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The Quantile Framework for Mathematics:

A System for Measuring Student Mathematical Understanding and Task Difficulty

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PROFESSIONAL PAPER

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INTRODUCTION

Mathematical knowledge and skills are fundamental to future success in college and career. With the adoption of the Common Core State Standards for Mathematics (CCSS-M) by 46 states and the District of Columbia, there is a new opportunity for students to experience focused and coherent mathematics instruction as preparation for future success (CCSSI, 2010). With significant shifts to the structure and pace of mathematics learning comes a need for change in the teaching and learning of mathematics (Sawchuk, 2012). New tools are needed to assist educators with a successful transition.

The Common Core State Standards for Mathematics have changed the grade level at which certain topics are taught. To optimize instructional time teachers need to be informed about what students know and are prepared to learn, teachers need information on what mathematics students have learned; the relative difficulty of the mathematical topics to be studied; and the prerequisite skills and concepts, supporting skills and concepts, and the content to follow the mathematics that the students are expected to learn. The Quantile Framework for Mathematics is a tool that provides teachers with this information. In this paper, we describe what the Quantile Framework is, how it was developed, and how teachers make use of this powerful resource to support students' success in school mathematics.

The Quantile Framework for Mathematics is also a tool for school leadership. The framework provides guidance with respect to tiered instructional placements, students' success in their placements, and overall instructional and school effectiveness with tools to measure and interpret student growth over time.

THE QUANTILE FRAMEWORK: AN OVERVIEW

What is the Quantile Framework?
The Quantile Framework is a developmental scale on which students' readiness for mathematics instruction, the difficulty of mathematics concepts and skills, and instructional materials for teaching mathematics are measured.

The Quantile Framework relies on a common metric, the Quantile®, to describe a student's readiness for instruction and to identify the difficulty of a mathematical skill or concept. Quantile measure is a numeric value represented as a number followed by a Q, e.g. 650Q. These measures comprise the Quantile scale, an equal-interval scale ranging from below 0Q to above 1400Q. Low values indicate emergent mathematical concepts and skills. Quantile measures that are less than 0Q are reported as EM, meaning Emerging Mathematician. For larger measures on the Quantile scale the mathematics associated with each measure is more complex. Students with higher Quantile measures are ready for instruction with more complex mathematics concepts and skills than are students with lower Quantile measures.

Additionally, widely-adopted textbooks and other curriculum materials have been calibrated to Quantile measures. With access to these measures, teachers can immediately understand the difficulty of the task at hand relative to the readiness of their students to receive that instruction.

The underlying assumption of the Quantile Framework is that when mathematics concepts and skills, curricular materials, and students are measured on the same scale, an educator can provide the best opportunities for student learning. The Quantile Framework aligns content, resources, and students, allowing educators to make the most of the limited time for school mathematics instruction.

How Measures Are Reported
A Quantile is the unit of measure on the Quantile scale. To interpret what a Quantile measure means for a specific student, two pieces of information are needed: the Quantile score, and the grade level during which the student received the Quantile score.

As stated, Quantile measures are reported as a numeric value, on an equal-interval scale that ranges from below 0 Quantile to above 1400 Quantile. The measure is represented by a number and is followed by a "Q" (e.g. 750Q). Low values indicate emergent mathematical skills and concepts (below 0Q measures are called EM for Emerging Mathematician), while higher values describe more complex and integrated skills and concepts.

Algebra readiness is denoted by the score of 1030Q. College and career readiness, or Pre-calculus, is denoted by the level of 1400Q.

The Value of the Quantile Framework

Research has determined that one major predictor of college and career success is the number of advanced secondary mathematics courses a student has completed (National Mathematics Advisory Panel, 2008). Research also suggests that students who take advanced mathematics courses in high school are more likely to find career success in higher paying jobs. (Achieve, 2006). Foundational to the idea that advanced math is critical to college and career success is other research that demonstrates success in Algebra I is the gateway to all advanced mathematics. Algebra is a critical step in developing the knowledge and skills needed for further success in mathematics, college, and work (Evan, Gray, & Olchefske, 2006).

