RESEARCH FOUNDATIONS:

EVIDENCE BASE

HMH Into AGA®

THE HMH RESEARCH MISSION STATEMENT

Houghton Mifflin Harcourt® (HMH®) is committed to developing innovative educational solutions and professional services that are grounded in learning science evidence and efficacy. We collaborate with school districts and third-party research organizations to conduct research that provides information to help improve educational outcomes for students, teachers, and leaders at the classroom, school, and district levels. We believe strongly in a mixed-methods approach to our research, an approach that provides meaningful and contextualized information and results.

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INTRODUCTION

Deep understandings of mathematics and well-honed abilities in mathematical thinking are critically relevant for today's students. Careers in science, technology, engineering and mathematics (STEM) are growing dramatically and grew much faster than employment in non-STEM occupations over the last decade (24.4 percent versus 4.0 percent, respectively). STEM occupations are projected to grow by 8.9 percent from 2014 to 2024, compared to 6.4 percent growth for non-STEM occupations. "For the United States to continue its technological leadership as a nation requires that more students pursue educational paths that enable them to become scientists, mathematicians, and engineers," as noted in *Adding It Up: Helping Children Learn Mathematics* (National Research Council [NRC], 2001, p.16).

The National Council for Teachers of Mathematics (NCTM®) continues to lead efforts to strengthen math teaching and learning. Most recently, NCTM has put a spotlight on high school mathematics with "Catalyzing Change in High School Mathematics: Initiating Critical Conversations" calling for all stakeholders to engage and improve teaching and learning mathematics at the high school level (NCTM, 2018. p. 6). NCTM enunciated the Curriculum Principle, which notes that excellent mathematics programs include curriculum that develops important mathematical concepts along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world (NCTM, 2014). In its expansive research in mathematics teaching and learning, NCTM promotes the idea that "knowing the mathematics curriculum for a particular grade level or course is sufficient to effectively teach the content to students" is an unproductive belief (NCTM, 2018). "Mathematics teachers need to have a clear understanding of the curriculum within and across grade levels—in other words, student learning progressions—to effectively teach a particular grade level or course in the sequence" (NCTM, 2014, p.72).

Houghton Mifflin Harcourt's ***Into AGA*™ © 2020** is an intentional, comprehensive, and inspiring mathematics solution for Algebra 1, Geometry, and Algebra 2 that centers on student growth. Growth is maximized when instruction, assessment, and professional learning are coordinated and tightly aligned, and *Into AGA* is structured to support growth in teaching and learning. The solution seeks to maintain the following:

- **Present multiple opportunities for students to persevere** using mathematical modeling practices and inspire them to utilize their mathematical toolkit to solve rigorous and relevant problems.
- **Design and present an intentional curriculum** in which the mathematics presented in a lesson is focused and structured to reflect the realities of managing a classroom and the challenges surrounding delivering effective instruction for each and every student.
- **Provide a comprehensive approach to teaching and learning** in which every aspect of the solution works collectively to drive growth while providing teachers with the tools and a clear pathway to personalize learning.
- **Encourage inspiring mathematics instruction that fosters a growth mindset** and supports students in experiencing success in mathematics.

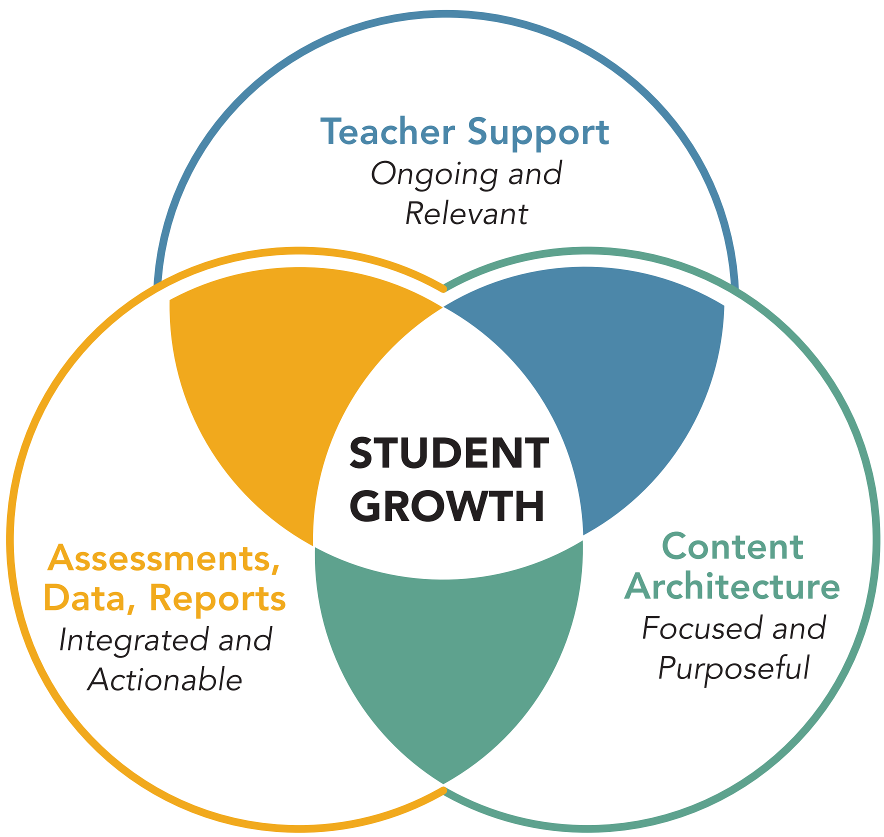
Built upon a foundation of mathematics education research and authored by leaders in the field of mathematics education, *Into AGA* is proven to be effective in raising students' achievement. This document highlights the features of this cohesive, innovative solution while demonstrating explicitly the research upon which it is based.

INSTRUCTIONAL MODEL AND THEORY OF ACTION

SOLUTION CONCEPT

Into AGA is a comprehensive Algebra 1, Geometry, and Algebra 2 learning system in which all the resources have a clear and intentional purpose that supports effectiveness. The solution concept unites a focused lesson structure with clear goals, measurements of student growth, and a teacher support system.

The **content architecture** is focused, purposeful, and coherent. Each lesson clearly outlines the standards and practices, objectives, and learning progressions so teachers can best support student learning. By incorporating a clear path for measuring student growth through **assessment, data, and reports**, the solution offers the best tools and choices for every situation and each goal for student success. ***Into AGA* professional learning empowers and supports teachers to be developers of high-impact learning experiences through embedded and ongoing professional learning**.



KEY THEMES

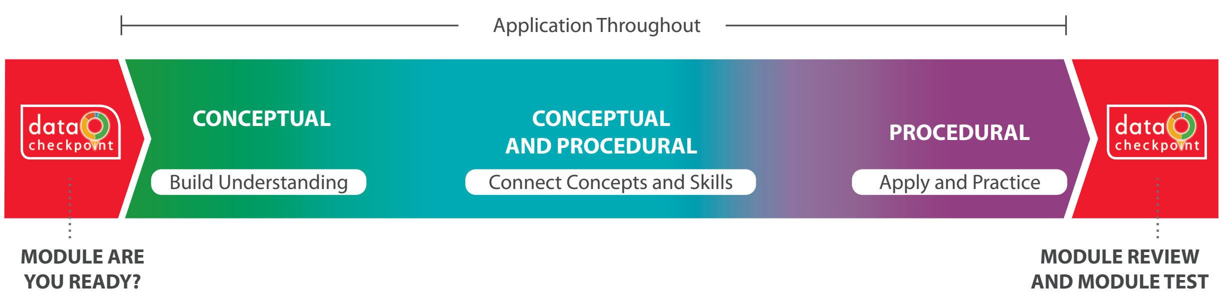
Into AGA presents several **key themes**, which are at the core of the solution. The themes back the solution concept and inspire the teaching and learning set forth for students. The **intentional design** of the solution provides a structure for students to keep them at the center of teaching and learning. There is a **clear path** outlined throughout each lesson of each module that integrates productive perseverance, small-group instruction, and a growth mindset through the inclusion of resources tailored to maximize student growth. The solution offers a **comprehensive assessment design** that focuses on connecting data and instruction so teachers are able to make decisions as necessary. *Into AGA* comprises resources, tasks, and practices designed to instill a **growth mindset**—students are encouraged to focus on expanding their understanding rather than performing (Dweck, 1999). The wide variety of embedded **professional growth** opportunities supports teachers throughout each lesson of each module.

