \qquad \underline{- x^2y - xy^2} \\ & \qquad\qquad\quad xy^2 \end{array}$$

Algebra 2, Lesson 103

Sample taken from Algebra 2 (Third Edition), page 427

Finally, xy^2 divided by x equals y^2. We record a y^2 above and multiply to finish.

$$
\begin{array}{r|l}
 & x^2 - xy + y^2 \\
\hline
x + y & x^3 \qquad\qquad\quad + y^3 \\
 & \underline{x^3 + x^2y} \\
 & \quad - x^2y \\
 & \quad \underline{- x^2y - xy^2} \\
 & \qquad\qquad xy^2 + y^3 \\
 & \qquad\qquad \underline{xy^2 + y^3}
\end{array}
$$

example 103.2 Divide $x^3 - y^3$ by $x - y$.

solution The procedure is the same, and the answer is the same, except that the sign of the middle term is different.

$$
\begin{array}{r|l}
 & x^2 + xy + y^2 \\
\hline
x - y & x^3 \qquad\qquad\quad - y^3 \\
 & \underline{x^3 - x^2y} \\
 & \quad x^2y \\
 & \quad \underline{x^2y - xy^2} \\
 & \qquad\qquad xy^2 - y^3 \\
 & \qquad\qquad \underline{xy^2 - y^3}
\end{array}
$$

practice Use long division to divide:

a. $8x^3 + 64y^3$ by $2x + 4y$ **b.** $8x^3 - 64y^3$ by $2x - 4y$

problem set 103

1. (101) A 60 percent markup of the purchase price was necessary to pay the rent, utilities, and the workers and still make a small profit. If an item sold for \$1424, what did the storekeeper pay for it?

2. (92) Sister Baby's boat could attain a speed of 18 miles per hour on a lake. If the boat took the same time to go 132 miles down the river as it took to go 84 miles up the river, how fast was the current in the river?

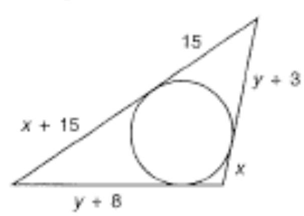
3. (74) Donna took twice as long to drive 720 miles as Maple took to drive 200 miles. Find the rates and times of both if Donna's speed exceeded that of Maple by 40 miles per hour.

4. (37) The initial pressure and temperature of a quantity of an ideal gas was 400 millimeters of mercury and 300 K. If the volume was held constant, what would the final temperature be in kelvins if the pressure was increased to 600 millimeters of mercury?

5. (6) David and Le Van found three consecutive multiples of 11 such that 4 times the sum of the first and third was 66 less than 10 times the second. What were the numbers?

6. (103) Use long division to divide $27x^3 + 8y^3$ by $3x + 2y$.

7. (7) Find x and y. Then find the perimeter of the triangle.



8. (102) Find $ab(2)$ where $a(x) = x - 5$; $D = \{\text{Reals}\}$, and $b(x) = x^2 + 4$; $D = \{\text{Negative integers}\}$.

Complete the square as an aid in graphing:

9. (100) $y = x^2 + 4x + 6$ **10.** (100) $y = -x^2 + 4x - 6$

Algebra 2, Lesson 103

Sample taken from Algebra 2 (Third Edition), page 428

11. Graph on a number line: $x + 3 \geq 5$; $D = \{\text{Reals}\}$

12. Find the number that is $\frac{2}{3}$ of the way from $\frac{1}{4}$ to $2\frac{1}{2}$.

13. Use substitution to solve: $\begin{cases} 4x + 3y = 17 \\ 2x - 3y = -5 \end{cases}$

Solve:

14. $\begin{cases} x^2 + y^2 = 6 \\ x - y = 2 \end{cases}$

15. $\begin{cases} x^2 + y^2 = 10 \\ 2x^2 - 2y^2 = 5 \end{cases}$

16. $\begin{cases} x + 2y + z = -1 \\ 3x - y + z = 6 \\ 2x - 3y - z = 8 \end{cases}$

17. Graph: $\begin{cases} x - 4y \leq -4 \\ x < 3 \end{cases}$

18. Graph on a number line: $-3 \leq x - 3 \not\geq 4$; $D = \{\text{Integers}\}$

Simplify:

19. $\dfrac{(x^{2a-2})^b}{x^{b/2}}$

20. $\dfrac{m}{m^2 + \dfrac{m}{m^2 + \dfrac{1}{m}}}$

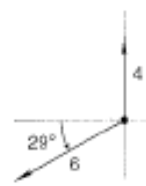
21. $\sqrt[5]{x^2y^3}\sqrt[4]{xy}$

22. $\dfrac{2i^2 + i^3}{i^3 + 2}$

23. $\dfrac{2i - 5}{5i^2 - 2i}$

24. $\dfrac{3 + 2\sqrt{5}}{5 - \sqrt{20}}$

25. The two vectors act on the point as shown. Find the resultant vector.



26. Find x: $a\left(\frac{b}{c} - \frac{1}{x}\right) = \frac{m}{p}$

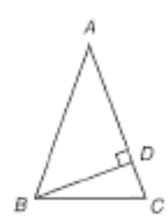
27. Solve: $\sqrt{z} + \sqrt{z + 33} = 11$

Simplify:

28. $3\sqrt{\frac{4}{3}} - 2\sqrt{\frac{3}{4}} + 5\sqrt{48}$

29. $\sqrt{-16} - \sqrt{-2}\sqrt{2}\sqrt{-3}\sqrt{-3} - i^5$

30. In this diagram, $\overline{AB} = \overline{AC}$, angle $\underline{A} = 40°$, and $\overline{BD}$ is perpendicular to $\overline{AC}$ at D. How many degrees are there in angle DBC?



Geometry

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LESSON 15

Introduction to Polygons

Warm Up

1. (2) **Vocabulary** A and B are the ________ of $\overline{AB}$.
2. (3) How many endpoints does a ray have?
3. (13) Classify each of the triangles in the diagram by both sides and angles.

New Concepts

A **polygon** is a closed plane figure formed by three or more segments. Each segment intersects exactly two other segments only at their endpoints. No two segments with a common endpoint are collinear.

The segments that form a polygon are called its **sides**. A **vertex of a polygon** is the intersection of two of its sides.

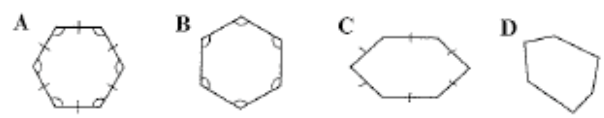
Polygons

Not Polygons

Hint

Equilateral and equiangular polygons have the same traits as equilateral and equiangular triangles, as introduced in lesson 13.

An **equiangular polygon** is a polygon in which all angles are congruent. An **equilateral polygon** is a polygon in which all sides are congruent. If a polygon is both equiangular and equilateral, then it is called a **regular polygon**. If a polygon is not equiangular and equilateral, then it is called an **irregular polygon**.



In the diagram, polygons A and B are equiangular. Polygons A and C are equilateral. Since polygon A is both equiangular and equilateral, it is a regular polygon. Polygons B, C and D are all irregular.

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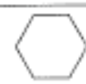
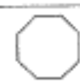
Polygons are named by the number of sides they have. The chart below shows some common polygons and their names.

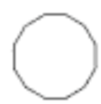
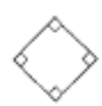
Math Language

An ***n*-gon** is a polygon with *n* sides. Problems may refer to *n*-gons when the number of sides of a polygon is not known, or when a solution is desired for all possible polygons.

Name	Sides	Regular Polygon	Irregular Polygon
Triangle	3		
Quadrilateral	4		
Pentagon	5		
Hexagon	6		
Heptagon	7		
Octagon	8		
Nonagon	9		
Decagon	10		
Hendecagon	11		
Dodecagon	12		

Example 1 Classifying Polygons

Classify each polygon. Determine whether it is equiangular, equilateral, regular, irregular, or more than one of these.

A  **B**  **C**  **D** 

SOLUTION

Polygon **A** has 5 sides, so it is a pentagon. It is equiangular but not equilateral, so it is irregular.

Polygon **B** has 7 sides, so it is a heptagon. It is equilateral and irregular.

Polygon **C** is a dodecagon. It is irregular.

Polygon **D** is a quadrilateral. It is equilateral and equiangular, so it is regular.

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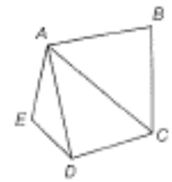
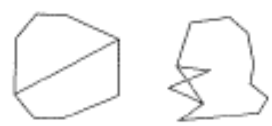
A **diagonal of a polygon** is a segment that connects two nonconsecutive vertices of a polygon. For example, pentagon $ABCDE$ has two diagonals, $\overline{AC}$ and $\overline{AD}$, from vertex A. Three other diagonals could be drawn: $\overline{BD}$, $\overline{BE}$, and $\overline{CE}$.



Math Language

The **exterior of a polygon** is the set of all points outside a polygon. The **interior of a polygon** is the set of all points inside a polygon.

Diagonals can help determine whether a polygon is concave or convex. In a **convex polygon**, every diagonal of the polygon lies inside it, except for the endpoints. In a **concave polygon**, at least one diagonal can be drawn so that part of the diagonal contains points in the exterior of the polygon.

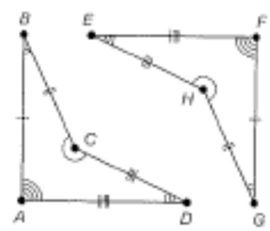


Convex polygon   Concave polygon

If two polygons have the same size and shape, they are **congruent polygons**.

Example 2 Identifying Polygon Properties

a. Find a diagonal that contains points in the exterior of polygon $ABCD$.



SOLUTION

Diagonal $\overline{BD}$ lies outside polygon $ABCD$, except for its endpoints.

b. Determine whether polygon $EFGH$ is convex or concave. Explain.

SOLUTION

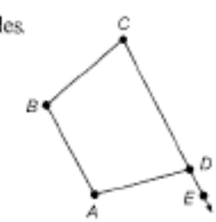
Diagonal $\overline{EG}$ contains points in the exterior of polygon $EFGH$. Therefore, polygon $EFGH$ is concave.

c. Are polygons $ABCD$ and $FGHE$ congruent? Justify your answer.

SOLUTION

Write a congruency statement for all corresponding sides and angles. Angle pairs $\angle A \cong \angle F$, $\angle B \cong \angle G$, $\angle C \cong \angle H$, and $\angle D \cong \angle E$. Sides $\overline{AB} \cong \overline{FG}$, $\overline{BC} \cong \overline{GH}$, $\overline{CD} \cong \overline{HE}$, and $\overline{DA} \cong \overline{EF}$. Therefore, $ABCD \cong FGHE$.

At each vertex of a polygon, there are two special angles. An **interior angle of a polygon** is an angle formed by two sides of a polygon with a common vertex. An **exterior angle of a polygon** is an angle formed by one side of a polygon and the extension of an adjacent side. In the diagram, $\angle CDA$ is an interior angle and $\angle ADE$ is an exterior angle.

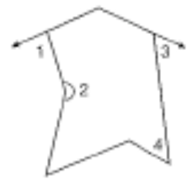


Math Reasoning

Generalize How many interior angles does a convex *n*-gon have? How many exterior angles does a convex *n*-gon have?

Example 3 Identifying Interior and Exterior Angles of Polygons

For each numbered angle in the polygon, determine whether it is an interior angle or an exterior angle.

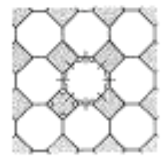


SOLUTION

Angles 2 and 4 are interior. Angles 1 and 3 are exterior.

Example 4 Application: Tile Patterns

This floor tile pattern uses polygonal tiles that fit together exactly.



a. Name the two types of polygons used in the pattern. Are they regular or irregular? Explain.

SOLUTION

Square and octagon; both types are regular, because they have all sides and all angles congruent, respectively.

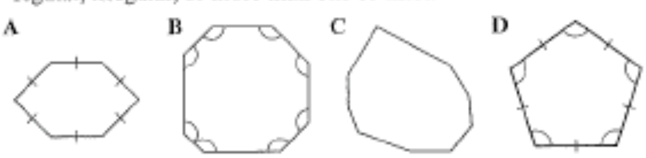
b. Pick any pair of unshaded polygons. Are they congruent? Are they convex or concave? Explain.

SOLUTION

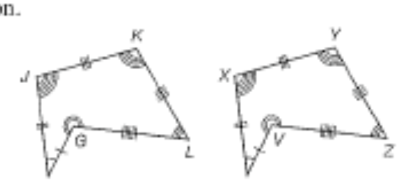
All pairs of unshaded polygons are congruent, because corresponding sides and angles are congruent. Each unshaded polygon is convex, because none of the polygon's diagonals contain points in its exterior.

Lesson Practice

a. Name each polygon. Determine whether it is equiangular, equilateral, regular, irregular, or more than one of these. *(Ex 1)*



b. Find a diagonal in polygon *GHJKL* that contains points in the exterior of the polygon. *(Ex 2)*



c. Determine whether polygon *VWXYZ* is convex or concave. Explain. *(Ex 2)*

d. Are polygons *GHJKL* and *VWXYZ* congruent? Justify your answer. *(Ex 2)*

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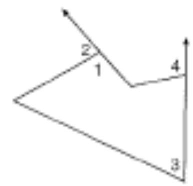
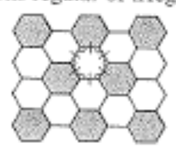
Sample taken from Geometry, page 93

e. (Ex 3) For each numbered angle in the polygon, determine whether it is an interior angle or an exterior angle.



f. (Ex 4) Name the type of polygon used in this pattern. Are the polygons regular or irregular? Explain.



g. (Ex 4) Pick any pair of polygons in this pattern. Are they congruent? Are they convex or concave? Explain.