The Quantile Framework measures the mathematical skills and concepts students need to be prepared for and successful in Algebra I (and High School Integrated Math I). With knowledge of these skills and concepts, math teachers can define a trajectory of student progress through the mathematical content needed to be successful in Algebra. Teachers can design instruction that improves weak foundational knowledge, while developing key early Algebra concepts. When used as a measurement tool, the Quantile Framework supports teachers as they tailor instruction to best meet the needs of all students and facilitate their progress to and through Algebra.

How are Quantile Measures Used?

The Quantile Framework for Mathematics is constructed on a developmental scale and can track student growth over time and across grade levels to indicate learning gains. This scale is also applied to the materials used in instruction, the assessments used to monitor instruction, and the results that are derived from assessments.

Quantile measures can be used in schools:

- **Conduct universal screening:** identify students' mathematical understandings.
- **Inform instruction:** target instruction for each student for differentiated instruction to close gaps in math.
- **Set Goals and Monitor Growth:** interpret growth over time and support a Response to Intervention (RTI) implementation.
- **Forecast future performance:** determine how well a student is likely to perform in the math curriculum and on high-stakes tests.

With the Quantile Framework, educators have tools to intervene, accelerate, and improve student math achievement. By using growth models, we can forecast the rate that students will move through the mathematics content through Algebra instruction. If students are moving at a rate higher or lower than expected, instruction can be tailored to meet their needs. Quantile measures are "actionable" because they identify how well students will be able to solve problems and apply mathematics; they allow educators to identify the skills and concepts that students are ready to learn.

When educators know the Quantile measures for their students and the mathematics the students are to about to learn, instructional time can be used more effectively.

On a school-wide level, Quantile measures can help educators make accurate placements into tiered instruction, and in the classroom, Quantile measures can support differentiation of instruction, thereby bolstering the quality of instruction in the core classroom. Quantile measures allow classroom teachers to provide instruction in large groups or small groups on content that students are ready to learn, and not on content that they already know or content for which they are unprepared. Further, tracking the growth of a student's Quantile measure over time provides an efficient and effective monitor of progress throughout the academic year. Understanding the progress a student is making and what can be expected for growth in Quantile measures from one grade to the next allows educators to determine if students are progressing toward college and career goals or if some form of intervention is necessary for the student.

THE DEVELOPMENT OF THE QUANTILE FRAMEWORK

The Quantile Framework was developed by MetaMetrics, Inc., an independent, educational measurement company, that develops assessment and measurement tools to inform instruction. The company's team of psychologists, psychometricians, and educational researchers also developed the widely-adopted Lexile Framework® for Reading. A goal in the development of the Quantile Framework for Mathematics was to develop a content progression of mathematical development while simultaneously considering the presence of conceptual understanding, computational fluency, and problem-solving skills.

Creating and Organizing the QTaxons

The Quantile Framework for Mathematics was created through a four-year process of research and development. The first step enacted was to develop the content taxonomy on which the framework was constructed. This process involved a review of state and national curriculum frameworks including the standards of the National Council of Teachers of Mathematics (NCTM) as described in *Principles and Standards for School Mathematics* (NCTM, 2000); the content standards of the National Assessment of Educational Progress (NAEP); and state curriculum standards from North Carolina, California, Florida, Illinois, and Texas. This review resulted in the development of a list of specific concepts and skills which spanned the mathematics content typically incorporated in Kindergarten through Geometry and Algebra II. In 2010, The Quantile Framework was aligned to the content standards of the Common Core State Standards - Mathematics.

The developed list of specific mathematical skills and concepts was ordered to comprise a taxonomy. The elements used to develop the taxonomic organization were mathematical content strands:

- Number and Operations
- Geometry
- Algebra
- Data Analysis
- Probability and Statistics
- Measurement

The grade at which the content typically first appears in the curriculum and a unique identifying number are assigned to the specific concept or skill. The identifying element is a number called the QTaxon. This moniker is a combination of the words "Quantile" and "Taxonomy". The name was created to facilitate streamlined examinations and efficient uses of the framework. Several examples of specific QTaxons are found in Table 1 and Table 2 below.