1	2	3	4	5
Intentional Design	**A Clear Path through Every Classroom Moment**	**Impactful Use of Data from Day One**	**Approach to Growth Mindset Grounded in Research**	**Building a Culture of Professional Growth**
Every piece of the solution works in concert with the functional lesson structure, keeping the student and the mathematics at the center	Grouping tools, aligned resources, and access to other grade-level assignments by standard	Strategic tools for a simple approach to formative assessment and cumulative data that inform and enrich the mathematics content	Based on research and implemented every day to inspire and evolve independent thinking	Embedded instruction for consistent teaching combined with personal assistance to professional learning

SOLUTION DESCRIPTION

Learning Arc

The **Learning Arc** maintains the intentional, consistent flow that supports students in making sense of mathematics. From start to finish, the arc offers strategic concepts—beginning with the **module diagnostic** to assess students' prior knowledge and foundational prerequisites. From the diagnostic assessment, the lessons build upon **conceptual and procedural understanding** and integrate applications to build understanding and check for understanding. Purposeful linking tasks connect conceptual understandings to upcoming procedural needs to create a stronger foundational understanding before moving to build procedural fluency that allows students to more easily access needed concepts and skills to persist with rigorous tasks. Formative measures are embedded throughout to support teachers' efforts to differentiate instruction. At the close of each module, there is a module test so teachers can measure mastery of what was taught in that module.



Lesson Design

Every lesson of the *Into AGA* solution follows an intentional design, and the routine is consistent to support familiarity for students—and teachers. There are three task types that align to the type of teaching and learning necessary: Build Understanding for conceptual lessons; Connect Concepts and Skills for bridging conceptual understandings to set the foundation for procedures; and Apply and Practice to help students understand each step in a procedure and its usage. Each task type follows a lesson design as described below. Each lesson includes all pieces of the learning arc. Lessons will maintain a focus on the part of the learning arc that is most appropriate for student understanding.

- **Spark Your Learning:** Teachers work on-level with students to gauge their readiness and to inspire and guide productive perseverance. Spark Your Learning tasks start each lesson. These tasks help students begin to build fluency, choose from the sets of learning strategies, and rely on their previously developed conceptual understanding to solve rigorous tasks. Spark Your Learning tasks provides real-world explorations of practical application of the mathematics to come within the lesson.

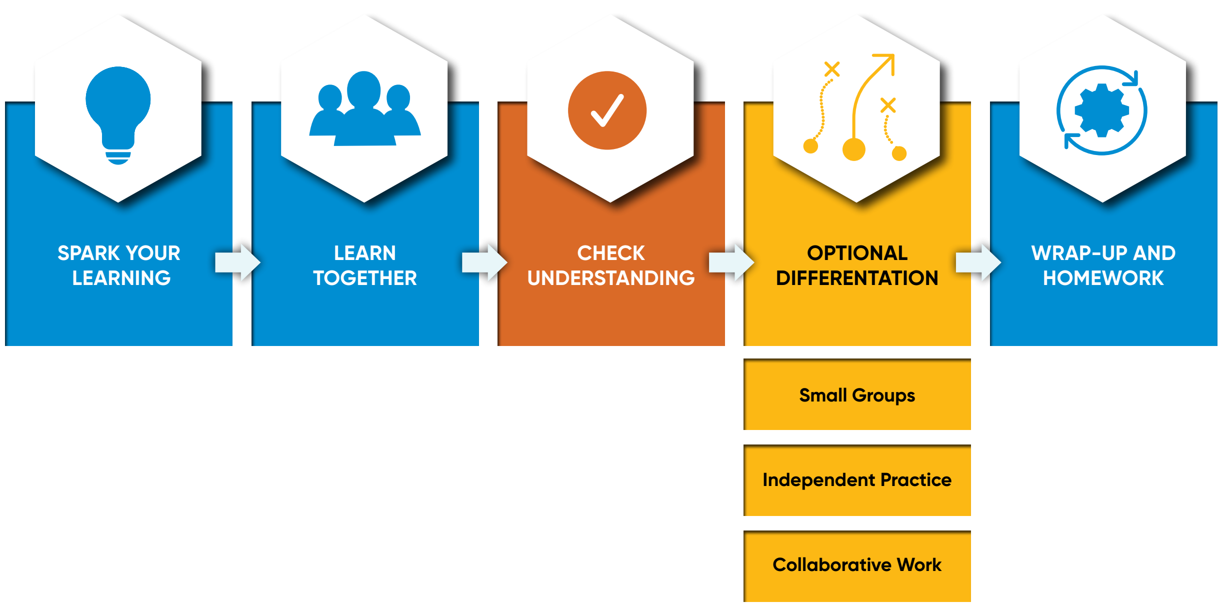
- **Learn Together:** Whole-group learning is facilitated in these Build Understanding and Step It Out tasks. Build Understanding tasks provide an opportunity to help students understand lesson concepts. Step It Out tasks promote procedural understanding.

- **Check Understanding:** After the learning tasks, these five- to ten-minute checkpoints provide a snapshot of what students know.

- **Differentiation Options:** According to student understanding, groups can be formed to ensure growth for each and every student by providing resources based on individual needs. Teachers can then decide how to best support students with differentiated resources for teacher-led groups, small collaborative work-groups, pairs, and individuals. Teachers have access to print and digital tools for use for differentiation.

- **Wrap-Up:** Here is the opportunity for additional practice, reteaching, or intervention. Teachers gauge student depth of understanding with exit tickets or suggested wrap-up ideas.

- **Homework or Practice:** Each lesson includes opportunities for practice and homework within the student print and digital materials. Teachers may also wish to assign additional practice from the Journal and Practice Workbook to strengthen procedural skills.



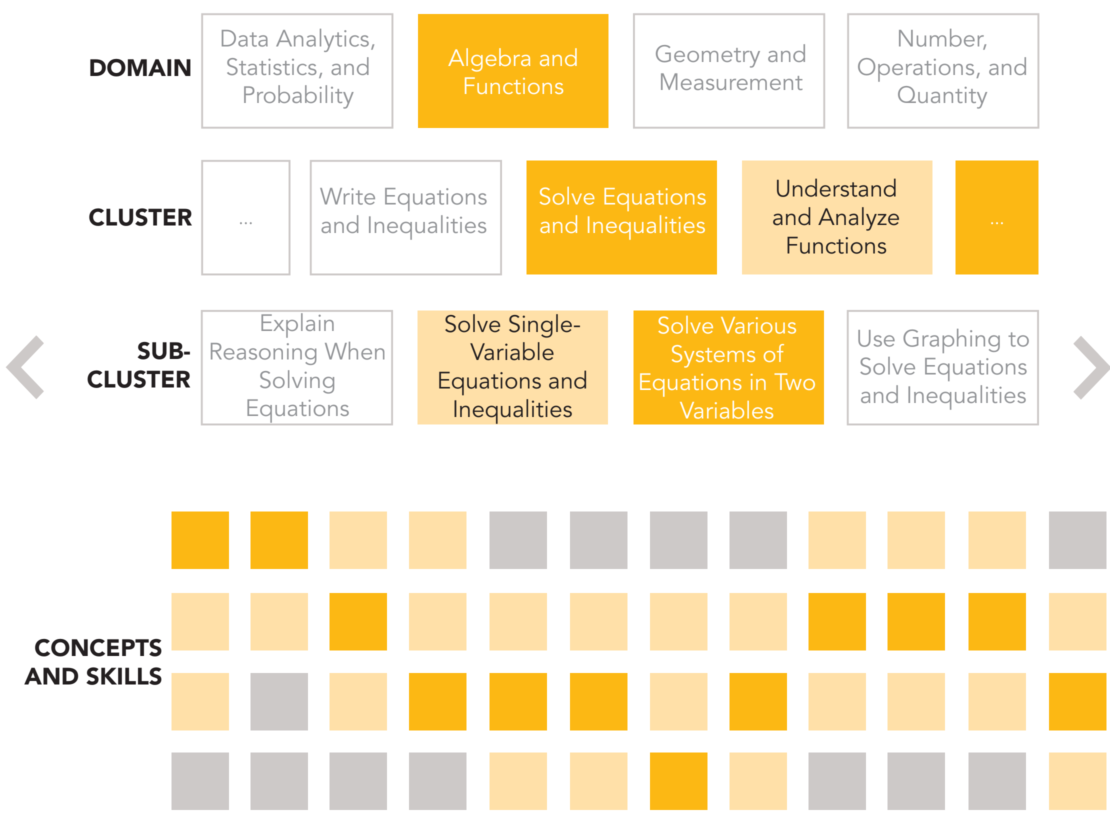
SOLUTION DESCRIPTION (CONTINUED)