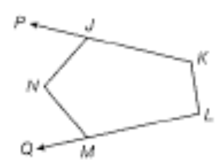
Practice Distributed and Integrated

1. (14) **Write** Explain why the pair of numbers 3 and 3 is a counterexample to this statement. *If the sum of two numbers is even, then both numbers are even.*

2. (5) **Agriculture** A farm is being divided so that each section of land has equal access to the canal running through the property for watering crops. If the road on the opposite side of the property runs parallel to the canal, explain how this can be done.

3. (8) **Wallpaper** A family wants to install wallpaper around the bottom half of a room. If the room has 12-foot tall ceilings and each wall is 14 feet long, calculate the area the wallpaper will cover.

4. (15) In polygon *JKLMN*, name each angle and identify it as an interior or exterior angle.



Write a conditional statement from each sentence.

5. (10) The absolute value of a number is a nonnegative number.

6. (10) A bilingual person speaks two languages.

7. (11) Determine the midpoint M of $\overline{XY}$ connecting $X(0, 2)$ and $Y(6, 1)$.

8. (14) **Model** Find a counterexample to this conjecture.
If two lines intersect, then any third coplanar line intersects both of them.

9. (7) Use inductive reasoning to determine the pattern in the following sequence:

2, 3, 5, 9, 17, 33, 65

10. (2) Identify the property that justifies this statement:
$a = b$ *and* $b = c$*, so* $a = c$.

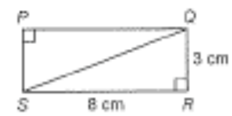
Geometry, Lesson 15

Sample taken from Geometry, page 94

11. (Inv 1) Name all the pairs of angles that are congruent when a transversal cuts a pair of parallel lines.

12. (9) Find the length of the segment connecting (1.3, 4.1) and (2.8, 6.1).

13. (13) **Verify** Rectangle *PQRS* is divided into two triangles by diagonal $\overline{QS}$.



a. Determine the area of rectangle *PQRS*.

b. Given that $\triangle PQS$ and $\triangle QRS$ have equal areas, use your answer to part **a** to determine the area of $\triangle PQS$.

c. Verify that the formula for the area of a triangle gives the same answer as part **b** for the area of $\triangle PQS$.

14. (15) Classify this polygon. Is it equiangular? Is it equilateral? Is it regular?

15. (12) **Home Renovation** Tasha is using two wooden rails to construct a pair of stair rails along the walls of a staircase. To make sure that the rails are parallel, she measures the acute angle each rail makes with the vertical edge of the wall at the base of the stairs.

a. What type of angles are these?

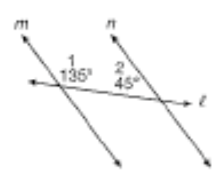
b. Tasha measures each angle to be 42°. Explain how she can make sure that the rails are parallel.

16. (14) **Verify** Confirm that this quadrilateral is a counterexample to the conjecture.

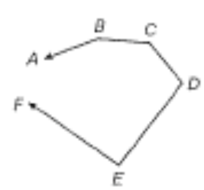
If a quadrilateral has two pairs of congruent sides, then both pairs of opposite sides are parallel.

17. (Inv 1) If two parallel lines are cut by a transversal and one pair of same-side interior angles has angles that measure $(10x + 90)°$ and $(4x + 6)°$, what is the measure of each angle?

18. (12) **Write** In this figure, the transversal line ℓ intersects lines m and n. Write a paragraph explaining how you know that m and n are parallel.



19. (15) This figure shows a polygon with one vertex and two sides missing.



a. Copy the figure and add a point *G* that makes *ABCDEFG* concave.

b. Make a second copy of the figure and add a point *H* that makes *ABCDEFH* convex.

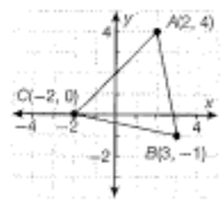
Geometry, Lesson 15

Sample taken from Geometry, page 95

20. (Inv 1) Name every pair of corresponding angles in the diagram.

21. (8) **Building** A builder wants to add a diagonal beam to add support to a structure. The beam needs to extend across a height of 15 feet and a distance of 28 feet. What will the length of the beam be to the nearest hundredth of a foot?

22. (11) Determine the midpoint of each side of $\triangle ABC$.



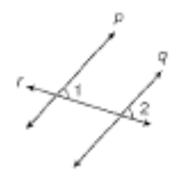
23. (13) **Algebra** The height of a triangle is 12.7 centimeters and its area is 31.75 square centimeters. Use the Triangle Area Formula to determine its base length.

24. (4) **Multiple Choice** Which statement is always true?

A Two planes intersect in a straight line.
B Two lines are contained in exactly one plane.
C Two lines can intersect at two points.
D Any four points can be contained in exactly one plane.

25. (12) **Multiple Choice** If $\angle 1$ and $\angle 2$ are congruent, which of these should be used to prove that lines p and q are parallel?

A Converse of the Alternate Exterior Angles Theorem
B Converse of the Alternate Interior Angles Theorem
C Converse of the Corresponding Angles Postulate
D Converse of the Same-Side Interior Angles Theorem



26. (7) **Predict** Use inductive reasoning to find the next term in this sequence. Explain the rule for the pattern.

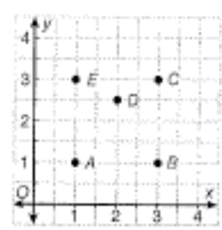
2, 3, 5, 9, 17, …

27. (8) **Multi-Step** Find the perimeter of a square if its area is 289 square centimeters.

28. (Inv. 1) If two parallel lines are intersected by a transversal, what is the sum of the measures of all four interior angles that are formed?

29. (8) Find the perimeter of a regular hexagon with side lengths of 6.8 inches

30. (15) If the points A, B, C, D, and E are connected, is polygon $ABCDE$ convex or concave?



Geometry, Lesson 54

Sample taken from Geometry, page 355

LESSON 54

Representing Solids

Warm Up

1. **Vocabulary** A prism with six square faces is called a ______. (49)
2. Name each of the pictured solids. If the solid is a prism or pyramid, classify it. (49)

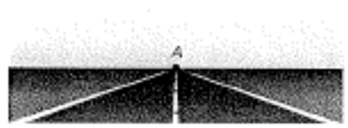
3. According to Euler's Formula, if a polyhedron has 7 faces and 10 vertices, how many edges does it have? (49)

New Concepts

In a **perspective drawing**, nonvertical parallel lines appear to meet at a point called a vanishing point. If you look straight down a highway, it appears that the edges of the highway eventually come together at a vanishing point, like point A in the diagram. In a perspective drawing, the **horizon** is the horizontal line that contains the vanishing point(s). A drawing with just one vanishing point is called **one-point perspective**.

Math Language

The **vanishing point** is the point in a perspective drawing on the horizon where parallel lines appear to meet.



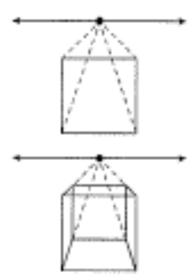
Example 1 **Drawing in One-Point Perspective**

Draw a rectangular prism in one-point perspective. Use a pencil with an eraser.

SOLUTION

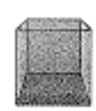
Step 1 Draw a square and a horizontal line above it representing the horizon. Mark a vanishing point on the horizon.

Step 2 Draw a dashed line from the vanishing point to each of the four corners of the square.



Step 3 Using the dashed lines drawn in Step 2, draw the sides of a smaller square.

Step 4 Connect the two squares and erase the reference lines and the horizon that are located behind the prism. This prism is drawn from a one-point perspective.



Online Connection
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Geometry, Lesson 54

Sample taken from Geometry, page 356

A drawing with two vanishing points is said to have two-point perspective. Look at the following example to see how a drawing can be made from a two-point perspective.

Example 2 **Drawing in Two-Point Perspective**

Draw a rectangular prism in two-point perspective in which the vanishing points are above the prism.

SOLUTION

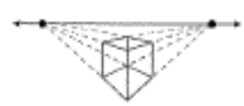
Step 1 Draw a horizontal line that represents the horizon. Place two vanishing points on the horizon. Draw a vertical line segment below the horizontal line and between the two vanishing points, representing the front edge of the prism.

Step 2 Draw dashed lines from each vanishing point to the top and bottom of the vertical line as shown.

Step 3 Draw vertical segments between the dashed lines from Step 2 as shown and draw segments to connect them to the first segment.

Step 4 Draw dashed perspective lines from the segments drawn in Step 3 to each of the vanishing points as shown.

Step 5 Draw a dashed vertical line between the two intersections of the perspective lines just drawn. Sketch the segments that make the top of the prism.



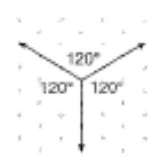
Step 6 Erase the horizon line and the dashed perspective lines. Keep the dashed lines inside the prism that represent the edges that are hidden.

This prism is drawn from a two-point perspective.

Math Reasoning

Model Could you also make a two-point perspective drawing by placing the vanishing points below the original line segment?

An **isometric drawing** is a way of drawing a three-dimensional figure using isometric dot paper, which has equally spaced dots in a repeating triangular pattern. The drawings can be made by using three axes that intersect to form 120° angles, as shown in the diagram.

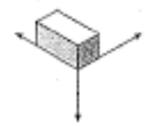


Geometry, Lesson 54

Sample taken from Geometry, page 357

Example 3 **Creating Isometric Drawings**

Create an isometric drawing of a rectangular prism.



SOLUTION

Draw the three axes on the isometric dot paper as shown above. Use this vertex as the bottom corner of the prism. Draw the box so that the edges of the prism run parallel to the three axes. Shading the top, front, and side of the prism will add the perception of depth.

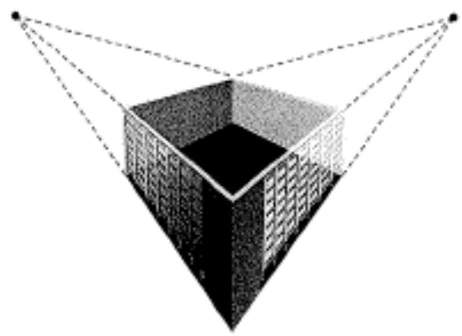
Example 4 **Application: Drafting**

Hint

In a two-point perspective drawing, it appears that one edge of the solid is the front of the diagram. In a one-point perspective drawing, it appears that a face of the solid is the front.

An architecture firm is planning to construct a rectangular building on a corner lot. The client would like a drawing that shows the building as though someone is looking at it from one edge. Should the drawing be from a one-point or two-point perspective? Make a sketch of what the drawing should look like.

SOLUTION



Since the front of the drawing will be an edge of the building, a two-point perspective drawing is appropriate. The diagram shows a completed view of the building.

Lesson Practice

a. Draw a rectangular prism in one-point perspective in which the vanishing point is to the left of the square. *(Ex 1)*

b. Draw a cube in two-point perspective with the vanishing points and horizon below the vertical line. *(Ex 2)*

c. Make an isometric drawing of a triangular prism. *(Ex 3)*

d. *(Ex 4)* **Drafting** Morgan wants to make a wooden bookshelf with two shelves. The bookshelf will be 1 meter wide, 1 meter deep, and 1.5 meters tall. To decide how much wood to buy, Morgan will draw his plans for the bookshelf. Should the drawing be from a one-point or two-point perspective? Sketch what Morgan's drawing should look like.

Geometry, Lesson 54

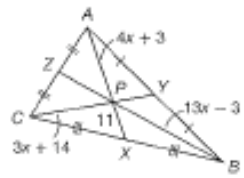
Sample taken from Geometry, page 358

Practice Distributed and Integrated

*1. (54) Draw a triangular prism in one-point perspective so that the vanishing point is below the prism.

2. (53) **Write** Explain why the following statement is true.
If a quadrilateral is a square, then it is a rectangle.

3. (32) **Algebra** Find the length of $\overline{ZP}$ in the diagram.



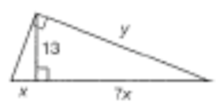
4. (42) What is the shortest distance from (5, 3) to the line $y = -2x + 8$?

*5. (54) (**Architecture**) An architect is creating different perspective drawings for a new building. The building is a rectangular prism and the client would like a drawing that focuses on the front façade of the building. Should the architect create the drawing using a one-point or two-point perspective? Sketch a sample drawing of the building.

6. (49) A figure has a hexagonal base and triangular lateral faces. Classify the figure.

7. (37) **Multi-Step** Graph the line and find the slope of the line that passes through the points $L(4, 1)$ and $M(3, -1)$. Then find a perpendicular line that passes through point $N(-2, -2)$.

8. (38) Find the value of x and y in the triangle shown to the nearest tenth.



9. (Inv 3) What is the sum of the exterior angles of a convex 134-sided polygon?

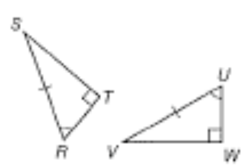
10. (53) Is the following statement always, sometimes, or never true?
A parallelogram is a rectangle.

*11. (54) Trace the figure at right on your paper. Then locate the vanishing point and the horizon line.

12. (30) **Algebra** In $\triangle ABC$, $m\angle ABC = 90°$, $AB = (3x - 7)$, and $m\angle BCA = 60°$, and in $\triangle DEF$, $m\angle DEF = 90°$, $DE = (5x - 17)$, and $m\angle EFD = 60°$. What value of x will make $\triangle ABC \cong \triangle DEF$?

13. (32) The point where three or more lines intersect is the ________________.

*14. (36) Use the Hypotenuse-Angle Congruence Theorem to prove that $\triangle RST \cong \triangle UVW$.