TABLE 1: Sample of QTaxons for Grade 5 by QTaxon Number

QTAXON	Strand	Grade	Description
QT 205	Geometry	5	Classify plane figures according to type of symmetry (line, rotation)
QT 208	Algebra	5	Solve one-step equations and inequalities
QT 211	Probability and Statistics	4	Organize, display and interpret information in stem-and-leaf plots

TABLE 2: Examples of Emerging Mathematician (EM) Number and Operations Skills and Concepts With an Associated Quantile Measure

Skill	Quantile Measure
Read, write, and count using whole numbers; rote count to 30	EM
Use ordinal numbers beyond tenths to describe order	EM

Determining the Difficulty of QTaxons

After developing the taxonomy with more than 500 specific mathematics skills and concepts, the next step was to determine the relative difficulty of each QTaxon and to ensure coherency.

To accomplish this goal, field tests were conducted. Test items were written for each QTaxon. During 2004, the field tests were administered in California, Florida, Illinois, New York, North Carolina, and Texas. The results from more than 9,800 students in Grades 1 through 11 were analyzed to determine which QTaxons were more difficult and which QTaxons required less mathematics understanding. Using this empirical data, the psychometricians at MetaMetrics were able to quantify the relative difficulty of each of the included mathematics skills and concepts. Additional field testing with more than 20,000 students was completed and results analyzed to further validate the actual taxonomy with evidence that allowed the difficulty of the QTaxons to be further organized into target, prerequisite, and impending skills and concepts.

The Role of the Knowledge Clusters

Mathematics as a discipline builds from a set of initial concepts and skills. As students expand their knowledge of developmental skills and concepts, the potential to learn new mathematics continues to grow. Similarly, in order to learn a new mathematics skill or concept, the prerequisite or underlying mathematical knowledge must be understood.

Given the interconnectivity on which mathematics is built, the designers of the Quantile Framework recognized that describing the links between discrete QTaxons would produce a much more useful tool. Thus, the notion of Knowledge Clusters was developed, and the QTaxons were further refined so that the prerequisite skills and concepts and the impending skills and concepts (the mathematics to come next) would be explicit.

Each QTaxon has a unique Knowledge Cluster that illustrates the different types of relationships and connections across the strands that create a cohesive mathematics. Each Knowledge Cluster includes:

1) **The Target Skill**

2) **Prerequisite Skills** (prerequisite knowledge, or the skills and concepts that are needed to work successfully with the target skill)

3) **Supporting (or Supplemental) Skills** (the skills and concepts that tend to emerge with the target skill)

4) **Impending Skills** (the skills and concepts that follow the target skill; the learning that follows the particular skills)

For example, in order to add two- or three-digit numbers without regrouping (QTaxon 79; EM), students first should understand addition and subtraction facts to ten (Prerequisite Skill) (QTaxon 41; EM).

Once mastered, the student is ready to learn how to add two- and three-digit numbers without regrouping (Impending QTaxon (QT598; 90Q).)

Each Knowledge Cluster reflects a specific skill, topic, or concept along with prerequisite, impending, and supplemental content (the mathematics that supports the learning of content of the target QTaxon.) Taken together, the information in a Knowledge Cluster provides coherent mathematical progression that can be used to make what students learn actionable.

By knowing where an important mathematical idea resides in the taxonomy, we can determine what mathematics students need to know prior to instruction on that topic, what mathematical ideas support the learning of that topic, and what mathematics students will be prepared to learn once that content is mastered. An example of a Knowledge Cluster for fraction multiplication is included in Table 3 on the following page.

TABLE 3: Sample Knowledge Cluster for Fraction Multiplication

QTaxon ID	Quantile Measure	Description
Targeted QTaxon		
QT-N-224	820Q	Multiply two fractions or a fraction and a whole number.
Prerequisite QTaxons		
QT-N-546	610Q	Model and identify mixed numbers and their equivalent improper fractions.
QT-N-160	600Q	Find the fractional part of a whole number with and without models and pictures
QT-N-668	590Q	Write and simplify equivalent fractions.
QT-N-121	180Q	Use multiplication facts through 144.
Supplemental QTaxons		
QT-N-155	710Q	Compare and order fractions using renaming strategies.
QT-N-222	690Q	Find factors, common factors, and the greatest common factor of numbers; explain.
QT-P-285	920Q	Determine the probability of compound events (with and without replacement).
QT-N-162	350Q	Know and use division facts related to multiplication facts through 144.