Learning Spine

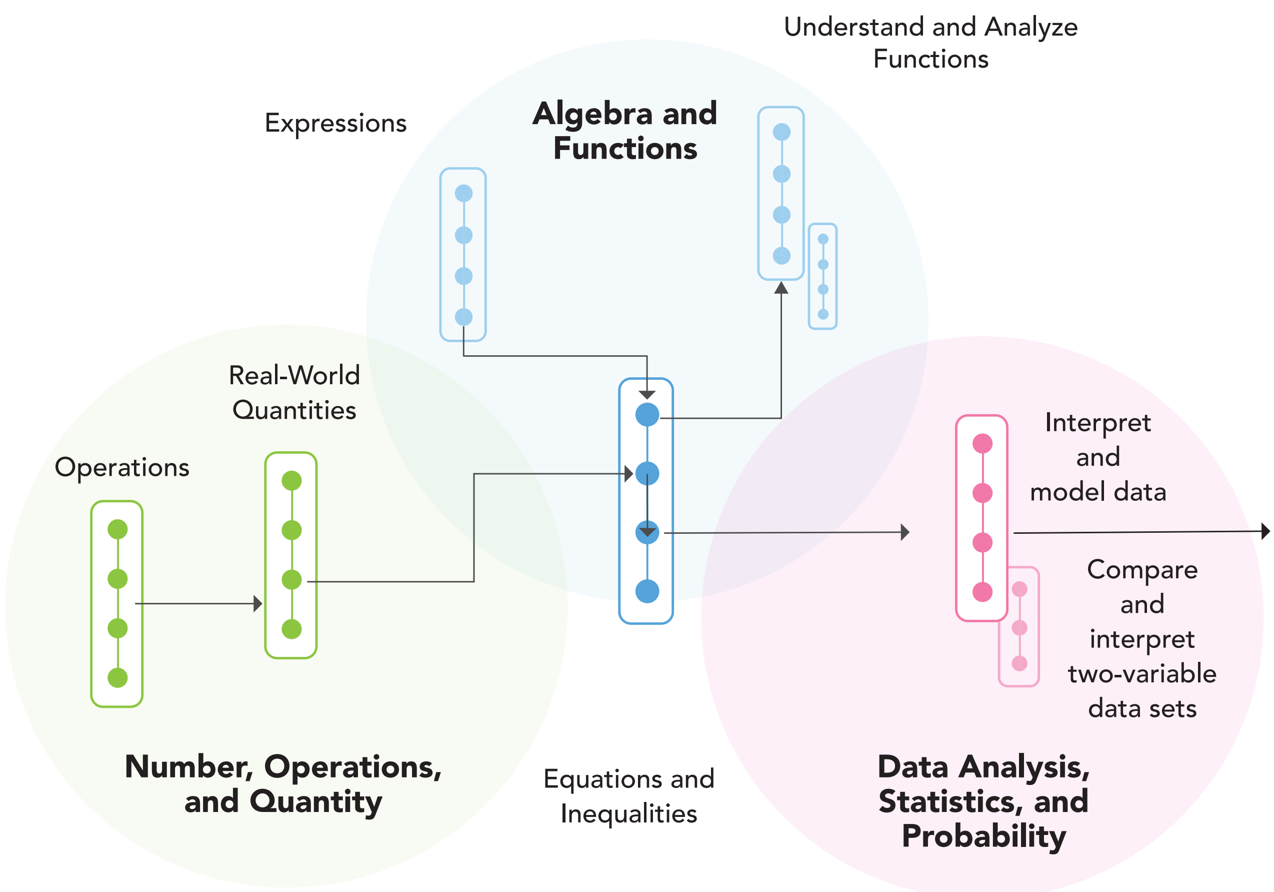
To follow the progression of student learning, the Into AGA solution references HMH's learning spine. A learning spine is a taxonomy of all of the concepts and skills taught across all domains and in all grade levels within a subject area. It also shows connections between concepts and skills, e.g., prerequisites, related skills, and follow-on skills. The learning spine for Into AGA was developed based on a combination of educational standards, HMH program content, and academic research on skill and knowledge development in a subject area.

The learning spine connects instruction and assessment to establish a data feedback loop that supports and optimizes features of each lesson's learning progression. The output of this loop results in data-based insights from student assessment and instructional practice being surfaced for teachers in order to help them plan responsive and differentiated instruction.

The Learning Spine also offers a deliberate look into the content organization and subsequent data which can be used to provide focused recommendations for teaching and learning. The hierarchy of domain, cluster, sub-cluster, and concepts and skills shows the comprehensive progression for acquiring knowledge for each grade. This approach correlates directly to "Achieve the Core" and offers the mapping of granularity necessary for student success.



In secondary grades, students conceptualize expressions by using numbers, symbols, and arithmetic operations. Students continue to build on this understanding to learn and apply relationships with expressions up to evaluating functions. Students apply their skills to model and solve real-world problems utilizing arithmetic expressions.

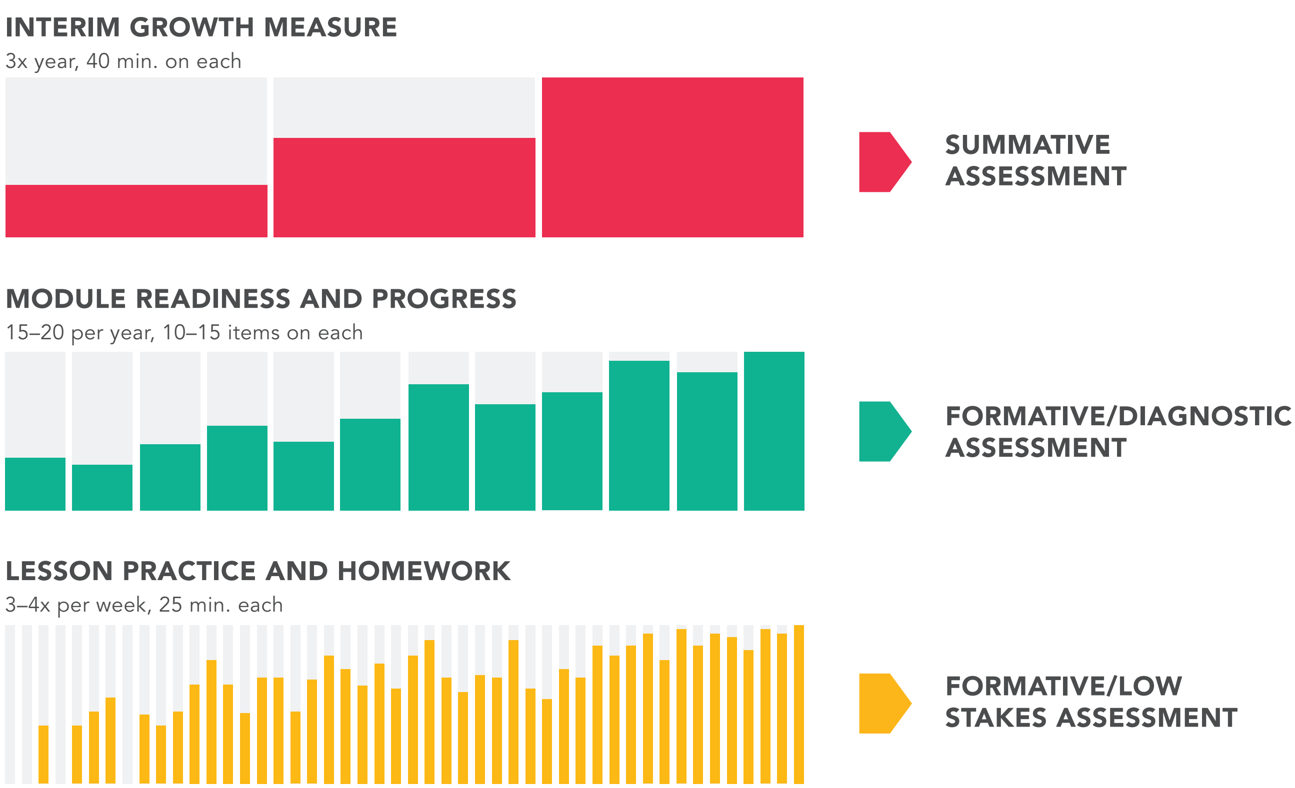


In high school, students build upon previous work with operations to extend to real-world quantities. Facility with real-world quantities enables students to use linear equations to model and solve real-world problems. Students then apply their understanding of linear equations to construct linear models to interpret two-variable data.

SOLUTION DESCRIPTION (CONTINUED)

Assessment Design

By offering an **integrated assessment design, including diagnostic, formative, and summative assessments**, the *Into AGA* solution provides the tools necessary to advance, differentiate, and personalize instruction for each and every student. The solution also adheres to the notion that data are key to effective teaching and to meeting the needs of learners. By incorporating a clear path for measuring, monitoring, and accelerating student growth through **assessment, data, and reports**, the solution offers the best tools and choices for today's classrooms with the goal of **student growth**. Most assessments are offered in paper-and-pencil format but without the technology-enhanced assessment items, data, analytics, and reporting that can be offered digitally. Teachers have access to all assessments online, which saves time on grading and saves efforts in printing materials. Students also receive instant feedback and can see all of their work in one place. The assessment system provides greater equity by enabling teachers to address targeted and specific needs of each and every student in an ongoing and regular manner.



Growth Mindset

Through partnership with Mindset Works®, HMH supports teachers and students with the latest research in growth mindset. Currently, most teachers think of mindset as “I have not gotten it yet,” but students need tools embedded and modeled for them, so mindset is key and seamless. With the *Into AGA* solution, teachers will see how to foster a growth mindset within the lessons, aligned to the content, so that it seeps into the language and practice of the mathematics lessons. Teachers can turn obstacles into success, and students can turn a lack of confidence into grit—knowing they can learn mathematics, even if they haven’t got it yet. Teachers are guided to support students as they learn and apply concepts with focus and tenacity.