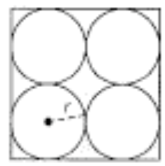


*15. (53) Find the exact length of the hypotenuse of a 45°-45°-90° right triangle with a leg that is 57 feet long.

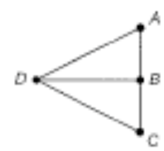
Geometry, Lesson 54

Sample taken from Geometry, page 359

16. (49) **Formulate** Four congruent circles are cut out of a square as shown. Write an expression for the area of the shaded region in terms of the radius of each circle, r.



17. (31) **Aviation** Four jet aircraft are flying in a triangular formation. Jets A, B, and C form a line perpendicular to the flight heading, while jet B is midway between the other two. Jet D flies directly in front of jet B. If $m\angle ADB = 37°$, what does the vertex angle of the triangular formation measure? Which theorem did you use?



***18.** (48) Use an indirect proof to prove that if two altitudes, $\overline{BX}$ and $\overline{CY}$ of $\triangle ABC$ are congruent, then the triangle must be isosceles.
Given: $\overline{BX} \cong \overline{CY}$, $\overline{BX}$ and $\overline{CY}$ are altitudes.
Prove: $\triangle ABC$ isosceles.

***19.** (53) Find the area, to the nearest hundredth, of a 45°-45°-90° right triangle with a hypotenuse of 17 centimeters.

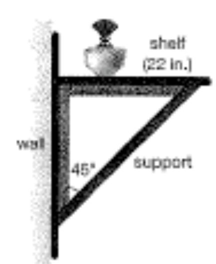
20. (43) **Algebra** If a chord perpendicular to a radius cuts the radius in two pieces that are 7 and 2 inches long, respectively, what are the two possible lengths of the chord to the nearest tenth?

***21.** (54) **Justify** How does a two-point perspective differ when the vanishing points are located close together compared with when they are located further apart? Justify your reasoning with drawings.

22. (50) Find the geometric mean of $\sqrt{2}$ and 5.

23. (34) **Multiple Choice** If the diagonals of parallelogram $JKLM$ intersect at P, which of the following is true?
A $JP = LP$ **B** $JP = KP$
C $JL = KM$ **D** $JM = KM$

***24.** (53) **Construction** The support of a shelf forms a 45°-45°-90° right triangle, with the shelf and the wall as the legs. Exactly how long is this support?

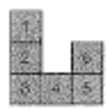


***25.** (44) **Analyze** $\triangle NPQ$ and $\triangle STV$ are similar isosceles triangles. How many of their six sides do you need numerical values for in order find all the other side lengths and the perimeters of both triangles? Explain.

26. (38) Using the diagram on the right, find the length of $\overline{MP}$ if $OP = 5$, $NO = 8$, and $MN = 18$.

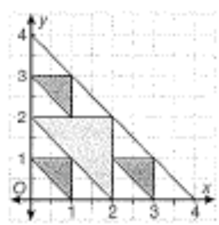
27. (Inv 4) **Cycling** Katya and Sareema start from the same location and bicycle in opposite directions for 2 miles each. Katya turns to her right 90° and continues for another mile. Sareema turns 45° to her left and continues for another mile. At this point, who is closer to the starting point?

28. (Inv 3) **Error Analysis** Darius drew this net of a number cube. Explain his error.



29. (45) **Analyze** Square $RSTU$ has vertices at $R(0, 4)$ and $S(0, 0)$. What are the possible coordinates of T and U?

30. (11) **Design** A white triangle with vertices at (0, 0), (4, 0), and (0, 4) is used to create a logo. A blue triangle is added to the design so that its vertices are the midpoints of the sides of the white triangle. The blue triangle divides the white triangle into three smaller white triangles. Smaller blue triangles are placed in each small white triangle so that their vertices are the midpoints of the sides of the small white triangles.



a. Find the coordinates of the vertices of the large blue triangle.

b. Find the coordinates of the vertices of each small blue triangle.

c. Which of the triangles are congruent, if any? Justify your answer.

Geometry, Lab 8

Sample taken from Geometry, page 387

LAB

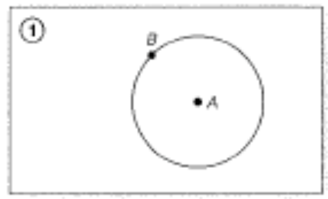
8

Tangents to a Circle

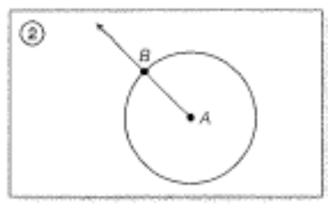
Construction Lab 8 *(Use with Lesson 58)*

Lesson 58 shows you how to identify lines tangent to a circle. This lab demonstrates how to construct lines tangent to a circle through a point on the circle.

1. To construct a tangent line through a given point on the circle, begin with $\odot A$ and a point on the circle, B.

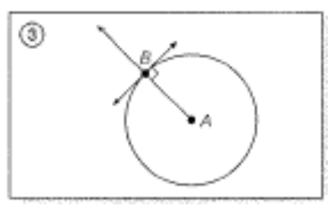


2. Draw $\overrightarrow{AB}$.



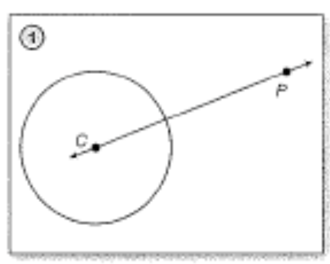
3. Using the method from Construction Lab 2, construct the line perpendicular to $\overrightarrow{AB}$ through B.

   The perpendicular line is tangent to $\odot A$ at B.



This lab demonstrates how to construct lines tangent to a circle through a point not on the circle.

1. To construct two tangent lines through a point external to a circle, draw a circle and label the center C. Choose a point exterior to the circle, and label it P. Draw $\overleftrightarrow{CP}$.



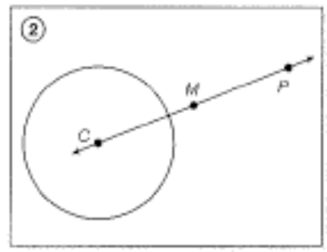
Geometry, Lab 8

Sample taken from Geometry, page 388

Hint

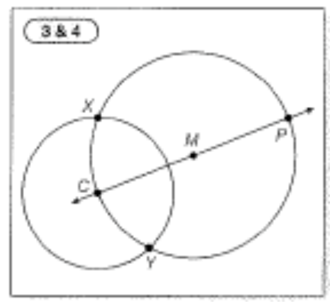
To construct the midpoint, use the method you learned in Construction Lab 3.

2. Construct the midpoint of $\overline{CP}$. Label the midpoint M.

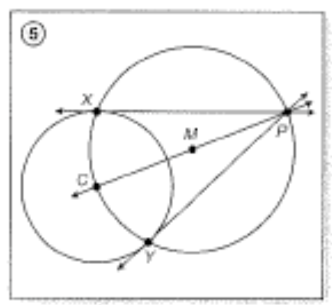


3. Draw a circle with a radius, CM centered on M. Notice that P is also on this circle.

4. Label the points of intersection of $\odot C$ and $\odot M$ as points X and Y.



5. Draw $\overleftrightarrow{XP}$. This line is tangent to $\odot C$ at point X. Draw $\overleftrightarrow{YP}$. Notice that $\overleftrightarrow{YP}$ is tangent to $\odot C$ at point Y.



Lab Practice

Use a compass to draw $\odot D$ and $\odot E$. Draw point A on $\odot D$. Draw points F and G outside, but near to, $\odot D$ and $\odot E$. Perform each construction indicated below.

a. a line tangent to $\odot D$ at point A

b. a line tangent to $\odot D$ from point F

c. a line tangent to $\odot E$ from point F

d. a line tangent to $\odot E$ from point G

Geometry, Investigation 8

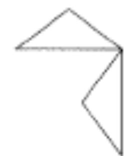
Sample taken from Geometry, page 529

INVESTIGATION

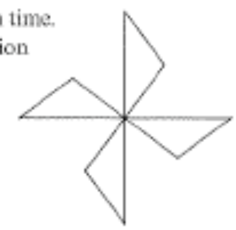
8

Patterns

Finding patterns is a valuable problem-solving skill. In this investigation, you will study patterns made by transforming a figure. The basic figure we will work with is an isosceles triangle like the one shown here. Copy this triangle by connecting the points (0, 0), (8, 0), and (4, 3) on a coordinate plane, and cut the triangle out so you can trace it onto paper.



Trace the cutout triangle onto a blank sheet of paper. First, transform the triangle by rotating it. With the triangle oriented as shown in the diagram above, rotate it 90° counterclockwise around either one of its acute angles. Trace the triangle again in its new position. You should now have a design like the one shown.



Continue to rotate the triangle 90° and trace it each time. Since the triangle will return to its original orientation after four rotations, the final design will look like the one shown.

1. What is the order of the rotational symmetry in the final pattern?

2. Does the final pattern have any lines of symmetry? If so, how many?

Trace your cutout triangle onto a blank sheet of paper. This time you will make a pattern by reflecting the triangle. Draw x- and y-axes and orient the triangle so its base lies on the x-axis and one of its acute angles lies on the origin.

3. Reflect the triangle over the y-axis and draw the resulting pattern.

4. Reflect the pattern from step 3 over the x-axis and draw the resulting pattern.

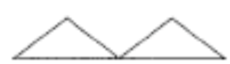
5. Does the final pattern have rotational symmetry? If so, what is the order of rotational symmetry?

6. Does the final pattern have any lines of symmetry? How many?

Math Reasoning

Model Could you have used different transformations of the triangle to obtain the same image as the one in step 4? Explain how.

Translation symmetry is a type of symmetry describing a figure that can be translated along a vector so that the image coincides with the preimage. A **frieze pattern** is a pattern that has translation symmetry along a line.



Trace your cutout triangle onto a blank sheet of paper. Orient the triangle so its base is parallel to the bottom edge of the paper. Translate the triangle to the right until the vertices of the opposite acute angles lie on the same point, as in the figure shown.

Geometry, Investigation 8

Sample taken from Geometry, page 530

Translate the triangle to the right again. Continue this process until you have 4 triangles in a row. This is a frieze pattern.

7. What other transformation(s) could have been used to create this same pattern?

8. Does the final pattern have rotational symmetry? If so, what is the order of rotational symmetry?

9. Does the final pattern have any lines of symmetry? How many?

Now we will explore some geometric patterns. Draw two points. There is only one segment that can be drawn connecting these two points. What about 3 points? Draw 3 noncollinear points and draw segments connecting them. You find that there are three segments that can be drawn.

10. Draw four non-collinear points. Draw a line segment between each pair of points. How many line segments do you have?

11. **Predict** Based on the pattern you have seen so far, predict how many line segments you can draw connecting 5 noncollinear points. What about 6 noncollinear points? … 7 noncollinear points?

12. **Formulate** Write a rule for the number of line segments, L, between n points, in terms of the number of line segments between $n - 1$ points (denoted L_{n-1}).

The numbers in the series you have just discovered are called triangular numbers. Triangular numbers are numbers that are equal to the sum of the first n whole numbers. The first 8 triangular numbers are: 1, 3, 6, 10, 15, 21, 28, and 36.

Can an algebraic expression be written for the n^{th} triangular number? The n^{th} triangular number is given by the formula below.

$x = 1 + 2 + 3 + 4 + \ldots + n - 1 + n$

To find an expression for the n^{th} triangular number, take this series and add it to itself. Instead of adding the terms in order though, add the first term to the last term, the second term to the second to last term, and so on.

$$\begin{array}{r} x = 1 + 2 + 3 + 4 \ldots + n - 1 + n \\ + \; x = 1 + 2 + 3 + 4 \ldots + n - 1 + n \\ \hline 2x = (1 + n) + (2 + n - 1) + (3 + n - 2) + \ldots \end{array}$$

Now notice that in the sum, the expressions in the parenthesis can be simplified. After being simplified, the sum becomes

$2x = (n + 1) + (n + 1) + \ldots$

Each expression in parenthesis is the same. Moreover, we know that the series has n terms. So it can be simplified further, resulting in $2x = n(n + 1)$. To solve for x, which is the n^{th} triangular number, divide by 2.

$$x = \frac{n(n + 1)}{2}$$

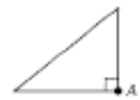
13. Using the expression above, what is the 50^{th} triangular number? What is the 100^{th} triangular number?

Geometry, Investigation 8

Sample taken from Geometry, page 531

Investigation Practice

a. Using the right triangle given here, sketch the result of rotating the figure 90° counterclockwise about the point A.



b. Continue to rotate the triangle 90° until it coincides with itself, sketching the result of each rotation. What is the order of rotational symmetry in the final figure? Does it have any lines of symmetry?

c. Return to the initial figure and reflect it over the vertical leg of the triangle. Then reflect it over the horizontal leg of the triangle. What kind of polygon is the resulting figure?

d. Does the resulting figure have any lines of symmetry? Does it have rotational symmetry?

e. Square numbers are whole numbers that could be the area of a square. The series begins: 1, 4, 9, 16, 25, ... Write an equation to find the n^{th} square number.

f. What is the 30th square number?

Advanced Mathematics

Table of Contents

***Advanced Mathematics,* Lesson 45**

Sample taken from Advanced Mathematics (Second Edition), page 319

LESSON 45 *Conditional Permutations • Two-Variable Analysis Using a Graphing Calculator*

45.A conditional permutations

Many problems about permutations (arrangements in a definite order) have conditions attached that make finding the solutions challenging. There is no formula that can be used, for the conditions seem to be different each time. We will find, however, that drawing a diagram is helpful, so the use of diagrams is recommended.

example 45.1 How many odd counting numbers can be formed from the digits 3, 4, and 5 if no repetition of digits is permitted?

solution We can have one-digit, two-digit, or three-digit odd numbers.