Unidimensionality of the Measure

The Quantile Framework, built on a taxonomy, provides a measure that describes a student's readiness for mathematics instruction within any mathematical content area. The Quantile Measure provides a student's level of readiness for math instruction in a single unit which does not fragment the essential connected nature of mathematics (cf. NCTM's Connections Standard as described in Principles and Standards for School Mathematics, 2000). By considering the complexity of the whole of school mathematics on a single scale, we can shift our thinking away from students excelling at different areas of mathematics towards providing level-appropriate instruction for all students across all content areas.

Advancing mathematics skills and concepts are dynamic and complex, usually involving concepts from multiple strands, therefore a Quantile measure can be viewed as a point in the continuum of skills and concepts inherent in mathematical development.

In terms of intervention, the unidimensionality of the Quantile measures provides an entry point to rebuild foundations. Because of the interconnected nature of mathematics, identifying deficits in learning by strand or sub skill alone will not sustainably serve to raise students' overall mathematical understanding. For this reason, the Quantile measure is a unidimensional measure which is not fragmented into strand-based scores.

USING THE QUANTILE FRAMEWORK TO INFORM INSTRUCTION

With the adoption of the Common Core State Standard – Mathematics (CCSS-M) comes a shift in what students are expected to learn and when. With a linking study to the CCSS-M completed, the Quantile Framework provides educators with student learning data that can assist them in multiple ways. According to MetaMetrics (2009), these include:

- monitoring student mathematics growth and learning progress
- forecasting student performance on end-of-year assessments
- matching students with appropriate instructional materials
- determining if students are ready to learn a new mathematics skill or concept
- linking big mathematical concepts with state curriculum objectives
- understanding the prerequisite skills in order to create an effective intervention
- adapting instructional methods in the classroom to ensure a greater level of understanding and application

The CCSS-M provides a coherent, focused curriculum designed to prepare students for success in college and career. Fundamental to this success is completion of Algebra I (NMAP, 2008). Quantile Measures allow teachers to set growth goals for students as they prepare for Algebra I, forecast learning outcomes for completion of important algebra ideas, and release content through appropriated scaffolded instruction.

Screening and Placement in Tiered Instruction

The Quantile Framework is a criterion-referenced and a norm-referenced tool. Its student measures can also be used to monitor growth and make placement recommendations.

Quantile Framework directly supports the practice of providing high-quality instruction and interventions that match students' learning needs. Learning communities that use a tiered model to deliver increasingly intense educational services will find that Quantile measures promote systematic, data-driven processes for determining the success of implementation strategies.

A tiered system of instruction or a Response to Intervention Framework (RtI) is fully complemented by the Quantile Framework, which through an aligned assessment tool supports at each tier, universal screening; instructional placement; and progress monitoring for core instructional programs (Tier I), strategic instructional programs (Tier II), and intensive intervention (Tier III).

By providing students with instruction centered on developmentally appropriate content, we can enhance the performance of all students, including those with learning disabilities.

Grade Level Expectations

Quantile Measures indicate content material that students should be ready to learn, and are designed to take into consideration grade-level expectations.

The linking studies conducted by MetaMetrics in 2004 observed the level of mathematics understanding of students in each grade. The table below shows the performance standards derived from these linking studies extrapolated in four categories across Grades 2-8.

By understanding where a student's score aligns in the national normed data table below, and by tracking his or her progress, teachers can begin to anticipate the level of success that the student will have in his or her current grade-level curriculum, and plan a course of intervention to raise math achievement.

TABLE 4: SMI Performance Standards Data

Grade	Below Basic	Basic	Proficient	Advanced
2	At or Below 100Q	105Q to 215Q	220Q to 420Q	At or Above 425Q
3	At or Below 215Q	220Q to 395Q	400Q to 520Q	At or Above 525Q
4	At or Below 350Q	355Q to 465Q	470Q to 720Q	At or Above 725Q
5	At or Below 550Q	555Q to 675Q	680Q to 820Q	At or Above 825Q
6	At or Below 640Q	645Q to 775Q	780Q to 950Q	At or Above 955Q
7	At or Below 700Q	705Q to 885Q	890Q to 1040Q	At or Above 1045Q
8	At or Below 800Q	805Q to 1025Q	1030Q to 1140Q	At or Above 1145Q

Differentiated Instruction

Differentiation is a philosophy about teaching and learning that allows educators to provide learning opportunities that best meet the needs of each of their students. Not all students at each age, in each grade, are ready for the same mathematics instruction at the same time. Students have different experiences and interests that impact how they learn (Tomlinson et al, 2003). Since all students have a right to a high-quality, demanding mathematics curriculum with the appropriate learning support (NCTM, 2000), the goal of differentiation is to provide instruction that works best for each student to prepare them student for future success in mathematics, college, and career.