Research and best practices drive embedded professional learning when introducing growth mindset.

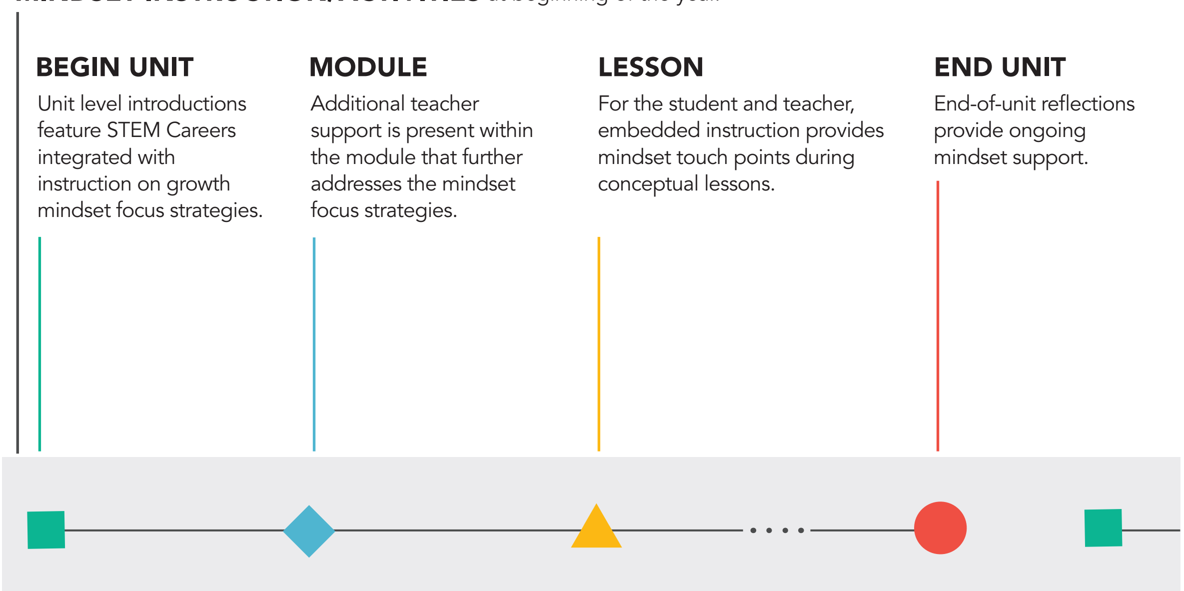
MINDSET INSTRUCTION/ACTIVITIES at beginning of the year.

BEGIN UNIT
Unit level introductions feature STEM Careers integrated with instruction on growth mindset focus strategies.

MODULE
Additional teacher support is present within the module that further addresses the mindset focus strategies.

LESSON
For the student and teacher, embedded instruction provides mindset touch points during conceptual lessons.

END UNIT
End-of-unit reflections provide ongoing mindset support.



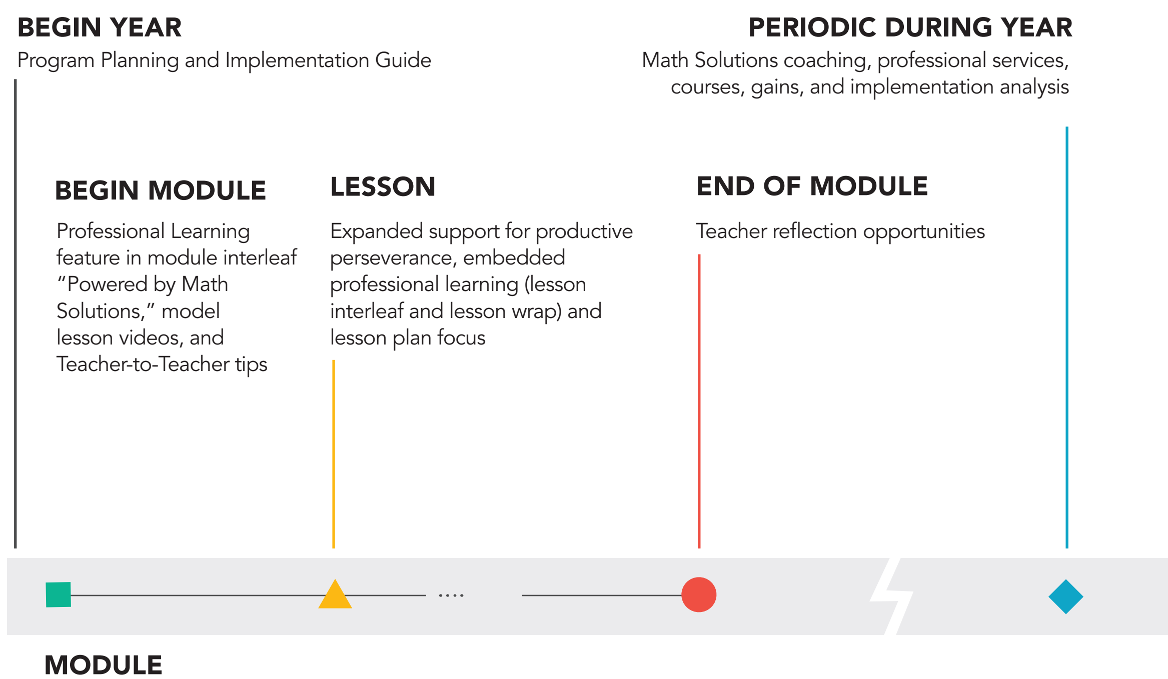
UNIT 1

SOLUTION DESCRIPTION (CONTINUED)

Professional Learning

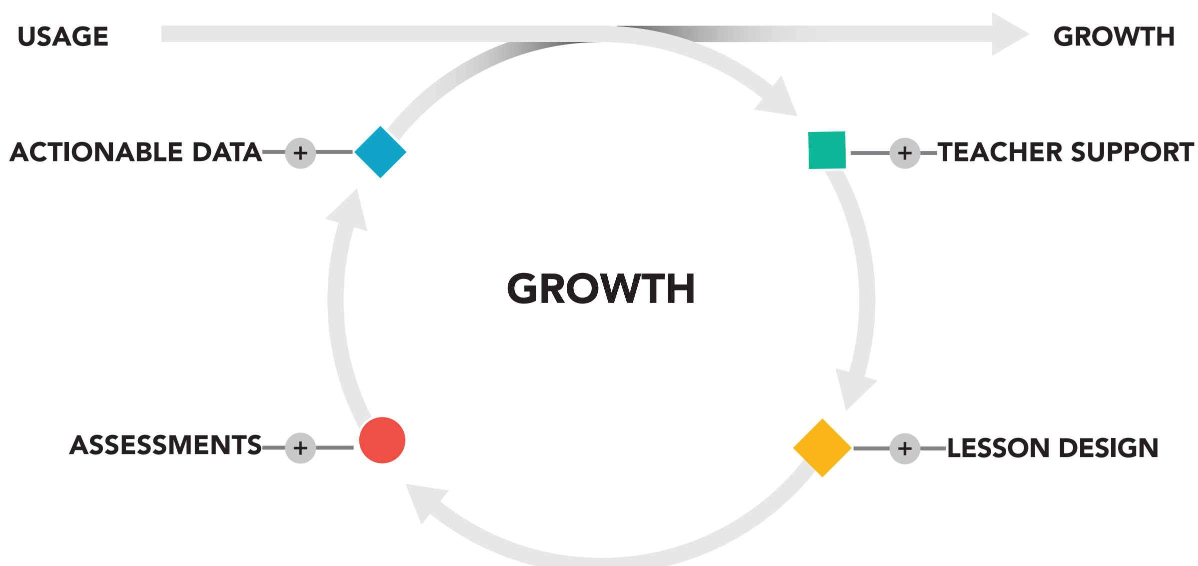
The *Into AGA* solution is uniquely positioned to deliver ongoing, timely, and relevant professional learning. The professional learning is embedded within *Into AGA* and includes high-impact strategies that are relevant to everyday teaching. Classroom videos for every module, on topics like productive perseverance, language routines, and talk moves, empower teachers to design meaningful learning experiences. Within lessons, embedded professional learning provides ongoing suggestions and strategies and provides teachers the opportunity to reflect on their own practice.

Courses and coaching from Math Solutions® support teachers as they gain a **deeper understanding of mathematics** and increase their overall effectiveness in the math classroom.



Coherent Experience

By focusing on growth, the *Into AGA* solution is built on consistency in practice and use. This leads to greater growth and better outcomes for students. Each piece of the solution leans on another—and when implemented with fidelity, they promote student growth. As noted, growth is maximized when instruction, assessment, and professional learning are coordinated and tightly aligned. *Into AGA* delivers on this notion and commits to yielding the best results for students.