The one-digit odd numbers are 3 and 5 total 2

The last digit in a two-digit odd number is an odd digit. Both 3 and 5 are odd digits, so there are two possible odd choices for the last digit.

$$\boxed{}\boxed{2}$$

The other digits may be even or odd, and two digits remain that can be used as the first digit.

$$\boxed{2}\boxed{2} \longrightarrow 2 \cdot 2 = 4 \qquad \text{total } 4$$

For three-digit numbers, we use the same process and get

$$\boxed{1}\boxed{2}\boxed{2} \longrightarrow 1 \cdot 2 \cdot 2 = 4 \qquad \text{total } 4$$

$$\text{Grand total} = 4 + 4 + 2 = 10$$

Thus, there are **10** possible odd counting numbers that can be formed using 3, 4, and 5 if no repetition of digits is permitted.

example 45.2 Find the number of odd three-digit counting numbers that are less than 600.

solution The statement of the problem indicates that repetition of digits is permissible. The 10 digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, and any of the five odd digits can be used in the last spot.

$$\boxed{}\boxed{}\boxed{5}$$

Advanced Mathematics, Lesson 45

Sample taken from Advanced Mathematics (Second Edition), page 320

The digit 0 cannot be used in the first box because the resulting number would have only two digits. Also 6, 7, 8, or 9 cannot be used in the first box or the number would not be less than 600. Thus, only 1, 2, 3, 4, or 5 can be used in the first box. There are 10 digits possible for the second box, so we have

5	10	5

$\rightarrow 5 \cdot 10 \cdot 5 = 250$

Thus, there are **250** odd three-digit counting numbers less than 600.

example 45.3 Five math books and four English books are on a shelf. How many permutations are possible if the math books must be kept together and the English books must be kept together?

solution If the math books come first, we get

MATH					ENGLISH			
5	4	3	2	1	4	3	2	1

$\rightarrow 5! \times 4! = 2880$

If the English books come first, we get

ENGLISH				MATH				
4	3	2	1	5	4	3	2	1

$\rightarrow 4! \times 5! = 2880$

If we add the two numbers, we get **5760** possible permutations.

example 45.4 How many different four-digit odd counting numbers can be formed if no repetition of digits is permitted?

solution The 10 digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, and five digits are even and five are odd. The number must end with an odd digit, so there are five choices for the last digit.

			5

A four-digit number cannot begin with 0 because if it did, it would be at most a three-digit number. Thus, there are only eight choices left for the first digit.

8			5

Zero can be the second digit, so there are eight choices for this digit and seven choices for the third digit.

8	8	7	5

$\rightarrow 8 \cdot 8 \cdot 7 \cdot 5 = 2240$

Thus, there are **2240** different four-digit odd counting numbers that can be formed if no repetition of digits is permitted.

example 45.5 An elf, a gnome, a fairy, a pixie, and a leprechaun were to sit in a line. How many different ways can they sit if the elf and the gnome insist on sitting next to each other?

solution Let's begin with the elf and the gnome in the first two seats.

E	G	3	2	1
G	E	3	2	1

$\rightarrow 3 \times 2 \times 1 = 6$

$\rightarrow 3 \times 2 \times 1 = 6$

Advanced Mathematics, Lesson 45

Sample taken from Advanced Mathematics (Second Edition), page 321

Now let's put them in the second two seats.

3	E	G	2	1	→	$3 \times 2 \times 1 = 6$
3	G	E	2	1	→	$3 \times 2 \times 1 = 6$

The other possibilities are

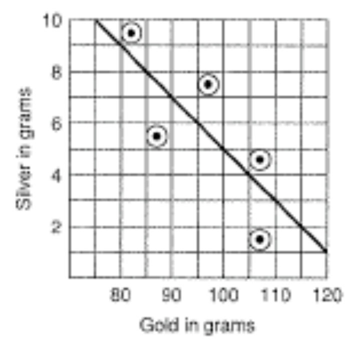
3	2	E	G	1	for 6	3	2	1	E	G	for 6
3	2	G	E	1	for 6	3	2	1	G	E	for 6

There are eight ways the gnome and the elf can sit side by side, and for each of these, there are six ways the other three little people can sit. Since $6 \times 8 = 48$, there are **48** different ways the little people can sit if the elf and the gnome sit next to each other.

45.B two-variable analysis using a graphing calculator

The data in the table on the left are from an experiment involving silver (Ag) and gold (Au). On the right we have graphed the data points and estimated the position of the line indicated by the data points.

Au	82	87	97	107	107
Ag	9.5	5.5	7.5	1.5	4.6



We will use the endpoints of the line (75, 10) and (120, 1) to determine the slope, which we see is negative.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{10 - 1}{75 - 120} = \frac{9}{-45} = -0.2$$

Now we find the intercept.

$\text{Ag} = -0.2\text{Au} + b$	equation
$10 = -0.2(75) + b$	used (75, 10) for Au and Ag
$b = 25$	solved for b

Now we can write the equation that gives silver as a function of gold.

$$\text{Ag} = -0.2\text{Au} + 25$$

The process of estimating the equation of a line that best fits the data is called **linear regression.**†

†This appellation is a misnomer because it has nothing to do with regression in the usual sense. It was introduced by Sir Francis Galton (1822–1911). Originally, he used the word *reversion*, but in an address in 1877 he used the word *regression*. See *Applied Regression Analysis* by Draper and Smith, John Wiley and Sons, 1966.

***Advanced Mathematics*, Lesson 45**

Sample taken from Advanced Mathematics (Second Edition), page 322

Graphing calculators use the *least squares algorithm* to do linear regressions. All you have to do is enter the x data points in one list, enter the y data points in another list, and press the proper key. The TI-82 gives the following display for these data:

```
LinReg
y=ax+b
a=-.2075
b=25.64
r=-.7824726541
```

This tells us that the equation of the line that the calculator estimates as the best fit for the data is

$$\text{Ag} = -0.21\text{Au} + 25.6$$

The last item on the calculator display is r, which is the correlation coefficient. If all the data points lie exactly on the line and the slope of the line is positive, the correlation coefficient r is +1. If all of the data points lie exactly on the line and the slope is negative, the correlation coefficient r is -1. If the data are so scattered that they do not determine a straight line, the correlation coefficient r is 0. For a rule of thumb for scientific data, we would like to have r values between -1 and -0.9 or between 0.9 and 1. For experiments in the social sciences, we would like to have r values between -1 and -0.7 or between 0.7 and 1. Our r value was -0.78. This negative correlation tells us that the amount of silver decreased as the amount of gold increased. We would have hoped for a correlation coefficient between -1 and -0.9. Maybe the relationship is not really linear. We are often not happy with this much scatter in scientific experimental data.

problem set 45

1. How many three-digit counting numbers are there that are less than 300 such that all the digits are even?

2. Six math books and three English books are on a shelf. How many ways can they be arranged if the math books are kept together and the English books are kept together?

3. In the factory k workers worked f hours to produce c articles. If x workers quit, how long would those that remained have to work to produce $c + 10$ articles?

4. The latitude of Princeton, New Jersey, is 40.5° north of the equator. How far is it from Princeton to the equator if the diameter of the earth is 7920 miles?

5. On the 24-mile trip to school, Brandon sauntered at a leisurely pace. Thus, he had to double his speed on the way back to complete the trip in 9 hours. How fast did he travel in each direction, and what were the two times?

6. Four thousand liters of solution was available that was 92% alcohol. How many liters of alcohol had to be extracted so that the solution would be only 80% alcohol?

7. The ratio of greens to blues was 2 to 1, and twice the sum of the number of blues and the number of whites exceeded the number of greens by 10. If there were 35 blues, greens, and whites in all, how many were there of each color?

8. The following data came from an experiment that involved lead (Pb) and copper (Cu). Use a graphing calculator to find the equation of the line which best fits this data and gives copper as a function of lead ($\text{Cu} = m\text{Pb} + b$). Also, find the correlation coefficient for this scientific data and discuss whether or not the line is a good model for the data.

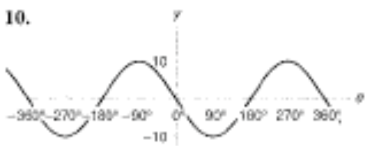
Pb	160	190	190	194	220
Cu	18	6	16	11	4

9. The following data came from an experiment that involved dysprosium (Dy) and rhodium (Rh). Use a graphing calculator to find the equation of the line which best fits this data and gives rhodium as a function of dysprosium (Rh = mDy + b). Also, find the correlation coefficient for this scientific data and discuss whether or not the line is a good model for the data.

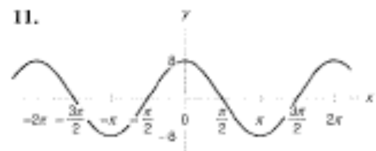
Dy	110	120	130	140	150	160
Rh	80	92	99	105	113	120

Write the equations of the following sinusoids:

10.



11.



12. Find the standard form of the equation of a circle whose center is (−2, 5) and whose radius is 6.

13. Sketch the graph of the function $f(x) = \left(\frac{1}{2}\right)^{-x+1}$.

Evaluate:

14. $\csc \frac{3\pi}{4} - \sec\left(-\frac{5\pi}{6}\right) + \cos \frac{9\pi}{4}$

15. $\sec\left(-\frac{19\pi}{6}\right) + \cos \frac{7\pi}{2} - \sin \frac{10\pi}{3}$

16. By how much does ${}_6P_3$ exceed ${}_6P_2$?

17. Simplify: $\log_3 9 - \log_5 5^3 + \log_7 7^2 - \log_{11} 1$

Solve for x:

18. $\log_5 7 + \log_5 8 = \log_5 (2x - 4)$

19. $\log_3 (x + 1) - \log_3 x = \log_3 15$

20. $\frac{3}{4} \log_{10} 10{,}000 = x$

21. Determine if $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x}$ are inverse functions by computing their compositions.

22. Write the quadratic equation with a lead coefficient of 1 whose roots are $2 + \sqrt{5}$ and $2 - \sqrt{5}$.

23. Find the equation of the line that is equidistant from the points (−4, −3) and (4, 6). Write the equation in slope-intercept form.

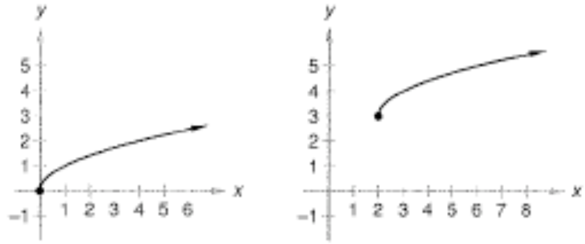
24. Find f where $g(x) = x^3$ and $(f \circ g)(x) = 2x^3 + 3$.

25. Find the domain of the function $f(x) = \frac{\sqrt{x}}{1 - |4x|}$.

***Advanced Mathematics*, Lesson 45**

Sample taken from Advanced Mathematics (Second Edition), page 324

26. The graph of the function $f(x) = \sqrt{x}$ is shown on the left below. The graph on the right is the same graph translated two units to the right and three units up. Write the equation of the graph on the right.



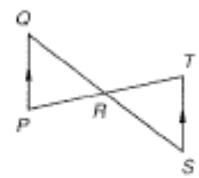
27. Given the function $f(x) = \sqrt{x}$, write the equation of the function g whose graph is the graph of f translated three units to the right and two units up. Then sketch the graph of the new function g.

28. Given: $\overline{PQ} \parallel \overline{ST}$

R is the midpoint of $\overline{QS}$

Write a two-column proof to prove:

$\overline{PR} \cong \overline{TR}$



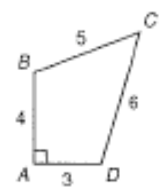
CONCEPT REVIEW PROBLEMS

29. Let x and y be real numbers. If $x < 1$ and $y > 3$, compare: A. $x + 1$ B. $y - 1$

30. In polygon $ABCD$, $m\angle A = 90°$.

Compare: A. Area of polygon $ABCD$

B. 16



Advanced Mathematics, Lesson 79

Sample taken from Advanced Mathematics (Second Edition), page 480

LESSON 79 De Moivre's Theorem • Roots of Complex Numbers

79.A De Moivre's theorem

We remember that when we multiply two complex numbers written in polar form, the absolute values are multiplied together and the angles are added. Therefore,

$$(r_1 \text{ cis } \theta_1)(r_2 \text{ cis } \theta_2) = (r_1 r_2) \text{ cis } (\theta_1 + \theta_2)$$

If $z = r \text{ cis } \theta$, we apply this rule to get

$$z^2 = (r \text{ cis } \theta)(r \text{ cis } \theta) = r^2 \text{ cis } 2\theta$$

$$z^3 = (r \text{ cis } \theta)(r \text{ cis } \theta)(r \text{ cis } \theta) = r^3 \text{ cis } 3\theta$$

$$z^4 = (r \text{ cis } \theta)(r \text{ cis } \theta)(r \text{ cis } \theta)(r \text{ cis } \theta) = r^4 \text{ cis } 4\theta$$

If we repeatedly multiply z by itself, we find that

$$z^n = (r \text{ cis } \theta)^n = r^n \text{ cis } n\theta$$

where n is a positive integer. This fact, considered to be one of the most important in the study of complex numbers, is known as **De Moivre's theorem.** This theorem is named after Abraham De Moivre (1667–1754), a French refugee who lived in London.

example 79.1 Find $(2 \text{ cis } 30°)^5$.

solution By De Moivre's theorem,

$$(2 \text{ cis } 30°)^5 = 2^5 \text{ cis } (5 \cdot 30°) = \mathbf{32 \text{ cis } 150°}$$

example 79.2 Use De Moivre's theorem to find $(1 + i)^{13}$. Express the answer in rectangular form.

solution To use De Moivre's theorem, we first need to put our complex number into polar form. We compute r and θ.

