MATH@WORK
CONNECTING MATH TO 21ST CENTURY CAREERS

Lesson 2

A Model Comparison

In this lesson, students will graph two equations on a coordinate plane to compare earnings over a given period.

TEACHER

GRADES 6-8 INSTRUCTIONAL FOCUS

- Generate 1-variable linear equations.
- Write equations for $x + p = q$ and $px = q$, where p, q and x are all nonnegative rational numbers.
- Solve problems leading to equations of the form $px + q = r$ and $p(x + q) = r$.
- Explain what a point (x, y) on the graph of a proportional relationship means.
- Solve problems leading to two 2-variable linear equations.

LANGUAGE SUPPORT

Math Terms

linear equation
an equation in two variables that results in a straight line on a graph.

slope
the steepness, or rate of change, of a line.

Academic Language

revenue
a person or company's income before paying expenses.

subscription model
a pricing model that has customers pay the same amount on a regular basis for continued access to a product.
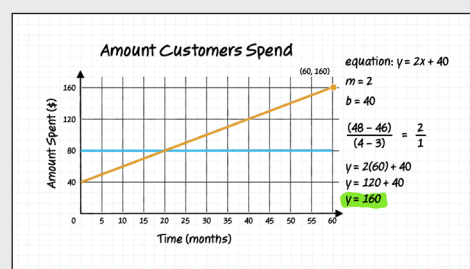
SET UP

Introduce Chapter 3 from *Math Meets Entrepreneurship.*

Ask questions to introduce Lesson 2.

For example: **Who can tell us what a linear equation is?** (A linear equation forms a straight line when it is graphed). **How can you find the slope of a linear equation?** (It is the x coefficient m when the equation is in the form $y = mx + b$).

Introduce the academic vocabulary word revenue. **Today, we'll compare two different pricing models to determine which provides more revenue over the course of 5 years.**

▶ Play Chapter 3: A Model Comparison

Amount Customers Spend

equation: $y = 2x + 40$
$m = 2$
$b = 40$
$\frac{(48 - 46)}{(4 - 3)} = \frac{2}{1}$
$y = 2(60) + 40$
$y = 120 + 40$
$y = 160$

[Pause at 6:55]

PLAN

Create a strategy to solve the problem.

In Plan A, a customer pays $80 per wristband. In Plan B, the customer pays $40 per wristband and an additional $2 per month for access to the app. Compare the plans over the course of 5 years.

- When is Plan A better than Plan B?
- When is Plan B better than Plan A?
- Which pricing model would you choose? Explain your reasoning.

Read the problem aloud to students.

Cover the numbers (e.g. with paper) in the problem. Lead a discussion about how students would solve this problem.

For example: **What is this problem asking us to do?** (compare the revenue that each plan would bring in over the course of 5 years.) **What is the first step you will take to solve this problem?** (Represent each plan mathematically.)

Point out that the words "per month" mean that this is a repeated charge that occurs every month.
For example: **How can we find the number of months in x years?** (multiply x by 12.)

Point out that you will do this by finding out how much a customer on each plan would spend over 5 years.

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Lesson 2

A Model Comparison (continued)

Mathematical Thinking: Attend to Precision

Students are careful about specifying units of measure and labeling axes to clarify the relationship between quantities in a problem.

SOLVE

Have student pairs solve the problem as you circulate.

Encourage students to come up with multiple strategies and represent the problem situation in different ways. Guide students to work backwards to check their work.

Support

Ask questions based on common errors to support student understanding.

- *What units of time are given in the problem? How will that affect your graph?*
- *How many months are in 5 years?*
- *What scale will you use for the x-axis?*

Extend

Ask questions to encourage students to expand their thinking.

- *How much do you think is reasonable to charge for the wristband and the app?*
- *Graph your pricing model. How does it compare to the other two after 1 year? 5 years? 20 years?*

SHARE

Have students present their solutions.

Ask students from each pair to explain their solutions to the class. Show at least two different approaches to solving the problem and one incorrect solution. To extend classroom discussion, call on students to explain the reasoning of the student who is presenting.