INTO AGA DIFFERENTIATORS

- Math Solutions
- Balance of Conceptual Understanding, Procedural Skills and Fluency, and Application
- Productive Perseverance and Discourse
- Spies and Analyst™ Tasks
- Growth Measure, 3x/year
- Module Tests, 15–20x/year
- Lesson-Level Homework, 3–4x/week
- Grouping
- Reports for Teachers, Admins, Students, and Parents

CONTEXT

MATHEMATICS ACHIEVEMENT

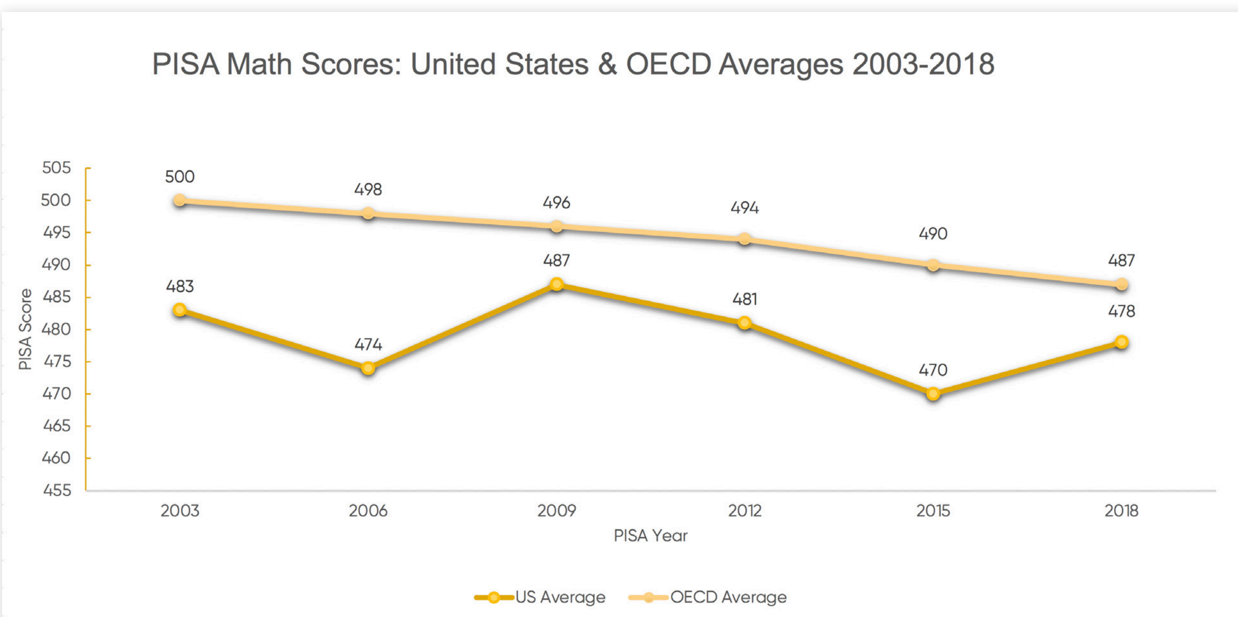
To keep pace with a rapidly changing world, our students need to be prepared for a future in mathematics. Although our nation has seen some increases and accomplishments in mathematics achievement, our students still score below average on international tests of mathematical knowledge and skills (Loveless, 2011).

PISA

In 2018, American students performed below the OECD average (487) in math with a mean score of 478. Scores were lower than 36 other countries. Citation: OECD (2019), PISA 2018 Results (Volume I): What Students Know and Can Do, PISA, OECD Publishing, Paris, https://doi.org/10.1787/5f07c754-en.

NAEP

When looking at national statistics, the picture is similar. The average mathematics score for twelfth-grade students decreased between 2013 and 2015, the most recent years of administration at the high school level. Growth has been stagnant since 2005, the first year the current assessment framework was implemented. Further, in 2015, only 25% of students completing K–12 education in the U.S. performed at or above the Proficient level (NCES, 2016). Less than half of our college-bound students are prepared for postsecondary education and beyond (College Board, 2011). Clearly, we need to reevaluate, redesign, and reinvent our approach to mathematics teaching and learning to address our continued challenges.



TEACHERS' ROLE

As we look to improve students' mathematical preparedness, we must emphasize the changing role of the teacher in the implementation. The advances in technology have changed the availability of information as well as access to it. There was a time when students needed a book or a teacher to learn, but now there is immediate access to information. Teachers employ practices they have learned and developed through participation in their own educational experiences as students (Ball, 1990; Cooney, 2001) and as practitioners (Cobb & McClain, 2001). Additionally, teacher practices are filtered through what teachers know and believe—resulting in standards implementation based upon negotiation of the objectives with teachers' held knowledge and beliefs and current teaching practices (Ball & Cohen, 1999). NCTM encourages implementing equitable instructional practices as an action that teachers should undertake to improve the experiences and learning outcomes of their students including supporting students to develop positive mathematical identities (NCTM, 2018, p.25). Those designing effective support for teachers must consider this reality (Lappan, 1997) and the challenges teachers and their students face in adapting to these new expectations.

Houghton Mifflin Harcourt's *Into AGA* efficiently collects data and provides reports for teachers to analyze the data that effectively guide their instructional decisions. Further, grouping and targeted, aligned resources along with additionally searchable prerequisite/extending standards-based assignments based on data ensure greater equity for each and every student.

TECHNOLOGY'S IMPACT ON MATHEMATICS ACHIEVEMENT

As the role of the teacher changes, education also experiences the swift impact of technology and its rapidly changing nature, which affects teaching and learning. As noted by Digital Promise Global (2016), "Without changes, the social and economic disparities and achievement gaps of people who historically are underserved will persist and grow, and we will be at risk of marginalizing more and more students" (p. 2).

Districts and teachers realize that the educational landscape is rapidly changing. Our students are changing—they have greater access to information than ever before. They expect technology to work for them. Classrooms are also more diverse; students have wide-ranging ability levels and unique needs. Teachers are overwhelmed—requirements from administrators, families, and testing make teaching challenging. Houghton Mifflin Harcourt's *Into AGA* invests in teachers, creates fearless problem solvers, and fosters a growth mindset by supporting a shift in pedagogical approach, supplying the technological tools to support the transition, and helping make change manageable—while seamlessly fitting into a complex and diverse learning ecosystem.

THE BIG QUESTION: HOW DO STUDENTS LEARN MATHEMATICS?

MATHEMATICS TEACHING PRACTICES

NCTM has extended and elaborated on the guiding mathematics principles (2000) in its 2014 publication *Principles to Actions: Ensuring Mathematical Success for All*. These principles are based on more than a decade of research and experience and continue to be a driving force behind a high-quality mathematics education for all students. NCTM further elaborated on the teaching practices in its most recent publication "Catalyzing Change in High School Mathematics: Initiating Critical Conversations" by highlighting that classroom instruction should be consistent with research-informed and equitable teaching practices. (NCTM, 2018, p. 7). Furthermore, equitable teaching practices are critically important because it is through equitable instruction that students develop positive mathematical identities (NCTM, 2018, p. 25). Each of the Mathematics Teaching Practices, part of the first guiding principle of *Principles to Actions*, is reflected in the components of Houghton Mifflin Harcourt's *Into AGA*.

Mathematics goals are focused, purposeful, and coherent. Each lesson clearly outlines the standards and practices, objectives, and learning progressions so teachers can best support and focus student learning. Identifying and clarifying what students are expected to learn and understand in a mathematics classroom is an essential component to success (Wiliam, 2011). Teachers and students benefit from establishing a shared foundation of what is being learned and why it is important to learn. "Formulating clear, explicit learning goals sets the stage for everything else" (Hiebert, Morris, Berk, & Jansen, 2007, p. 57).

Tasks require a high level of cognitive demand, which is necessary when promoting reasoning and problem solving in the mathematics classroom. In a study that compared students exposed to teaching strategies that promoted higher-order thinking with those who were taught more traditionally, researchers found that experimental-group students outperformed control-group students, showing significant improvement in critical-thinking skills; "Our findings suggest that if teachers purposefully and persistently practice higher-order thinking strategies, for example dealing in class with real-world problems, encouraging open-ended class discussions, and fostering inquiry-oriented experiments, there is a good chance for a consequent development of critical thinking capabilities" (Miri, David, & Uri, 2007, p. 353). Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural (Boaler & Staples, 2008; Stein & Lane, 1996).

Connections among mathematical ideas are an essential part of mathematics learning. Making connections between new information and students' existing knowledge—knowledge of other content areas and of the real world—has proved to be more effective than learning facts in isolation (Beane, 1997; Bransford, Brown, & Cocking, 1999; Caine & Caine, 1991; Kovalik & Olsen, 1994). Further, connecting mathematics to science, social studies, and business topics can increase students' understanding of and ability with mathematics (Russo, Hecht, Burghardt, & Saxman, 2011). Students maximize learning when they make connections among ideas, both within mathematics and outside of the mathematics content.