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$\text{Tan } \theta = \frac{1}{1} = 1$ and the number lies in the first quadrant, so

$$\theta = 45°$$

Thus, $1 + i = \sqrt{2} \text{ cis } 45°$. Applying De Moivre's theorem and remembering that cis of an angle equals cis of an integer multiple of 360° added to that angle, we get

$$\begin{aligned}(1 + i)^{13} &= (\sqrt{2} \text{ cis } 45°)^{13} \\ &= (\sqrt{2})^{13} \text{ cis } (13 \cdot 45°) \\ &= 2^{13/2} \text{ cis } (13 \cdot 45°) \\ &= 2^6 2^{1/2} \text{ cis } 585° \\ &= 64\sqrt{2} \text{ cis } (360° + 225°) \\ &= 64\sqrt{2} \text{ cis } 225°\end{aligned}$$

Converting back to rectangular form gives us the final answer.

$$\begin{aligned}64\sqrt{2} \text{ cis } 225° &= 64\sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) \\ &= \mathbf{-64 - 64i}\end{aligned}$$

Advanced Mathematics, Lesson 79

Sample taken from Advanced Mathematics (Second Edition), page 481

79.B roots of complex numbers

If we use 2 as a factor three times, the product is 8.

$$2 \cdot 2 \cdot 2 = 8$$

This is the reason we say that a third root of 8 is 2. Also, we can write

$$\sqrt[3]{8} = 2$$

Because the notation $8^{1/3}$ means the same thing as $\sqrt[3]{8}$, this expression also has a value of 2.

$$8^{1/3} = 2$$

There are really three different cube roots of 8, two of which are complex. In fact, every nonzero number has n distinct roots. In this section, we will learn how to determine all n roots of a number. Above, when we used the cube root notation, $\sqrt[3]{\ }$, and wrote $8^{1/3}$, we understood the answer to be 2 instead of either of the complex third roots. This is because, by convention, we assume that the nth root notation when applied to a real number refers to the real nth root. Should there be a positive and negative choice of real roots, the nth root notation refers to the positive real root. For example, the number 4 has two square roots, but when we write $\sqrt{4}$, we mean the positive square root, 2.

Now we will illustrate how to find all the roots of a complex number expressed in polar form. Suppose we use the complex number 2 cis 12° as a factor three times. The product is 8 cis 36°

$$\text{(a)}\quad (2 \text{ cis } 12°)(2 \text{ cis } 12°)(2 \text{ cis } 12°) = 8 \text{ cis } 36°$$

because we multiply complex numbers in polar form by multiplying the numerical coefficients and adding the angles. Thus, a third root of 8 cis 36° is 2 cis 12°.

There are two other third roots of 8 cis 36°.

$$\begin{aligned}\text{(b)}\quad (2 \text{ cis } 132°)(2 \text{ cis } 132°)(2 \text{ cis } 132°) &= 8 \text{ cis } 396° = 8 \text{ cis } (360° + 36°)\\ &= 8 \text{ cis } 36°\end{aligned}$$

$$\begin{aligned}\text{(c)}\quad (2 \text{ cis } 252°)(2 \text{ cis } 252°)(2 \text{ cis } 252°) &= 8 \text{ cis } 756° = 8 \text{ cis } (720° + 36°)\\ &= 8 \text{ cis } 36°\end{aligned}$$

In (b) the angle of the product is 396°, which is once around (360°) and 36° more. In (c) the angle of the product is 756°, which is twice around (720°) and 36° more. Thus, the three third roots of 8 cis 36° are

$$2 \text{ cis } 12° \qquad 2 \text{ cis } 132° \qquad 2 \text{ cis } 252°$$

To get the first root, we took the third root of 8 and divided the angle by 3. The angle of the next root is 360°/3, or 120°, greater, and the angle of the next root is 2(360°/3), or 240°, greater.

$$\text{A third root of 8 cis } 36° = 8^{1/3} \text{ cis } \frac{36°}{3} = 2 \text{ cis } 12°$$

$$\text{A second third root of 8 cis } 36° = 8^{1/3} \text{ cis } \left(\frac{36°}{3} + 120°\right) = 2 \text{ cis } 132°$$

$$\text{A third third root of 8 cis } 36° = 8^{1/3} \text{ cis } \left(\frac{36°}{3} + 240°\right) = 2 \text{ cis } 252°$$

If we continue the process, the roots will begin to repeat. The next step would be to add $3 \times 120°$, or 360°. If we do this, the result is 2 cis 12° again.

$$8^{1/3} \text{ cis } \left(\frac{36°}{3} + 360°\right) = 2 \text{ cis } 372° = 2 \text{ cis } 12°$$

Every complex number except zero has two square roots, three cube roots, four fourth roots, five fifth roots, and, in general, *n* *n*th roots. The angles of the third roots differ by 360°/3, or 120°. The angles of the fourth roots differ by 360°/4, or 90°. The angles of the fifth roots differ by 360°/5, or 72°; etc. The angles of the *n*th roots differ by 360°/*n*.

***Advanced Mathematics*, Lesson 79**

Sample taken from Advanced Mathematics (Second Edition), page 482

example 79.3 Find the four fourth roots of 16 cis 60°. Check the answers by multiplying.

solution The first root is $16^{1/4}$ cis (60°/4) = **2 cis 15°**. Angles in the polar form of the fourth roots differ by 360°/4, or 90°, so the other three roots are

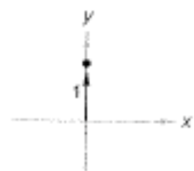
$$\mathbf{2\text{ cis }105°,\ \ 2\text{ cis }195°,\ \ 2\text{ cis }285°}$$

Now we check:

$$\begin{aligned}
(2\text{ cis }15°)(2\text{ cis }15°)(2\text{ cis }15°)(2\text{ cis }15°) &= 16\text{ cis }60° \\
(2\text{ cis }105°)(2\text{ cis }105°)(2\text{ cis }105°)(2\text{ cis }105°) &= 16\text{ cis }420° \\
&= 16\text{ cis }(60° + 360°) = 16\text{ cis }60° \\
(2\text{ cis }195°)(2\text{ cis }195°)(2\text{ cis }195°)(2\text{ cis }195°) &= 16\text{ cis }780° \\
&= 16\text{ cis }(60° + 720°) = 16\text{ cis }60° \\
(2\text{ cis }285°)(2\text{ cis }285°)(2\text{ cis }285°)(2\text{ cis }285°) &= 16\text{ cis }1140° \\
&= 16\text{ cis }(60° + 1080°) = 16\text{ cis }60°
\end{aligned}$$

example 79.4 Find five fifth roots of i.

solution **We write a complex number in polar form with a positive coefficient to find the roots.**



$$0 + i = 1\text{ cis }90°$$

The real fifth root of 1 is 1, so we get

$$\text{A fifth root of } i = 1^{1/5}\text{ cis }\frac{90°}{5} = \mathbf{1\text{ cis }18°}$$

Successive angles of the polar form of the fifth roots differ by 360°/5, or 72°, so the other roots are

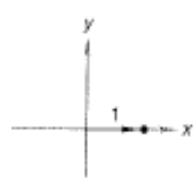
$$\mathbf{1\text{ cis }90°,\ \ 1\text{ cis }162°,\ \ 1\text{ cis }234°,\ \ 1\text{ cis }306°}$$

Now we check the angles.

$5 \times 18° = 90°$	check
$5 \times 90° = 450° = 90° + 360°$	check
$5 \times 162° = 810° = 90° + 720°$	check
$5 \times 234° = 1170° = 90° + 1080°$	check
$5 \times 306° = 1530° = 90° + 1440°$	check

example 79.5 Find two square roots of 1.

solution The polar form of $1 + 0i$ is 1 cis 0°.



$$1 + 0i = 1\text{ cis }0°$$

***Advanced Mathematics,* Lesson 79**

Sample taken from Advanced Mathematics (Second Edition), page 483

The positive real square root (known as the **principal square root**) of 1 is 1, so we get

$$\text{A square root of } 1 = 1^{1/2} \text{ cis } \frac{0°}{2} = \mathbf{1 \text{ cis } 0°}$$

The angles of square roots of complex numbers differ by 360°/2, or 180°, so the other square root of 1 cis 0° is **1 cis 180°.** Now we check our answers.

$$(1 \text{ cis } 0°)(1 \text{ cis } 0°) = 1 \text{ cis } 0° \qquad \text{check}$$

$$(1 \text{ cis } 180°)(1 \text{ cis } 180°) = 1 \text{ cis } 360° = 1 \text{ cis } (0° + 360°) = 1 \text{ cis } 0° \qquad \text{check}$$

Of course, if we wish, we could write the answers in rectangular form as 1 and −1.

example 79.6 Find three third roots of −1.

solution **We always begin by writing the complex number in polar form with a positive coefficient.**

$$-1 + 0i = 1 \text{ cis } 180°$$

The first angle is 180°/3, or 60°. The angles of the third roots differ by 120°, so the three roots are as shown.

$$\text{The three third roots of } -1 = \mathbf{1 \text{ cis } 60°, 1 \text{ cis } 180°, 1 \text{ cis } 300°}$$

These roots can also be written in rectangular form as

$$\frac{1}{2} + \frac{\sqrt{3}}{2}i, \; -1 + 0i, \; \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

example 79.7 Find the four fourth roots of 42 cis 40°.

solution We use the root key on the calculator to find that $42^{1/4}$ is about 2.55. The angles of the fourth roots differ by 360°/4, or 90°, so the four fourth roots are as shown.

2.55 cis 10°, 2.55 cis 100°, 2.55 cis 190°, 2.55 cis 280°

problem set 79

1. There were 12 people present. How many committees of 9 could be selected from the 12 people?
2. How many distinguishable ways can 8 flags be lined up along a wall if 2 of the flags are identical?
3. The cost of finishing the contract varied linearly with the number of men who worked. If 10 men worked, the cost was \$5100. If only 5 men worked, the cost was \$2600. What would be the cost if only 2 men worked?
4. The still-water speed of the boat was 3 times the speed of the current in the river. If the boat could go 16 miles downstream in 2 hours less than it took to go 32 miles upstream, how fast was the boat in still water and what was the speed of the current in the river?
5. A crew of 81 workers can do 1 job in 24 days. In order to finish on time, the contractor increased the size of the work force by one third. How many days will be saved by adding the additional workers?
6. Find $(3 \text{ cis } 35°)^3$ and write the answer in polar form.
7. Use De Moivre's theorem to find $(1 - \sqrt{3}i)^5$. Write the answer in rectangular form. Give an exact answer.

Advanced Mathematics, Lesson 79

Sample taken from Advanced Mathematics (Second Edition), page 484

8. Find the three cube roots of $8i$ and express them in polar form.

9. Find the two square roots of -1 and express them in rectangular form. Give an exact answer.

10. Given the hyperbola $\frac{x^2}{36} - \frac{y^2}{9} = 1$, find the vertices and the equations of the asymptotes. Graph the hyperbola.

11. Given the hyperbola $9x^2 - 4y^2 = 36$, write the equation in standard form and find the coordinates of the vertices and the equations of the asymptotes. Graph the hyperbola.

12. Write all seven terms of $(x + y)^6$.

13. Write the sixth term of $(a + b)^9$.

Show:

14. $\dfrac{\tan\theta}{\sec\theta} = \sin\theta$

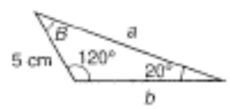
15. $\sin(90° - \theta)\sec(90° - \theta) = \cot\theta$

16. Use Cramer's rule to solve: $\begin{cases} 5x - 3y = 8 \\ 4x + 2y = 5 \end{cases}$

17. What is the radius of the circle that can be circumscribed about a 7-sided regular polygon (regular heptagon) whose perimeter is 49 feet?

18. The perimeter of a 12-sided regular polygon is 96 feet. What is the length of one of the sides of the polygon? What is the area of the polygon?

19. Solve this triangle for the unknown parts.



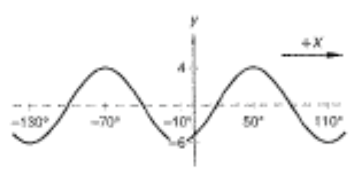
20. Write the equation in standard form and graph the ellipse: $16x^2 + 4y^2 = 64$

21. The birth weights of babies at a particular hospital are found to be approximately normally distributed with a mean of 6.8 pounds and a standard deviation of 0.2 pound. What is the approximate percentage of babies born at this hospital who weigh more than 6.9 pounds?

22. Solve for x: $\begin{vmatrix} x + 2 & 2x \\ x - 1 & x - 3 \end{vmatrix} + 8 = 0$

23. A parabola has its vertex at $(-4, 2)$ and its focus at $(-4, 6)$. Write the equations of the parabola, the directrix, and the axis of symmetry. Graph the parabola.

24. Write the equation of the sinusoid as a cosine function.



25. Multiply $[4 \text{ cis } (-300°)](2 \text{ cis } 30°)$ and express the answer in rectangular form. Give an exact answer.

Solve the following equations given that $0° \le \theta < 360°$:

26. $2\sqrt{2}\sin^2\theta - 12\sin\theta + 5\sqrt{2} = 0$

27. $2\cos 4\theta + 1 = 0$

28. Find the distance from the point $(1, 3)$ to the line $x - 3y + 5 = 0$.

29. Solve for x: $\ln(x + 2) - \ln(3x - 4) = \ln 3$

30. Use the midpoint formula method to find the equation of the perpendicular bisector of the line segment with endpoints $(-6, -2)$ and $(4, -8)$. Write the equation in slope-intercept form.