Three components of instruction should be differentiated for students: content, process, and product. That is, educators should consider what mathematics students are to learn, how they will learn it, and how their learning will be assessed (Tomlinson, 2001).

The use of Quantile measures in instruction support the practice of differentiated instruction. Once a student's Quantile measure is known, instruction can start with materials and resources that learners are forecasted to understand with ease. The Quantile Framework defines readiness for instruction at the threshold where students will understand the correspondingly calibrated material 50% of the time without prior instruction on the topic.

Quantile measures help teachers as they prepare to instruct on grade-level expectations, but with access to the Knowledge Cluster that is linked to the target topic, the teachers now have tools to differentiate math in the core instruction classroom.

Students who are not prepared to learn the content will need reinforcement of prerequisite content. The associated Knowledge Clusters include all prerequisite mathematics skills and concepts students should know before they are ready for instruction on the target skill and concept. If a student does not have the prerequisite knowledge needed for intended instruction, the teacher can differentiate instruction by modifying the pace and delivery of the prerequisite mathematical concepts and skills prior to instruction on new content. When teachers use instructional time to rebuild understanding of the prerequisite skills first, they avert unnecessary struggle when the lesson at hand is introduced. Additionally, with Quantile measures used in the differentiation of math instruction, the teachers can avoid repeated instruction on skills and concepts a student has already mastered. Instruction can be targeted to accelerate learning.

Consider a classroom where the Quantile Framework is utilized. Here teachers can align instruction with student results to provide optimal learning experiences for all students. For example:

i. Proficiency Students

When student's Quantile measures are in the optimal range for learning, between 50Q above and below the content to be taught, they are ready for grade-level instruction and few adjustments are needed to the curriculum materials. Instruction should be geared to their interests and mathematical self-concepts. Also, effective differentiation employs multiple instructional methods. Consider how to use small groups, large groups, and individualized practice in a way that engages these students in multiple ways.

ii. Advanced or Highly Proficient Students

Advanced or highly proficient students' Quantile measures are more than 50Q above the content to be taught. These students tend to possess the foundational knowledge and skills that comprise the instructional path. In these cases, instruction can be adapted in one of four ways:

1. Acceleration: Exploring similar content more deeply
2. Enrichment: Extending the mathematical content being studied
3. Sophistication: Infusing more complex, related ideas into study
4. Novelty: Incorporating new content into study that is developmentally appropriate but typically off-curriculum (Van de Walle, Karp, & Bay-Williams, 2010)

If advanced students are presented with mathematics below their ability level, little or no learning will occur. Therefore, it is important to present them with meaningful mathematics that is moderately challenging for their level of mathematical development (Tomlinson et al., 2003).

iii. Basic or Below Basic Students

Underperforming students (basic or below basic students) typically receive a Quantile measure more than 50Q below the skills or concepts to be taught. Where gaps between students' readiness for instruction and the expected Quantile range of the mathematics of that grade level are widely disparate, an accommodation (perhaps outside the core instruction classroom) should be made. Given that other measures of student performance exist, each of these should be weighed in making a decision about which students may need extending activities and which may need intervention services.

Instruction on mathematics at an appropriate level is an excellent way to motivate students to engage more with mathematics; and by engaging more, students can ultimately achieve more (Tomlinson, 2008).

Finding Student Measures and Math Resources

Student Quantile Measures can be found in assessments that are linked to the Quantile Framework, such as the *Scholastic Math Inventory,* (SMI). This research-based, computer-adaptive math assessment assesses students in Grades 2–12 as they prepare for instruction through Algebra I, the course identified as the gateway to college and career readiness. SMI is recommended to be used three to five times a year to monitor the growth of students' mathematics learning.