Students use **mathematical models and representations** to help make mathematical concepts more concrete. "Because of the abstract nature of mathematics, people have access to mathematical ideas only through the representations of those ideas" (NRC, 2001, p. 94). For young students, representations are especially important because they can be physical objects or actions students perform as they are trying to solve problems (NRC, 2001). As students create representations, they have the opportunity to internalize and process what they are doing, what they are creating, and what they are seeing. These actions, in turn, allow students to take an active role in their learning.

Students engage in mathematical discourse throughout each lesson. When students write about and discuss math concepts, they have the chance to think through, defend, and support their ideas. A review of studies conducted by the National Council of Teachers of Mathematics revealed that "the process of encouraging students to verbalize their thinking—by talking, writing, or drawing the steps they used in solving a problem—was consistently effective. . . . Results of these students were quite impressive, with an average effect size of 0.98" (Gersten, Clarke, & Mazzocco, 2007, p. 2). Communicating about math improves learning; "encouraging students to verbalize their current understandings and providing feedback to the student increases learning" (Gersten & Chard, 2001).

"Effective teaching of mathematics uses **purposeful questions** to assess and advance student reasoning and sense making about important mathematical ideas and relationships" (NCTM, 2014, p. 35). While types of questions vary from asking students to recall facts to requesting an explanation for an answer, presenting questions throughout the learning process is necessary to understanding how students are making sense of the math. As noted in research by Weiss and Pasley (2004), questions are critical in helping students make connections and learn important mathematics concepts—especially questioning that effectively gauges student understanding.

To achieve understanding, each lesson recognizes and supports the relationship between **procedural fluency and conceptual understanding**. Specifically, "Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems" (NCTM, 2014, p. 42). Effective mathematics instruction cannot have one without the other as "procedural knowledge and conceptual understandings must be closely linked" (NRC, 2005, p. 232).

Students have the opportunity to practice and engage in **productive perseverance** with mathematical problems and ideas that encourages them to consider their own thinking and to discover that learning can happen without rushing to simply find the correct answer. Teachers who guide students through "productive perseverance" are supporting the development of student learning and understanding (Hiebert & Grouws, 2007). Students receive reassurance as they grapple with ideas, and teachers support them through the process rather than give them the answers (Hiebert et al., 1996).

Teachers are prompted to **elicit and use evidence of student thinking**. To discover what students know or don't know, what they do well or poorly, the teacher must closely examine the students' work. "Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning" (NCTM, 2014, p. 53).

MATHEMATICAL PROGRESSIONS

The Common Core State Standards are research and evidence based. The Math Standards progressively develop content each year by providing a greater focus on fewer topics, coherence of topics and thinking across the grades, and the pursuit of conceptual understanding, procedural skills and fluency, and application with equal intensity.

The coherence of standards, as illustrated by the logical progression across grade levels, is an essential element of effective standards. **The standards build on the foundations of earlier years**, with new learning extending upon what has already been learned. This focus on progressions is intentional, develops deep content knowledge, and builds the complexity of student skills over time.

The Mathematics Standards reflect the most important mathematical topics. They comprise related ideas, concepts, skills, and procedures that form the foundation for understanding and using mathematics and lasting learning (NCTM, 2006).

CURRICULUM DESIGN AND STANDARDS

Focused content, clear goals, and intentional design are consistent cornerstones throughout the *Into AGA* solution. As noted in research, standards-based learning environments appear to have significant impacts on student achievement (Tarr et al., 2008). Standards correlate with higher student achievement, perhaps because standards ensure alignment between what experts in the field identify as essential skills and knowledge; provide the focus of classroom teachers' instruction; clearly detail what students are supported in learning; and inform the content of assessments. Every lesson of the *Into AGA* solution articulates the content focus and goals, as well as the mathematical practices to support students as they develop ways of thinking about and doing mathematics. Every lesson also follows an intentional design that is predictable and logical to best shape the planning and delivery of instruction.

CURRICULUM DESIGN AND STANDARDS

FOCUSED CONTENT

For students to achieve understanding and acquire mathematics skills, **identifying and clarifying what those students are expected to learn and understand in a mathematics classroom is an essential component to success** (Wiliam, 2011). Standards offer a guide for teachers to ensure that they are helping students build the foundations they need to move on to the next grade level and be ready for college and work. Standards can help ensure that teachers are appropriately targeting instruction and can help students set clear academic goals for learning. For a few decades, across the U.S. new standards were implemented to raise expectations in secondary mathematics. Over the same period, many states increased the number of credits of mathematics required for students to receive a high school diploma and specified particular mathematics course requirements that students must complete, such as Algebra I and Geometry. For a few decades, across the U.S. new standards were implemented to raise expectations in secondary mathematics. Over the same period, many states increased the number of credits of mathematics required for students to receive a high school diploma and specified particular mathematics course requirements that students must complete, such as Algebra I and Geometry. Yet many of today's graduates are still unprepared for college-level coursework and require remediation, especially in mathematics. (The American Diploma Project, 2004; The College Board, 2011; Long, 2009).

Reviews of the mathematics curricula in top-performing countries find that these "present fewer topics at each grade level but in greater depth" (National Mathematics Advisory Panel, 2008, p. 20). Standards must "promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible" (Achieve, 2010, p. 1). Cobb and Jackson (2011) came to the same conclusion: that it is beneficial for mathematics teaching and learning to focus on a small number of core mathematical ideas at each grade.

Although the standards detail the knowledge and skills that students are expected to learn at each grade, they do not describe the instructional approaches teachers will take to help students meet the standards. Thus, an effective instructional solution is needed to bridge the expectations set out by the standards and the desired student outcomes. These instructional solutions must also be focused.

One approach noted by Dr. Juli Dixon is to consider the following four questions:

1. What is the learning goal?
2. Is the lesson conceptually based?
3. What do I want students to be able to answer by the end of the lesson?
4. How can I zoom out from the student-facing I Can statement to protect the discovery? Taking the essential idea into consideration allows teachers to set the stage for students to make valuable connections as they learn.

HOW *INTO AGA* DELIVERS

Into AGA is rigorous, focused, and cohesive, which is necessary for effective mathematics teaching and learning. Throughout the solution, students build their conceptual understandings, improve their procedural fluency, and apply their knowledge in meaningful contexts and real-world applications. *Into AGA* clearly outlines essential content and skills. With more focused learning, students can achieve at higher levels. The curricula also reflect a sequence of content within and across grade levels in a way that best reflects the hierarchical and logical structures of mathematics—for deep understanding. The Common Core Standards for Mathematics are noted, as are the mathematical practices in which students will engage and develop mathematical habits. The *Into AGA* solution commits to a concise, logical curriculum, tightly focused on building deep conceptual understanding connected to procedural fluency and thorough application. The solution's Learning Arc supports students in making connections and bridging the conceptual to the procedural, providing them with better access to the concrete models associated with the procedures when they need those procedures to complete more complex tasks.

At the beginning of each lesson, teachers can pose the **I Can Objective** and reinforce it throughout with the projectable I Can scale. "Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships" (NCTM, 2014, p. 35). Teachers and students kick off each lesson with a shared understanding. Activating prior knowledge and understanding helps support moving forward with the lesson. *Into AGA* gives teachers guidance to ask students meaningful questions related to the critical concepts they are learning.

Mathematics learning and language objectives can be challenging to create. *Into AGA* provides teachers with **coherent objectives for each lesson of each module**. The objectives contribute to the targeted, coherent solution set forth, and the prompts give teachers the ability to ensure that instruction is aligned.

The Mathematical Practices and Processes provide guidance as students engage, learn, and use mathematics. Problem solving, reasoning, proving, reflecting, choosing tools and strategies, connecting, representing, and communicating are all part of the solution. Students learn to make this happen through the I Can Objective.

LESSON FOCUS AND COHERENCE

Mathematics Standards

- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity.

Mathematical Practices and Processes MP

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and make use of structure.

I Can Objective

I can write an algebraic expression, interpret the parts of the expression, and use the Distributive Property to simplify the expression.

Learning Objective

Write, interpret, and simplify linear expressions in one variable, and use linear expressions and compatible units to model real-world situations.

Language Objective

Explain how to write and interpret a linear expression that models a real-world situation and use the Distributive Property to simplify the expression.

Vocabulary

Review: **coefficient**, **equivalent expressions**, **expression**, **like terms**, **term**

CLEAR GOALS

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions (NCTM, 2014). As noted by Marzano (2010) in *Designing and Teaching Learning Goals and Objectives*, identifying goals is the first step in building a unit of instruction.

Teachers and students benefit from establishing a shared foundation of what is being learned and why it is important to learn. “Formulating clear, explicit learning goals sets the stage for everything else” (Hiebert et al., 2007, p. 57). Specific goals articulate a clear path for behavior and desired performance and serve as motivation for learning (Marzano, 2010). Setting clear goals and expectations increases motivation by encouraging student involvement in and responsibility for their own learning (Ames, 1992; Bransford, Brown, & Cocking, 1999).