Advanced Mathematics, Lesson 95

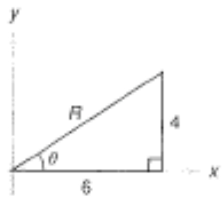
Sample taken from Advanced Mathematics (Second Edition), page 551

LESSON 95 Advanced Complex Roots

We have restricted our work with the roots of complex numbers to problems where the number is written in polar form or can be easily written in polar form. A calculator will permit us to find the roots of any complex number if we remember that the first step is to write the complex number in polar form.

example 95.1 Find the four fourth roots of $6 + 4i$. Express the roots in polar form.

solution **We begin by writing the number in polar form.** We will use a calculator and round our answers.



$$R = \sqrt{6^2 + 4^2} \qquad \tan\theta = \frac{4}{6}$$

$$R = 7.21 \qquad \theta = 33.69°$$

So we find that $6 + 4i = 7.21 \text{ cis } 33.69°$. Now the first of our fourth roots is $(7.21)^{1/4} \text{ cis } (33.69°/4)$. We use the y^x key or the $\sqrt[x]{y}$ key to find the fourth root of 7.21.

$$(7.21)^{1/4} = 1.64$$

Next, we divide 33.69 by 4 to get the angle, so now we have

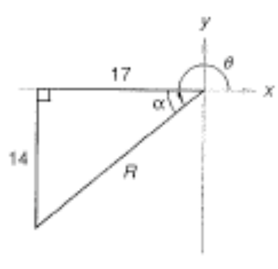
$$\text{One fourth root of } 7.21 \text{ cis } 33.69° = 1.64 \text{ cis } 8.42°$$

Now we add 360°/4, or 90°, to get the second root, add 180° to get the third root, and add 270° to get the fourth root; our four roots are

$$\mathbf{1.64 \text{ cis } 8.42°, \ 1.64 \text{ cis } 98.42°, \ 1.64 \text{ cis } 188.42°, \ 1.64 \text{ cis } 278.42°}$$

example 95.2 Find the five fifth roots of $-17 - 14i$. Express the roots in polar form.

solution The first step is to write the complex number in polar form.



$$R = \sqrt{17^2 + 14^2} \qquad \tan\alpha = \frac{14}{17}$$

$$R = 22.02 \qquad \alpha = 39.47°$$

Therefore, since θ lies in the third quadrant,

$$\theta = 180° + 39.47° = 219.47°$$

Now we restate the problem as

$$\text{One fifth root of } 22.02 \text{ cis } 219.47° = 22.02^{1/5} \text{ cis } \frac{219.47°}{5} = 1.86 \text{ cis } 43.89°$$

There are four more roots, which we get by adding $\frac{n \cdot 360}{5}$ to 43.89° ($n = 1, 2, 3, 4$).

$$\mathbf{1.86 \text{ cis } 43.89°, \ 1.86 \text{ cis } 115.89°, \ 1.86 \text{ cis } 187.89°, \ 1.86 \text{ cis } 259.89°, \ 1.86 \text{ cis } 331.89°}$$

Advanced Mathematics, Lesson 95

Sample taken from Advanced Mathematics (Second Edition), page 552

problem set 95

1. There were 90 people in the room. Half were girls and one third were redheads. Two thirds of the people either were redheads or were girls. If one person is chosen at random, what is the probability of choosing a redheaded girl?

2. An urn contains 4 green marbles, 3 white marbles, and 3 blue marbles. A marble is drawn at random and then replaced. Then 2 more marbles are randomly drawn without replacement. What is the probability that all 3 are white?

3. There were 600 at first, but they increased exponentially. After 60 minutes there were 1000. How many minutes had elapsed before there were 5000?

4. In another room there were also 600, but they decreased exponentially. After 60 minutes, they had decreased in number to only 580. What was the half-life of these creatures?

5. Rondo could carry the 140 liters on his back. If the solution was 20% alcohol, how much pure alcohol must he add to get a solution that is 44% alcohol?

6. Find the four fourth roots of $3 + 4i$ and express the roots in polar form.

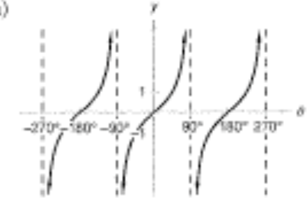
7. Find the three third roots of $2 + 3i$ and express the roots in polar form.

Sketch the graphs of the following:

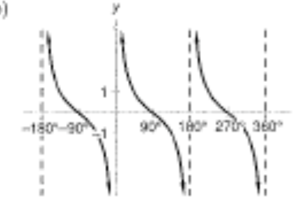
8. $y = 3 + 11 \cos \frac{3}{2}(x - 100°)$ 9. (a) $y = \sec x$ (b) $y = \csc x$

10. Write the equations of these trigonometric functions:

(a)



(b)



Show:

11. $\dfrac{\sin x}{1 + \cos x} + \dfrac{1 + \cos x}{\sin x} = 2 \csc x$ 12. $(1 + \tan x)^2 = \sec^2 x + 2 \tan x$

13. $\dfrac{\sin^4 x - \cos^4 x}{2 \sin^2 x - 1} = 1$

14. Develop the identity for $\cos \frac{1}{2}x$ by using the identity for $\cos (A + B)$.

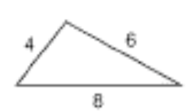
15. Find $\cos 285°$ by using the sum identity for the cosine function and the fact that $285° = 240° + 45°$. Use exact values.

16. Use a sum identity to find an expression for $\sin \left(x + \frac{\pi}{4}\right)$. Use exact values.

Solve the following equations given that $0° \le x < 360°$:

17. $3 \tan^2 x + 5 \sec x + 1 = 0$ 18. $-1 + \tan 4x = 0$

19. Find the angle with the smallest measure in the triangle shown.



20. Compute ${}_7P_2$ and ${}_7C_2$.

21. Find the two geometric means between 3 and −24.

22. Find the first five terms of the arithmetic sequence whose fourth term is 4 and whose thirteenth term is 28.

23. A horizontal ellipse has a major axis of length 10 and a minor axis of length 4. If its center is at the origin, write the equation of the ellipse in standard form. Graph the ellipse.

Advanced Mathematics, Lesson 95

Sample taken from Advanced Mathematics (Second Edition), page 553

24. Given the hyperbola $32x^2 - 18y^2 = 288$, write the equation in standard form and find the coordinates of its vertices and the equations of the asymptotes. Graph the hyperbola.

25. A seven-sided regular polygon has a perimeter of 35 inches. What is the area of the polygon?

Solve for x:

26. $\dfrac{9^{3x-2}}{3^{2x-1}} = 3$

27. $\dfrac{1}{3}\log_2 27 - \log_2 (2x - 1) = 2$

28. $x = \log_{1/3} 18 - \log_{1/3} 6$

29. Simplify: $5^{\log_5 7 - \log_5 3} - \log_5 5^2$

30. Let $f(x) = \sqrt[3]{x}$ and $g(x) = x - 1$. Find $(g \circ f)(x)$.

Calculus

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Sample taken from Calculus (Second Edition), page 138

LESSON 24 *New Notation for the Definition of the Derivative • The Derivative of x^n*

24.A new notation for the definition of the derivative

Many modern calculus books use the letter h instead of Δx in the definition of the derivative. Thus they use the notation on the right-hand side instead of the notation on the left-hand side.

$$\frac{d}{dx}f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \qquad \frac{d}{dx}f(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

example 24.1 Use the h notation for the definition of the derivative to find $g'(x)$ given that $g(x) = \sqrt{x}$.

solution First we write

$$\frac{d}{dx}\sqrt{x} = \lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h}$$

We must find a way to get the h out of the denominator so we can let h approach zero. Sometimes one algebraic procedure works, and sometimes another algebraic procedure works. In this case, if we multiply above and below by the conjugate of the numerator, we get a factor of h in the numerator that cancels the h in the denominator.

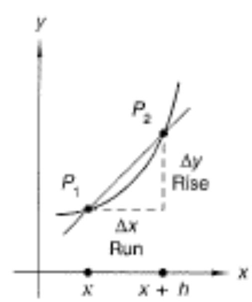
$$\begin{aligned}
\frac{d}{dx}\sqrt{x} &= \lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x + h} + \sqrt{x}}{\sqrt{x + h} + \sqrt{x}} && \text{conjugate} \\
&= \lim_{h \to 0} \frac{x + h - \sqrt{x + h}\sqrt{x} + \sqrt{x + h}\sqrt{x} - x}{h\left(\sqrt{x + h} + \sqrt{x}\right)} && \text{multiplied} \\
&= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x + h} + \sqrt{x}\right)} && \text{simplified numerator} \\
&= \lim_{h \to 0} \frac{1}{\sqrt{x + h} + \sqrt{x}} && \text{canceled}
\end{aligned}$$

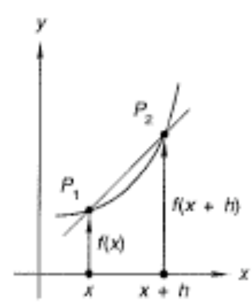
In this expression we still have an h in the denominator, but h is not a factor of the denominator, so h can approach zero without the denominator approaching zero. Thus

$$\frac{d}{dx}\sqrt{x} = \lim_{h \to 0} \frac{1}{\sqrt{x + h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$$

24.B the derivative of x^n

We remember the definition of the derivative of a function by remembering the geometrical interpretation. The slope of the secant through P_1 and P_2 in the figure on the left below is the rise divided by the run. In the figure on the right the y-value of P_2 is $f(x + h)$ and the y-value of P_1 is $f(x)$.





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Sample taken from Calculus (Second Edition), page 139

The value of the rise Δy is the difference of these two expressions. The run is Δx; so we can write the rise over the run as

$$\frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

The derivative is the limit of this expression as h approaches zero. We remember that the trick is to rearrange the expression algebraically so that h is not a factor of the denominator when h approaches zero. If the function whose derivative we seek is x^3, we would proceed as follows.

$$\frac{d}{dx}x^3 = \lim_{h\to 0}\frac{(x+h)^3 - x^3}{h}$$

When we expand $(x + h)^3$ by using the binomial formula, we get x^3 as the first term, which cancels with the $-x^3$ term in the numerator.

$$\frac{d}{dx}x^3 = \lim_{h\to 0}\frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

Furthermore, we note that the second term has h as a factor and that every other term has h^2 as a factor. If we divide by h, we no longer have an h in the denominator.

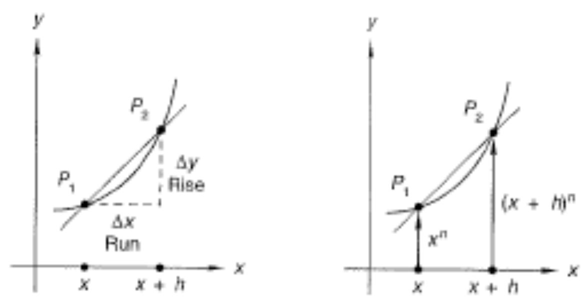
$$\frac{d}{dx}x^3 = \lim_{h\to 0}\left[3x^2 + (3xh + h^2)\right]$$

In this expression all of the terms after the first term have h as a factor. If we let h approach zero, then the value of all of these terms approaches zero. Thus,

$$\frac{d}{dx}x^3 = 3x^2$$

example 24.2 Find the derivative of x^n where n is 1, 2, 3, 4, ...

solution We use the same diagrams to remember that the definition of the derivative is an algebraic expression of the limit of the rise over the run as the run approaches zero.



$$\frac{d}{dx}x^n = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h}$$

When we expand the numerator, we get $x^n + nx^{n-1}h$ plus other terms whose coefficients we represent with empty boxes since their value is of no interest. The last term in the numerator is the last term in the numerator above.

$$\frac{d}{dx}x^n = \lim_{h\to 0}\frac{x^n + nx^{n-1}h + \Box x^{n-2}h^2 + \Box x^{n-3}h^3 + \cdots + h^n - x^n}{h}$$

Every term except the first two terms and the last term has h^2 as a factor. Thus

$$\frac{d}{dx}x^n = \lim_{h\to 0}\frac{x^n + nx^{n-1}h + [\text{terms that have } h^2 \text{ as a factor}] - x^n}{h}$$

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Sample taken from Calculus (Second Edition), page 140

The sum of the first term and the last term in the numerator is zero. If we divide the rest by h, the h in the second term is eliminated, and every term in the parentheses still has h as a factor.

$$\frac{d}{dx}x^n = \lim_{h\to 0}\left[nx^{n-1} + (\text{terms that have } h \text{ as a factor})\right]$$

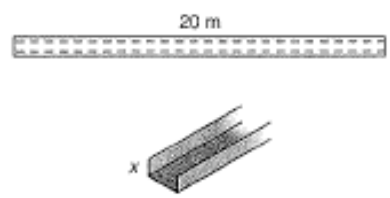
When h approaches zero, the values of all of the terms in the parentheses approach zero, which means

$$\boxed{\frac{d}{dx}x^n = nx^{n-1}}$$

In this example we used the binomial expansion of $(x + h)^n$ to prove that the derivative of x^n is nx^{n-1} if n is a positive integer. This proof is only valid if n is a natural number. It does not work for a rational number such as $\frac{2}{3}$ or an irrational number such as π, because the binomial expansion cannot be used to expand expressions such as $(x + h)^{2/3}$ or $(x + h)^{\pi}$. **The rule above is valid, however, for any real number value of n.** We will use this fact even though the complete proof is not presented. This rule is called the **power rule** for derivatives.

problem set 24

1. (5) A rectangular sheet of metal measuring 1 meter by 20 meters is to be made into a gutter by bending its two sides upward at right angles to the base. If both vertical sides of the gutter have the same height x, what is the capacity of the gutter in terms of x? (The capacity of the gutter is the maximum amount of fluid it could hold if it were closed at both ends.)