POSSIBLE STUDENT WORK:

Let x = months
Let y = revenue ($)

Plan A: $y = 80$
Plan B: $y = 40 + 2x$

$$80 = 40 + 2x$$
$$80 - 40 = 2x$$
$$40 = 2x$$
$$20 = x$$

20 months = 1 year 8 months
The revenue for both plans is equal at 1 year 8 months.

Before 1 year 8 months:
Plan A: $y = 80$
Plan B: $y = 40 + 2(19)$
$\quad\quad = 40 + 38$
$\quad\quad = 78$
78 < 80

After 1 year 8 months:
Plan A: $y = 80$
Plan B: $y = 40 + 2(21)$
$\quad\quad = 40 + 42$
$\quad\quad = 82$
80 < 82

So, Plan A is better before 1 year 8 months and Plan B is better after 1 year 8 months.

▶ **Play the Chapter 3 Solution from** *Math Meets Entrepreneurship.*

PRACTICE

Have students complete the Practice and Reflect sections on Student Page 2 in class or as a homework assignment.

Students choose a pricing model for a GPS dog collar and app.

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Name: _____

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PLAN

Create a plan to solve the problem with your partner.

1. Write each plan as an equation.
2. Create a table showing revenue at different times.
3. Compare the revenue between 0 and 5 years.

SOLVE

Use your strategy to solve the problem.

POSSIBLE STUDENT WORK:

Plan A	$y = 80$	horizontal line because y is a constant
Plan B	$y = 40 + 2x$	line through the point (0, 40) with a slope of 2 line is going up

5 years = 12 × 5 months = 60 months

Months	Plan A Revenue	Plan B Revenue
0	$80	$40 + $2(0) = $40
20	$80	$40 + $2(20) = $80
40	$80	$40 + $2(40) = $120
60	$80	$40 + $2(60) = $160

I would choose Plan B because it creates more revenue after 20 months.

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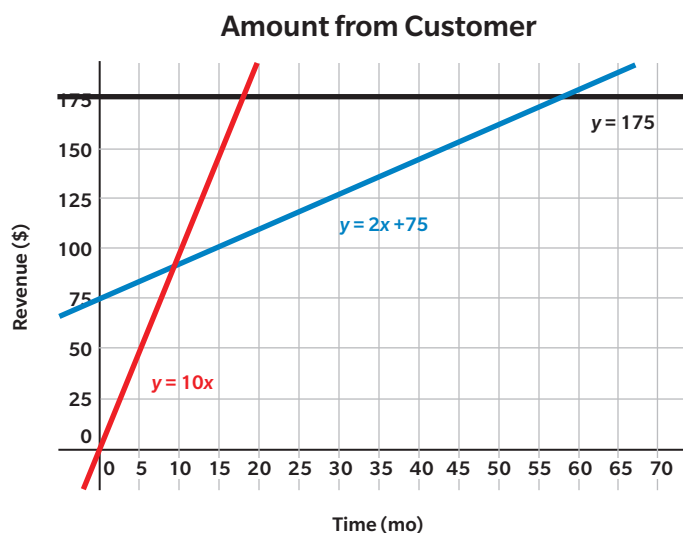
A Model Comparison (continued)

PRACTICE

Apply your skills to solve another problem.

You've designed a GPS collar and app that tracks and displays a dog's activity throughout the day. Other GPS collars on the market cost between $150 and $200. Graph all three pricing models. Choose the pricing model that will work best for your company and explain your choice.

Model 1: $175 collar, free subscription

Model 2: $75 collar, $3/month subscription

Model 3: free collar, $10/month subscription

Amount from Customer

Graph: Revenue ($) vs. Time (mo). Lines shown: $y = 175$ (horizontal), $y = 2x + 75$, $y = 10x$.

POSSIBLE STUDENT WORK:

I would charge $175 because it gives the revenue right away instead of having to wait 4 years for it, and I don't think people will pay $10 a month for the app.

REFLECT

Explain how you made sense of the math by listing pros and cons of models.

One-Time Sale Model		Subscription Model	
Pros	**Cons**	**Pros**	**Cons**
• Get the money right away even if people stop using the device.	• Not able to continue making money on one sale • Need more customers	• Can keep making money off one sale • Don't need as many customers	• Have to wait for the money • Customers might unsubscribe

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Lesson 2 — A Model Comparison (continued)

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