By looking at the goals within mathematics learning progressions (Charles, 2005), teachers have the opportunity to examine and monitor student progress and needs in order to adjust instruction as necessary (Clements, Sarama, & DiBiase, 2004; Sztajn, Confrey, Wilson, & Edgington, 2012). Teachers can support learners as they build on what they know, develop more complex understandings, and realize that mathematics is not a set of discrete parts—it is coherent and connected (Fosnot, Twomey, & Jacob, 2010; Ma, 2010).

Research also shows that setting and sharing learning goals with students has a positive impact on their learning. Work by Haystead & Marzano (2009) and Hattie (2009) shows that students in classrooms where learning goals are clearly articulated perform at higher levels than students who are unaware of the expectations. When expectations are discussed with students, they are able to find value in their work and understand the greater purpose of what they are learning (Black & Wiliam, 1998; Marzano, 2010). Establishing goals allows students to focus on the expectation that is set and become more aware of their own thinking and learning (Clarke, Timperley, & Hattie, 2004; Zimmerman, 2001).

As noted by Dixon (2018), while it is important for learning goals to be clear, it is equally important that students are the ones doing the “sense-making.” Goals should be posted after the exploration phase of a lesson, where conceptual understanding is being built, to avoid undermining student engagement or depriving them of the rigor of the task and the opportunity to make valuable personal connections from previous learning to new understandings (Dixon, 2018).

HOW *INTO AGA* DELIVERS

All learning is shown in the context of **mathematical progressions** so students and teachers can clearly see the expected progression of learning—**the learning goals**. Strong learning progressions help students develop a deep understanding of mathematical content. Each lesson of each module includes a graphic representation of prior learning, current development, and future connections for learning. Throughout the solution, students build to more complex concepts, skills, and applications of content knowledge over time.

Mathematical Progressions

Prior Learning	Current Development	Future Connections
Students: • explained and justified each step required to solve a multi-step linear inequality. **(Gr7, 8.2)** • used linear inequalities to represent and solve real-world problems. **(Gr7, 8.3)**	**Students:** • write compound linear inequalities in one variable using both "and" and "or." • represent solutions to compound inequalities on a number line. • use compound inequalities to represent and solve real-world problems.	**Students:** • will write and solve systems of equations and inequalities. **(9.1–10.2)** • will graph solutions to linear inequalities in two variables. **(10.1 and 10.2)**

PROFESSIONAL LEARNING

Using Mathematical Practices and Processes

Look for and make use of structure.

Compound inequalities can use two inequality symbols, or the word "AND" or "OR." The symbols or words communicate the structure of the inequality. When the solutions are graphed on a number line, there is a visual reminder as to whether the solutions satisfy both one simple inequality AND the other or whether the solutions satisfy one simple inequality OR the other (or both), by whether the graph of the compound inequality shows an overlap or separate parts. However, the structure of the graph of an inequality can also show that the solutions are all real numbers or that there are no solutions.

To further elaborate on the goals, the Teacher's Editions include an **Unpacking the Standards** component that notes the math standard(s) addressed in the lesson, as well as an additional explanation of what the standard means. Providing a more thorough description of what is expected in each lesson allows teachers and students to have a shared understanding of learning.

Learning Objective

Write and solve compound linear inequalities in one variable using both "and" and "or," represent solutions of compound inequalities on a number line, and use compound inequalities to model and solve real-world problems.

Language Objective

Explain how to write and solve compound inequalities.

The Teacher's Editions also include Language Objectives. These objectives support students as they learn mathematical concepts and language and practice communicating mathematically.

INTENTIONAL DESIGN

To achieve understanding, **using a logical and intentional design in both the planning and delivery of instruction has been shown to positively impact student outcomes**. In their work *Understanding by Design*, Wiggins and McTighe (2005) describe effective instructional design in the classroom centered on guiding questions, such as these: What should students know, understand, and be able to do? How will we know if students have achieved the desired results? How will we support learners as they come to understand important ideas and processes? Thus, the three stages in their model include identifying desired results, determining assessment evidence, and planning learning experiences and instruction (Wiggins & McTighe, 2005).

Stage 1 will clarify "goals, examine established content standards (national, state, province, and district), and review curriculum expectations. Because there is typically more content than can reasonably be addressed within the available time, teachers are obliged to make choices. This first stage in the design process calls for clarity about priorities" (Wiggins & McTighe, 2005).

In Stage 2, it is important to look back to Stage 1, as the assessment evidence is based on the desired results. "Thus, we consider in advance the assessment evidence needed to document and validate that the targeted learning has been achieved. Doing so invariably sharpens and focuses teaching" (Wiggins & McTighe, 2005).

Stage 3 requires teachers to consider the most appropriate and effective approaches to instruction after they assess whether students have achieved the goal(s). Critical to this stage is that teachers need to teach for understanding. Teaching for understanding requires that students be given numerous opportunities to draw inferences and make generalizations for themselves (with teacher support) (Wiggins & McTighe, 2005).

Identifying what students will learn is only one aspect of intentional design in lessons. It is critical that classroom experiences also connect to what students need to know. Students must understand what they are learning and why they are learning it. "When you know your why, your what has more impact because you are walking in and toward your purpose" (Kanold, 2018).

HOW *INTO AGA* DELIVERS

Each lesson of each module follows the sequence noted previously. Concepts are introduced, and students engage in productive perseverance to explore the concepts. The teacher then assesses student understanding and guides differentiated activities to further develop the concepts for some students and to clarify for others. Then the lesson closes, and students are able to further practice the concepts and procedures, preparing for the next lesson. The solution is intentionally designed to reflect the realities of real classrooms while achieving the goals of focused topical content with actionable assessment guiding differentiated instruction.

While *Into AGA* does follow the three stages, it also refines teaching and learning, and the routine is consistent to support familiarity for students—and teachers. In addition to incorporating application throughout as students bridge conceptual and procedural understanding, each lesson includes the following:

- **Spark Your Learning:** Teachers work on-level with students to gauge their readiness and to inspire and guide productive perseverance. In Apply and Practice tasks, students begin to build fluency, learn to choose from multiple available strategies, and rely on the conceptual understanding developed previously to solve rigorous tasks.
- **Learn Together:** Whole-group learning is facilitated in these Build Understanding and Step It Out tasks. Build Understanding tasks provide an opportunity to help students understand lesson concepts. Step It Out tasks promote procedural understanding.
- **Check Understanding:** After the learning tasks, these five- to ten-minute checkpoints provide a snapshot of what students know.
- **Differentiation Options:** According to student understanding, groups are formed to ensure growth for each and every student by providing resources based on individual needs. Teachers can then decide how to best support students with differentiated resources such as independent practice, Collaborative Groups, or working in small, teacher-facilitated groups.
- **Wrap-Up:** Here is the opportunity for additional practice, reteaching, or intervention. Teachers gauge student depth of understanding with exit tickets and suggested wrap-up ideas.
- **Homework or Practice:** Each lesson includes homework/practice opportunities for students to practice the concepts just introduced within the Journal and Practice Workbook.

MATHEMATICS KNOWLEDGE AND TEACHING

Research shows that effective teaching is the driving force behind powerful mathematics instruction and deep understanding. *Into AGA* empowers teachers by providing them with the tools, resources, and professional learning they need to improve learning outcomes and create an engaging classroom culture. Teachers are guided in helping students develop procedures and concepts iteratively as they engage in productive perseverance and practice routines for reasoning to deepen their knowledge and understanding. *Into AGA* reinforces the importance of mathematical discourse through communicating mathematically—with immersive opportunities to listen, speak, read, and write about mathematics—and also provides math-specific vocabulary activities for further support. Every *Into AGA* teacher receives professional learning resources to support fostering a growth mindset in the classroom—equating effort with achievement.

MATHEMATICS KNOWLEDGE AND TEACHING

CONCEPTS AND PROCEDURES

In order to achieve understanding in mathematics in particular, students need instruction that recognizes the relationship between procedural fluency and conceptual understanding, which requires students to practice procedural skills that apply the conceptual underpinnings. Specifically, **"Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems"** (NCTM, 2014, p. 42). Effective mathematics instruction cannot have one without the other as "procedural knowledge and conceptual understandings must be closely linked" (NRC, 2005, p. 232).

A large number of reports recognize the critical relationship and balance between concepts and procedures in mathematics instruction for student learning (National Mathematics Advisory Panel, 2008; NRC, 2001). Also evidenced by results from a study conducted by Rittle-Johnson and Alibali (1999), concepts and procedures develop iteratively—and gains in one area lead to gains in the other. Research by Franke, Kazemi, and Battey (2007) suggests that students need an environment to develop both concepts and skills in order to become flexible when engaging with mathematical ideas and to develop as critical thinkers. Being proficient in math involves knowing and doing. To effectively solve problems in mathematics, students need to be capable of demonstrating both conceptual understanding and procedural fluency. When students are able to connect procedures and concepts, their retention improves, and they are better able to apply what they know in different situations (Fuson, Kalchman, & Bransford, 2005).