2. (24) Find $\dfrac{dy}{dx}$ where $y = x^3$.

3. (24) Find $f'(x)$ where $f(x) = \sqrt[3]{x}$.

4. (24) Find $\dfrac{ds}{dt}$ where $s = \dfrac{1}{t^3}$.

5. (24) Find $D_x y$ where $y = \sqrt[4]{x^3}$.

6. (24) Find $\dfrac{dy}{dx}$ where $y = \dfrac{1}{x^2}$.

7. (14,24) Let $f(x) = x^2$ and define g by $g(x) = \dfrac{f(2 + x) - f(2)}{x}$.

(a) Graph g on a graphing calculator.

(b) Use the trace feature or the table feature to determine the value $g(x)$ approaches as x approaches 0.

(c) Find $f'(x)$ and evaluate f' at $x = 2$.

(d) How do the answers to (b) and (c) compare?

8. (23) Solve: $\cos(3\theta) = -\dfrac{1}{2}$ $(0 \le \theta < 2\pi)$

9. (23) Use the graphing calculator to graph $4y^2 + 8y - x + 5 = 0$.

10. (22) Find the coefficient of x^4y^3 in the expansion of $(x - 2y)^7$.

11. (7) Let $f(x) = e^x$ and $g(x) = f(-x)$. Graph f and g on the same coordinate plane.

12. (7,21) Let $f(x) = \cos x$ and $h(x) = 1 + f\left(x - \dfrac{\pi}{4}\right)$. Graph h.

Calculus, Lesson 24

Sample taken from Calculus (Second Edition), page 141

13. (21) Sketch the graph of $y = \dfrac{1}{x^2}$.

14. (20) Rewrite $y = \log x$ in terms of the natural logarithms.

15. (19) Use the definition of the derivative to calculate $f'(x)$ where $f(x) = 2x^2$.

16. (18) Let $f(x) = x^2$ and $g(x) = \sqrt{x - 4}$.

(a) Write the equations of $f \circ g$ and $g \circ f$.

(b) Find the domain and range of f, g, $f \circ g$, and $g \circ f$.

Evaluate the limits in problems 17 and 18.

17. (14) $\lim\limits_{x \to 3} \dfrac{x^3 - 27}{x - 3}$

18. (17) $\lim\limits_{x \to \infty} \dfrac{2x^3 - x^4}{2x^2 - 1}$

19. (2) (a) Find the coordinates of the midpoint of the line segment joining the points $(2, -1)$ and $(4, 2)$.

(b) Write the equation of the line that passes through the points $(2, -1)$ and $(4, 2)$.

(c) Find the equation of the line consisting of all the points that are equidistant from the points $(2, -1)$ and $(4, 2)$.

20. (12) Sketch the graph of $y = -\log_6 x$.

21. (13) Given that $f(x) = \begin{cases} 2 & \text{when } x \geq 1 \\ -2 & \text{when } x < 1, \end{cases}$ sketch the graph of f and find:

(a) $\lim\limits_{x \to 1^+} f(x)$

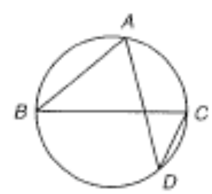
(b) $\lim\limits_{x \to 1^-} f(x)$

22. (9) Let L represent a constant. Use interval notation to describe the values of y for which $|y - L| < 0.001$.

23. (6) Show that $(1 - \cos^2 x)\csc^2 x + \tan^2 x = \sec^2 x$ for all values of x where the functions are defined.

24. (4) Evaluate: $3 \tan^2 \dfrac{\pi}{6} + 2 \sin^2 -\dfrac{\pi}{4}$

25. (8) If m$\angle ABC$ in the figure shown is 40°, then what is the measure of angle ADC? Justify your answer.

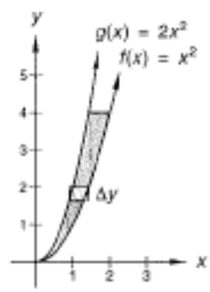


Calculus, Lesson 81

Sample taken from Calculus (Second Edition), page 414

LESSON 81 *Solids of Revolution II: Washers*

Some solids of revolution have cavities, and their volumes can be computed as the difference of two volumes, each of which can be found by stacking disks. These volumes can also be found by stacking washers. The volume of the solid formed by rotating the first-quadrant region shown below about the y-axis is the volume formed by revolving the region bounded by the graph of $f(x) = x^2$ about the y-axis, and then removing the volume formed by revolving the graph of $g(x) = 2x^2$ about the y-axis.



Representative washer

The solid formed is depicted in the center figure, and its volume can be approximated by a stack of circular washers similar to the representative washer shown. The volume of the representative washer is the product of its thickness (Δy) and the area of the whole disk (πR^2) reduced by the area of the hole in its center (πr^2).

$$\text{Volume} = (\pi R^2 - \pi r^2)\Delta y$$

Since this result is exactly the same as the difference in the volumes of two representative disks,

$$\text{Volume} = \pi R^2 \Delta y - \pi r^2 \Delta y$$

we see that the volume-by-washer method is a difference-of-two-disks method in disguise. The washers make it easier to visualize the problem, which is why we use them. The total volume is given by a definite integral that sums all of these smaller volumes.

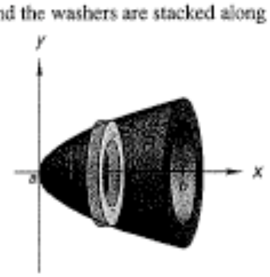
$$\boxed{\text{Volume} = \int_a^b (\pi R^2 - \pi r^2)\,dy}$$

This is the case when the region has been rotated about the y-axis. R and r are represented in terms of y, and a and b are the smallest and largest y-values denoting the locations of the vertically stacked washers.

We can write a similar integral for volumes of solids of revolution rotated about the x-axis:

$$\boxed{\text{Volume} = \int_a^b (\pi R^2 - \pi r^2)\,dx}$$

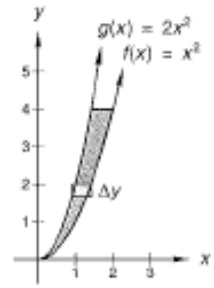
Here, R and r are in terms of x, and the washers are stacked along the x-axis from a to b.



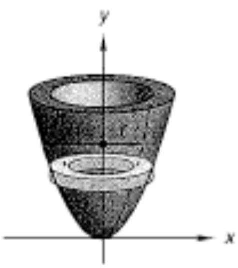
Calculus, Lesson 81

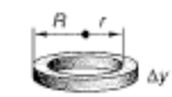
Sample taken from Calculus (Second Edition), page 415

example 81.1 Find the volume of the solid formed by revolving the region shown about the y-axis.



solution First we generate the solid and draw a representative washer.





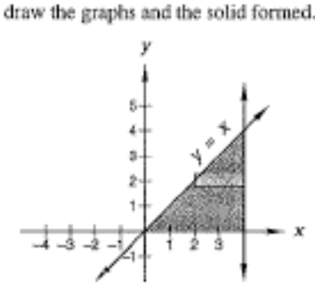
Representative washer

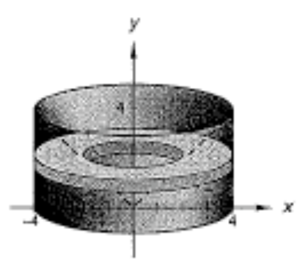
The volume of the representative washer is $(\pi R^2 - \pi r^2)\Delta y$ where R is the outer radius and r is the inner radius of the washer. In this case R is determined by the input for the function f, so $y = R^2$. On the other hand, r is determined by the input for the function g, so $y = 2r^2$ or $r^2 = \frac{y}{2}$. These washers are stacked from $y = 0$ to $y = 4$. Thus the volume of the solid in question is

$$
\begin{aligned}
V &= \int_a^b (\pi R^2 - \pi r^2)\, dy \\
&= \int_0^4 \left(\pi y - \pi \frac{y}{2}\right) dy \\
&= \frac{\pi}{2} \int_0^4 y\, dy \\
&= \frac{\pi}{2}\left[\frac{y^2}{2}\right]_0^4 \\
&= \mathbf{4\pi \text{ units}^3}
\end{aligned}
$$

example 81.2 Find the volume of the solid formed by revolving about the y-axis the region bounded by $y = x$, $x = 4$, and the x-axis.

solution First, we draw the graphs and the solid formed.





Calculus, Lesson 81

Sample taken from Calculus (Second Edition), page 416

Note that the volume in question is that of a cylinder with a cone removed. The volume of the entire cylinder is

$$V_{\text{cyl}} = \pi r^2 h = \pi(4)^2 4 = 64\pi$$

while the volume of the removed cone is

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h = \frac{1}{3}(64\pi) = \frac{64\pi}{3}$$

Therefore, the volume of the resulting object is $V_{\text{cyl}} - V_{\text{cone}} = 64\pi - \frac{64\pi}{3} = \frac{128\pi}{3}$.

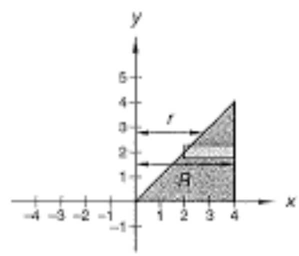
To confirm this, we compute a definite integral that describes the volume of the object.

$$V = \int_{y=0}^{y=4} (\pi R^2 - \pi r^2)\, dy$$

Here $R = 4$ for all washers, while $r = x$. But since this region is revolved about the y-axis, we must write the equations for R and r in terms of y.

$$R = 4$$

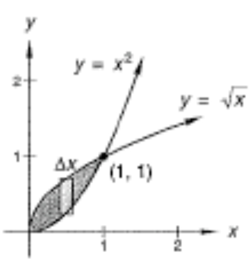
$$r = y \qquad \text{(since } y = x\text{)}$$



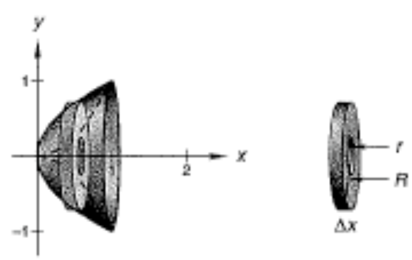
Now we find the volume.

$$\begin{aligned} V &= \int_0^4 \pi(4^2 - y^2)\, dy \\ &= \pi\left[16y - \frac{y^3}{3}\right]_0^4 \\ &= \pi\left[64 - \frac{64}{3}\right] \\ &= \mathbf{\frac{128\pi}{3}\ units^3} \end{aligned}$$

example 81.3 Find the volume of the solid formed by revolving the region shown about the x-axis.



solution First we draw the solid and show a representative washer.



The volume of the representative washer is

$$\begin{aligned} V &= (\pi R^2 - \pi r^2)\,\Delta x \\ &= \left[\pi(\sqrt{x})^2 - \pi(x^2)^2\right]\Delta x \\ &= (\pi x - \pi x^4)\,\Delta x \end{aligned}$$

Calculus, Lesson 81

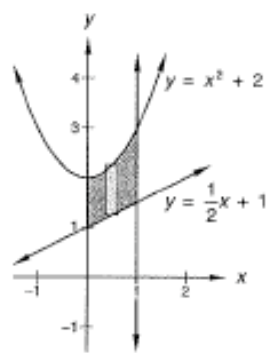
Sample taken from Calculus (Second Edition), page 417

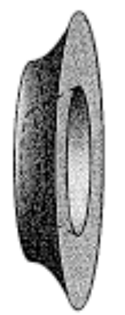
Moreover, the washers are stacked from $x = 0$ to $x = 1$. Therefore the volume in question is

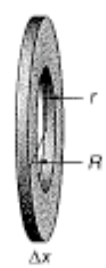
$$\begin{aligned} V &= \int_0^1 (\pi x - \pi x^4)\,dx \\ &= \left[\frac{\pi x^2}{2} - \frac{\pi x^5}{5}\right]_0^1 \\ &= \pi\left(\frac{1}{2} - \frac{1}{5}\right) \\ &= \mathbf{\frac{3}{10}\pi\ units^3} \end{aligned}$$

example 81.4 The region between the graphs of $y = x^2 + 2$ and $y = \frac{1}{2}x + 1$ on the interval $[0, 1]$ is revolved about the x-axis. Find the volume of the solid of revolution generated.

solution We begin by drawing the graph and shading the region to be revolved. Next we show the solid that is generated, as well as a representative washer.







The rectangle in the graph is a profile of a section of the washer on the right. The width of the washer is Δx, and we stack these washers from $x = 0$ to $x = 1$. The radius r of the inside of the washer is $\frac{1}{2}x + 1$, and the radius R of the outside of the washer is $x^2 + 2$. So the volume in question is

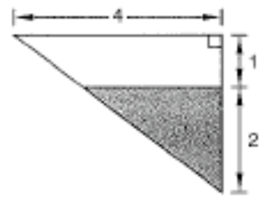
$$\begin{aligned} V &= \int_0^1 (\pi R^2 - \pi r^2)\,dx \\ &= \int_0^1 \left[\pi(x^2 + 2)^2 - \pi\left(\frac{1}{2}x + 1\right)^2\right] dx \\ &= \pi \int_0^1 \left[x^4 + 4x^2 + 4 - \left(\frac{x^2}{4} + x + 1\right)\right] dx \\ &= \pi \int_0^1 \left(x^4 + \frac{15}{4}x^2 - x + 3\right) dx \\ &= \pi\left[\frac{x^5}{5} + \frac{5x^3}{4} - \frac{x^2}{2} + 3x\right]_0^1 \\ &= \pi\left[\frac{1}{5} + \frac{5}{4} - \frac{1}{2} + 3\right] \\ &= \mathbf{\frac{79\pi}{20}\ units^3} \end{aligned}$$

Calculus, Lesson 81

Sample taken from Calculus (Second Edition), page 418

problem set 81

1. (77) A 10-meter-long trough with a right triangular cross section is partially filled with a fluid whose weight density is 9000 newtons per cubic meter. The level of the fluid is 1 meter below the top rim of the trough. Find the work done in pumping all the fluid out of the top of the tank.