SMI has an item bank of more than 5,000 questions across five strands of mathematics: number and operations, geometry, measurement, algebra, and data analysis and probability. Results are reported in both criterion-referenced and norm-referenced terms, indicating a student's mathematical performance on the Quantile scale and how his or her test results compare to those of other students.

Upon completing the assessment, each student receives a Quantile measure, which identifies his or her performance level. SMI reports provide instructional recommendations for each student and for groups of students. These recommendations are in a range of 50 Quantile above and 50 Quantile below the student's Quantile measure. This range represents the student's learning frontier – the range where topics and concepts are located for which the student has the greatest likelihood of showing success.

Additionally, Quantile alignments are available for many basal textbooks and math interventions. MetaMetrics has developed a searchable textbook database that provides alignment of textbook lessons to QTaxons. The database can be found at www.scholastic.com/SMI

BECOMING COLLEGE AND CAREER READY

Researchers have observed that one major predictor of college and career success is the number of advanced high school mathematics courses a student has completed. http://www.nsf.gov/statistics/nsb0602/

In addition, students who take advanced mathematics courses in high school also perform significantly better in college science courses. http://www.nsta.org/publications/news/story.aspx?id=51939

Those same students who take advanced mathematics courses in high school are more likely to find career success in higher paying jobs that require application of advanced math concepts. By 2016, more jobs will be added in computer and mathematical occupations than in any other category (ACT, 2005; Pearson, 2009). To better understand the mathematical demands on workers in the future, the mathematical skills needed to be successful as an Information Technology Technician have been identified by Achieve, Inc. (2006) as part of the MathWorks project.

The skills needed are listed below.

TABLE 5: Math Demand for Information Technology Technician

QTaxons Needed	Quantile Measure
Use rules of exponents to simplify numeric and algebraic expressions.	1000Q
Evaluate expressions and use formulas to solve problems involving exponential functions (e.g., growth and decay).	1050Q
Compute probabilities using combinations and permutations.	1070Q
Use linear programming (systems of three or more inequalities) to solve problems.	1300Q
Identify appropriate mathematical models to represent real data sets (constant, linear, quadratic, cubic, square root, absolute value, reciprocal, trigonometric, exponential, logarithmic, piecewise, and greatest integer).	1400Q

While the skills listed are necessary to be successful, the level of proficiency with the skills needs to be at a higher level than if a person were simply ready for instruction on the topic.

An Information Technology Technician would need a mathematics achievement level in the range of about 1300-1450Q, which is an estimate much higher than the typical range of students in high school.

Since the 2005 National Education Summit on High Schools, states have made significant progress closing the gap between what is expected of students in high school and the expectations they face in college and 21st century jobs. Many states, and now the nation as a whole, are actively discussing standards that describe what it means for a student to be "college- and career-ready."

The National Governors Association, the Council of Chief State School Officers, Achieve, The College Board, and ACT have developed these standards and published them in conjunction with the Common Core State Standards Initiative in September 2009. Among the goals of this initiative, many coincide with the purpose of the Quantile Framework and its partner assessment SMI. These standards:

- Align with college and work expectations
- Are clear, understandable, and consistent
- Include rigorous content and application of knowledge through high-order skills
- Build upon strengths and lessons of current state standards
- Are informed by other top performing countries, so that all students are prepared to succeed in our global economy and society
- Are evidence and research-based

Using SMI and the Quantile Framework, educators are able to place both students and mathematical resources on the same scale, making it possible to define and close gaps between a student's level of achievement and the level of mathematical understanding members of society need in order to be productive and successful.

SUMMARY

Many times in school mathematics a teacher introduces a new concept or skill that all students are expected to learn. This concept or skill may be identified in the adopted content standards, found in the district's scope and sequence for the course, or the teacher may simply judge this concept or skill to be fundamental to the mathematics of the particular course. A diligent teacher presents this new mathematics and then uses any number of methods to determine which students have learned the new material and which students have struggled and not learned the target topic or concept. Often, teachers have useful insights they can use to make their assessment of students' new learning more efficient, but the fact remains it is necessary to determine what students have and have not learned. Then, given this knowledge, a teacher must make decisions about what instructional next steps are most appropriate for particular students. The Quantile Framework can support teachers in this process by providing student data that better allows them to do the following:

- Forecast student performance
- Monitor student growth
- Differentiate instruction
- Motivate students through goal-setting
- Communicate student learning

Additionally, the Quantile Framework provides a tool to identify and match resources within the school to meet the needs of every learner. In general, educators are required to teach specific skills and concepts (QTaxons) at specific times or in specific grade levels. Teachers now have tools to differentiate instruction in core math classroom.