2. (75) Suppose the function f is defined as $f(x) = \begin{cases} x^3 + 2x & \text{when } x \le 2 \\ 3x + b & \text{when } x > 2 \end{cases}$. Find the value of b for which f is continuous everywhere.

3. (81) Let f be a quadratic function. The slope of the line tangent to the graph of f at $x = 1$ is 1, and the slope of the line tangent to the graph of f at $x = 2$ is 5. The graph of f passes through the point $(0, 1)$. Find the equation of f.

In problems 4 and 5, let R be the region in the first quadrant between $y = x^2$ and the x-axis on the interval $[0, 3]$.

4. (71) Find the volume of the solid formed when R is revolved about the x-axis.

5. (81) Find the volume of the solid formed when R is revolved about the y-axis.

6. (81) Let R be the region in the first quadrant enclosed by the graphs of $y = x^2$, $y = \frac{1}{4}x^2$, and $y = 4$. Find the volume of the solid formed when R is rotated around the y-axis.

7. (81) Let R be the first-quadrant region completely bounded by the graph of $y = \sqrt{x}$ and $y = x^3$. Find the volume of the solid formed when region R is revolved about the x-axis.

8. (81) Let R be the region bounded by the graphs of $y = x^2 + 1$, $y = x$, $x = 0$, and $x = 2$. Find the volume of the solid formed when region R is rotated around the x-axis.

9. (80) Write the equations of the asymptotes of the graph of the function $y = \dfrac{2x^2 - 2x - 4}{x - 1}$.

Graph the functions in problems 10 and 11. Clearly indicate all zeros and asymptotes.

10. (80) $y = \dfrac{x^2 + 1}{2x}$

11. (80) $y = \dfrac{x^2 + x - 2}{x + 1}$

Evaluate the limits in problems 12 and 13.

12. (79) $\displaystyle\lim_{x \to 0} \frac{\sin(3x)}{x}$

13. (79) $\displaystyle\lim_{x \to 2} \frac{x^3 - x^2 - x - 2}{x - 2}$

14. (72) Write the equation of the line tangent to the graph of $y = 2^x$ at $x = 2$.

15. (75) If $\displaystyle\lim_{x \to 2} f(x) = 7$, which of the following must be true?

A. f exists at $x = 2$.

B. $f(2) = 7$

C. f is continuous at $x = 2$.

D. None of the above

16. (64,72) Differentiate $y = x \ln|x^3 - x| + 2^{2x-3} + \arctan x$ with respect to x.

17. (64,73) Antidifferentiate: $\displaystyle\int \left(2^x + \frac{1}{1 + x^2}\right) dx$

Calculus, Lesson 81

Sample taken from Calculus (Second Edition), page 419

18. (66) Evaluate $\int_0^{\pi} \frac{\cos x}{\sqrt{\sin x + 1}}\, dx$ by changing the variable of integration.

19. (45) Which of the following functions has a graph that is concave upward everywhere?

A. $y = x^3$ B. $y = -x^2$ C. $y = e^x$ D. $y = \sin x$

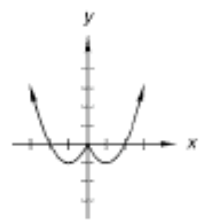
20. (43) If f is a function that is continuous and increasing for all real values of x, which of the following must be true?

A. The graph of f is always concave up.
B. The graph of f is always concave down.
C. $f(x_1) < f(x_2)$ if $x_1 > x_2$
D. $f(x_2) > f(x_1)$ if $x_2 > x_1$

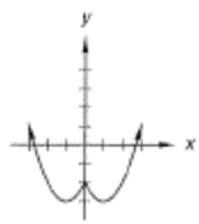
21. (38) Let $f(x) = e^{3x}$. Find the value of $f^{-1}(1)$.

22. (13,27) For what values of k does the graph of $y = \frac{4}{3}x^3 + 2kx^2 + 5x + 3$ have two tangent lines parallel to the x-axis.

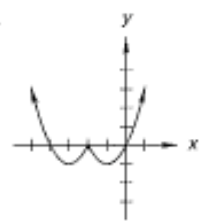
23. (21) The graph of the function f is shown at the right. The graph of $g(x) = f(x + 2)$ most resembles which of the following graphs?



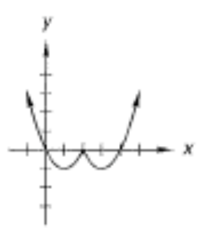
A.



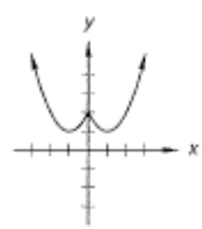
B.



C.



D.



24. (18) Determine the domain and range of $y = \sin\left(\sqrt{x - 1}\right)$.

25. (2,27) Let $f(x) = -x^2 - 4x + 12$ on the interval $[-3, 1]$. Find the point(s) on the curve where the tangent line is parallel to the line segment joining the point corresponding to $x = -3$ to the point corresponding to $x = 1$.

Calculus, Lesson 116

Sample taken from Calculus (Second Edition), page 597

LESSON 116 *Series*

Lesson 105 introduced the concept of a sequence, an infinite and ordered list of terms. We now discuss the concept of the sum of infinitely many terms, which is called an **infinite series** or **series.** If $\{a_i\}$ is a sequence of terms for $i = 1, 2, 3, \ldots,$ we can form a series S by summing these terms.

$$S = a_1 + a_2 + a_3 + \cdots \qquad \text{or} \qquad S = \sum_{i=1}^{\infty} a_i$$

Unfortunately S is represented as an infinite summation. If it has a value, that value cannot be determined by adding all the a_i's, because the process never ends. However, it is possible to add the first n terms. Therefore the nth **partial sum** of S, denoted S_n, is defined by

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n$$

All partial sums are finite, since each is a sum of a finite number of terms.

$$\begin{aligned} S_1 &= a_1 \\ S_2 &= a_1 + a_2 \\ S_3 &= a_1 + a_2 + a_3 \\ &\vdots \end{aligned}$$

Notice that the partial sums of S form a sequence $S_1, S_2, S_3, \ldots$. Thus, we define the **sum** of a series S to be the limit of the sequence of its partial sums.

$$S = \sum_{i=1}^{\infty} a_i = \lim_{n \to \infty} S_n$$

Moreover, we say the infinite series S **converges** if $\lim_{n \to \infty} S_n$ converges. Otherwise S is said to **diverge.**

example 116.1 Let $S = \sum_{n=1}^{\infty} \frac{1}{2^n}$. Find the first five partial sums of S. That is, find $S_1, S_2, S_3, S_4,$ and S_5.

solution The first five terms of S are $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16},$ and $\frac{1}{32}$. The partial sums are as follows:

$$\begin{aligned} S_1 &= \mathbf{\frac{1}{2}} \\ S_2 &= \frac{1}{2} + \frac{1}{4} = \mathbf{\frac{3}{4}} \\ S_3 &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{3}{4} + \frac{1}{8} = \mathbf{\frac{7}{8}} \\ S_4 &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{7}{8} + \frac{1}{16} = \mathbf{\frac{15}{16}} \\ S_5 &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = \frac{15}{16} + \frac{1}{32} = \mathbf{\frac{31}{32}} \end{aligned}$$

example 116.2 Does the infinite series $S = \sum_{n=1}^{\infty} \frac{1}{2^n}$ converge or diverge?

solution To answer such a question regarding infinite series, we must consider

$$\lim_{n \to \infty} S_n$$

Calculus, Lesson 116

Sample taken from Calculus (Second Edition), page 598

Therefore we must find a formula for S_n, the nth partial sum of S. We seek a pattern in the partial sums S_1, S_2, S_3, S_4, and S_5. Notice that the denominators are powers of 2.

$$S_1 = \frac{1}{2} = \frac{1}{2^1}$$
$$S_2 = \frac{3}{4} = \frac{3}{2^2}$$
$$S_3 = \frac{7}{8} = \frac{7}{2^3}$$
$$S_4 = \frac{15}{16} = \frac{15}{2^4}$$
$$S_5 = \frac{31}{32} = \frac{31}{2^5}$$

Moreover, the numerators are one less than the denominators.

$$S_1 = \frac{2^1 - 1}{2^1}$$
$$S_2 = \frac{2^2 - 1}{2^2}$$
$$S_3 = \frac{2^3 - 1}{2^3}$$
$$S_4 = \frac{2^4 - 1}{2^4}$$
$$S_5 = \frac{2^5 - 1}{2^5}$$

From these we conjecture that

$$S_n = \frac{2^n - 1}{2^n} = 1 - \frac{1}{2^n}$$

It turns out we can prove that this formula for S_n is correct for all positive integers n. (Usually, it is more difficult to find an explicit formula for S_n.) Thus, we can determine whether the series converges or diverges.

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \frac{1}{2^n}\right) = 1 - 0 = 1$$

He nce $\sum_{n=1}^{\infty} \frac{1}{2^n}$ **converges** and $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$.

example 116.3 Find the first four partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

solution The partial sums are as follows:

$$S_1 = \frac{1}{1} = \mathbf{1}$$
$$S_2 = \frac{1}{1} + \frac{1}{2} = \mathbf{\frac{3}{2}}$$
$$S_3 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \mathbf{\frac{11}{6}}$$
$$S_4 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \mathbf{\frac{25}{12}}$$

While these partial sums do not appear to grow large, this series actually diverges. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is known as the **harmonic series.** It will be discussed more in Lesson 127.

Calculus, Lesson 116

Sample taken from Calculus (Second Edition), page 599

problem set 116

1. (93) Approximate to ten decimal places the x-coordinate of the first-quadrant point of intersection of the graphs of $y = x$ and $y = \cos x$.

2. (57) A solid has a base bounded by $y = 4 - x^2$ and the x-axis. Each cross section perpendicular to the base and parallel to the x-axis is a rectangle of height 2. Find its volume.

3. (89) Determine the average value of $f(x) = \sin x$ on the closed interval $[0, \pi]$. Confirm the Mean Value Theorem for Integrals using f on this interval.

4. (116) Define: *series*

5. (62) A variable force $F(x) = xe^{x^2}$ newtons is applied to an object to move it along a straight line in the direction of the force. Find the work done by the force on the object in moving it from $x = 0$ to $x = 3$ meters.

6. (91) Evaluate: $\lim_{x \to 0^+} [\sin x \ln (\sin x)]$

Antidifferentiate in problems 7 and 8.

7. (115) $\int \frac{6x + 1}{x(x + 1)(x + 2)}\, dx$

8. (115) $\int \frac{-x - 7}{(x + 1)(x - 2)}\, dx$

9. (107) Write the polar form of the rectangular equation $x^2 + y^2 = 4$.

10. (114) Find the length of the curve whose graph is defined by the parametric equations $x = e^t \sin t$ and $y = e^t \cos t$ on the interval from $t = 0$ to $t = 2$.

11. (114) A particle moves along the path defined by the parametric equations $x = \frac{t^2}{2}$ and $y = \frac{1}{3}(2t + 1)^{3/2}$. Find the distance the particle travels between times $t = 0$ and $t = 4$.

Graph the equations in problems 12 and 13 on a polar coordinate system.

12. (110) $r = 2 \sin \theta$

13. (110) $r = 3 \sin (3\theta)$

Integrate in problems 14–17.

14. (66) $\int \frac{2x}{4 + 9x^2}\, dx$

15. (38) $\int \frac{4 + 9x^2}{2x}\, dx$

16. (66) $\int \frac{2x}{\sqrt{4 + 9x^2}}\, dx$

17. (113) $\int \frac{2}{\sqrt{4 + 9x^2}}\, dx$

18. (116) List the first six terms of $\sum_{n=1}^{\infty} \frac{2n}{3}$.

19. (116) Find the first six partial sums of the series $\sum_{n=1}^{\infty} \frac{2n}{3}$.

20. (116) Would you guess that the series $\sum_{n=1}^{\infty} \frac{2n}{3}$ converges or diverges? If you say it converges, to what would you guess it converges?

21. (116) List the first six terms of $\sum_{n=1}^{\infty} \frac{3}{2^n}$.

22. (116) Find the first six partial sums of the series $\sum_{n=1}^{\infty} \frac{3}{2^n}$.

Calculus, Lesson 116

Sample taken from Calculus (Second Edition), page 600

23. (116) Would you guess that the series $\sum_{n=1}^{\infty} \frac{3}{2^n}$ converges or diverges? If you say it converges, to what would you guess it converges?

24. (58, 64) Differentiate $y = \frac{1}{\sqrt{x}} - x \ln |\sin x| + \arcsin \frac{x}{2}$ with respect to x.

25. (95) An experiment confirms that there is a relationship between two quantities, which we represent by the variables x and y. The experiment produced the correspondences between x and y indicated in the following table:

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0
y	2.7	3.5	4.1	4.0	3.8	3.2	2.4

Though we do not have the equation $y = f(x)$, we know that $\int f(x)\,dx$ has an important physical meaning. Approximate $\int_2^5 f(x)\,dx$ using this data and the trapezoidal rule with $n = 6$ subintervals.

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