Quantile measures are valuable in the hands of educators who know how to evaluate and use the information they provide and who create an environment that fosters each student's mathematical development.

Quantile measures are particularly useful as a mathematical indicator when they are tracked over time to show a growth in students' mathematical understanding. Quantile measures can be used to set measurable goals, monitor, and evaluate mathematics curriculum and instruction, and show quantifiable growth.

FAQS

Q: What is a QTaxon?
A: A taxon is the root of the word taxonomy, meaning a taxonomic category or group. In terms of the Quantile Framework, a QTaxon defines a specific mathematical concept or skill and is used to annotate the framework.

Each QTaxon has a Quantile measure (expressed as a number followed by the letter "Q") which estimates its solvability (how well an individual will likely understand the concept or skill prior to instruction) in the taxonomy of the framework. QTaxons are linked to supplemental and prerequisite QTaxons which illustrate the interconnectivity of the Quantile Framework and the natural progression of mathematical skills needed to solve increasingly complex problems. The Quantile Framework comprises approximately 500 QTaxons which educators can use to monitor progress and target instruction by comparing a student's Quantile measure with the measure of a particular QTaxon. Each QTaxon aligns with one of five content strands—Numbers and Operations, Geometry, Algebra/Patterns & Functions, Data Analysis & Probability, and Measurement.

Q: How is a Quantile measure assigned to a skill or concept?
A: When given for a math skill, the Quantile measure refers to the understanding of this skill or concept, or how difficult tasks related to the skill are compared to other tasks. Actual performance by students on an assessment was used to determine the Quantile measure of a QTaxon empirically. Two large field studies were conducted with more than 35,000 students. The items initially associated with each QTaxon were reviewed by subject matter experts and accepted for inclusion in the set of items, moved to another QTaxon, or not included in the set. Grade levels for items were required to match the grade level of the introduction of the skill or concept as derived from review of national curricular frameworks. The Quantile measure of a QTaxon is defined as the mean Quantile measure of items that met the criteria.

Q: Why the emphasis on readiness and introductory problems?
A: The Quantile theory predicts the solvability of individual concepts. Introductory problems tend to be straightforward assessments of concept knowledge. More advanced problems that blend with other concepts cloud the picture in terms of predicting the difficulty of the primary concept.

Q: Why is there not a Quantile measure for each strand of mathematics?
A: A student's Quantile measure describes overall readiness for mathematics instruction. A student will not receive a separate Quantile measure for each of the five strands—the strands are so interconnected (correlated) that only one score will be reported. This one score can be used to describe how a student is performing across the five strands.

Q: Should a student's Quantile measure exactly match a skill or concept's Quantile measure before instruction takes place?
A: The Quantile measure for a student is the level at which he or she is ready for instruction and has knowledge of the prerequisite mathematical concepts and skills necessary to succeed. When a student's Quantile measure does not match the skill or concept's Quantile measure, then additional

support (e.g., review and/or instruction with prerequisite QTaxons, more individualized/differentiated teaching, small-group instruction) may be required for a successful instructional experience.

TABLE 6: Probable Level of Success Prior to Instruction Based on Quantile Match

Quantile measure of the skills and concept	Student's Quantile measure	Probable success prior to instruction
750Q	250Q	10%
750Q	500Q	25%
750Q	750Q	50% <- Learning Frontier
750Q	1000Q	75%
750Q	1250Q	90%

When targeting instruction, the optimal range to present material is 50Q below and 50Q above the student's Quantile measure. This range, often called the learning frontier, is where a student is expected to have 50% success with the materials prior to instruction, with reliable knowledge of the prerequisite mathematical skills and concepts needed for the instruction. The learning frontier is associated with the target QTaxon —the immediate skills and concept the teacher is to convey.

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