Mathematics helps students understand and solve real-world problems. While teachers present concepts and procedures, students first conceptualize the mathematical nature of a problem and then carry out the corresponding mathematical procedure. Teachers define and link mathematical concepts with their respective procedures to maximize student learning. "While balance is important, so is the order with which these are addressed. Concepts must be taught before procedures otherwise there is no motivation to make sense of the mathematics prior to using more efficient processes" (Dixon, in press-a).

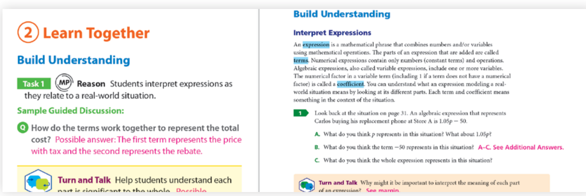
All students need to have a deep and flexible knowledge of a variety of procedures, along with an ability to make critical judgments about which procedures or strategies are appropriate for use in particular situations for best success in the mathematics classroom (NRC, 2001, 2005; Star, 2005).

Into AGA builds a bridge between conceptual and procedural learning—something previously absent from mathematical learning. Teachers have been supporting students in building concepts and moving into the procedural; however, the critical bridging aspect was not as clearly defined within the actual lesson architecture. With *Into AGA*, bridging lessons makes concepts sink in as students delve into more rigorous tasks.

HOW *INTO AGA* DELIVERS

Houghton Mifflin Harcourt's *Into AGA* fully addresses the expectations for content learning in mathematics content and practice standards. Lessons within each module incorporate instruction on the Mathematics Standards and the Standards for Mathematical Practice so that students are prepared for the challenges of higher-level mathematics courses. Additionally, all course-level Mathematics Standards are explicitly stated at the opening of each lesson, as well as at the point of use, in order to provide continual focus during instruction.

The *Into AGA* solution provides balanced instruction on mathematical content and practice. Teachers are given guidance to develop both concepts and procedures in mathematics learning. Every lesson incorporates the Standards for Mathematical Practice, and they are fully embedded and integrated throughout the solution. The lesson structure integrates productive perseverance, small-group instruction, and growth mindset supported by the right resources. Highlights of alignment include the following:


② Learn Together

Build Understanding

Task 1 MP Reason Students interpret expressions as they relate to a real-world situation.

Sample Guided Discussion:

Q How do the terms work together to represent the total cost? Possible answer: The first term represents the price with tax and the second represents the rebate.

Turn and Talk Help students understand each

Build Understanding

Interpret Expressions

An expression is a mathematical phrase that combines numbers and/or variables using mathematical operations. The parts of an expression that are added are called terms. Numerical expressions contain only numbers (constant terms) and operations. Algebraic expressions, also called variable expressions, include one or more variables. The numerical factor in a variable term (including 1 if a term does not have a numerical factor) is called a coefficient. You can understand what an expression modeling a real-world situation means by looking at its different parts. Each term and coefficient means something in the context of the situation.

1 Look back at the situation on page 31. An algebraic expression that represents Carlos buying his replacement phone at Store A is $1.05p - 50$.

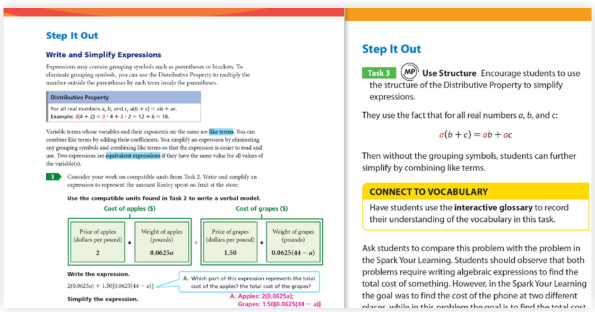
A. What do you think p represents in this situation? What about $1.05p$?

B. What do you think the term -50 represents in this situation? A–C. See Additional Answers.

C. What do you think the whole expression represents in this situation?

Turn and Talk Why might it be important to interpret the meaning of each part

Build Understanding—Activities provide students with opportunities to model with mathematics, use appropriate tools, reason abstractly and quantitatively, analyze patterns and structures, and make conjectures.


Step It Out

Write and Simplify Expressions

Expressions may contain grouping symbols such as parentheses or brackets. To eliminate grouping symbols, you can use the Distributive Property to multiply the number outside the parentheses by each term inside the parentheses.

Distributive Property
For all real numbers a, b, and c, $a(b + c) = ab + ac$.
Example: $3(4 + 2) = 3 \cdot 4 + 3 \cdot 2 = 12 + 6 = 18$.

Variable terms whose variables and their exponents are the same are like terms. You can combine like terms by adding their coefficients. You simplify an expression by eliminating any grouping symbols and combining like terms so that the expression is easier to read and use. Two expressions are equivalent expressions if they have the same value for all values of the variable(s).

3 Consider your work on compatible units from Task 2. Write and simplify an expression to represent the amount Keeley spent on fruit at the store.

Use the compatible units found in Task 2 to write a verbal model.

Write the expression.

$2(0.0625a) + 1.50[0.0625(44 - a)]$

A. Which part of this expression represents the total cost of the apples? the total cost of the grapes?

A. Apples: $2(0.0625a)$; Grapes: $1.50[0.0625(44 - a)]$

Simplify the expression.

Step It Out

Task 3 MP Use Structure Encourage students to use the structure of the Distributive Property to simplify expressions.

They use the fact that for all real numbers a, b, and c:

$$a(b + c) = ab + ac$$

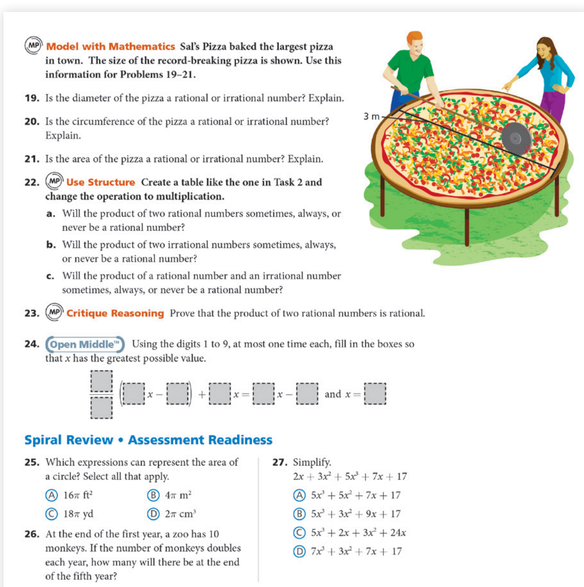
Then without the grouping symbols, students can further simplify by combining like terms.

CONNECT TO VOCABULARY

Have students use the **interactive glossary** to record their understanding of the vocabulary in this task.

Ask students to compare this problem with the problem in the Spark Your Learning. Students should observe that both problems require writing algebraic expressions to find the total cost of something. However, in the Spark Your Learning the goal was to find the cost of the phone at two different

Step It Out—Prompts encourage students to analyze solution methods, explain concepts in their own words, construct arguments, justify their own reasoning, and critique the reasoning of others.


MP **Model with Mathematics** Sal's Pizza baked the largest pizza in town. The size of the record-breaking pizza is shown. Use this information for Problems 19–21.

19. Is the diameter of the pizza a rational or irrational number? Explain.

20. Is the circumference of the pizza a rational or irrational number? Explain.

21. Is the area of the pizza a rational or irrational number? Explain.

22. MP **Use Structure** Create a table like the one in Task 2 and change the operation to multiplication.
    a. Will the product of two rational numbers sometimes, always, or never be a rational number?
    b. Will the product of two irrational numbers sometimes, always, or never be a rational number?
    c. Will the product of a rational number and an irrational number sometimes, always, or never be a rational number?

23. MP **Critique Reasoning** Prove that the product of two rational numbers is rational.

24. Open Middle™ Using the digits 1 to 9, at most one time each, fill in the boxes so that x has the greatest possible value.

$$\frac{\square}{\square}(\square x - \square) + \square x = \square x - \square \text{ and } x = \square$$

Spiral Review • Assessment Readiness

25. Which expressions can represent the area of a circle? Select all that apply.
    Ⓐ 16π ft² Ⓑ 4π m² Ⓒ 18π yd Ⓓ 2π cm³

26. At the end of the first year, a zoo has 10 monkeys. If the number of monkeys doubles each year, how many will there be at the end of the fifth year?

27. Simplify. $2x + 3x^2 + 5x^3 + 7x + 17$
    Ⓐ $5x^3 + 5x^2 + 7x + 17$
    Ⓑ $5x^3 + 3x^2 + 9x + 17$
    Ⓒ $5x^3 + 2x + 3x^2 + 24x$
    Ⓓ $7x^3 + 3x^2 + 7x + 17$

Concepts and Skills—Open Middle® problems further highlight the program's rigor by having students optimize their problem solving techniques to analyze the various ways they may arrive at a solution.

PRODUCTIVE PERSEVERANCE

The National Council of Teachers of Mathematics (NCTM) policy document, *Principles to Actions: Ensuring Mathematical Success for All*, notes that "an effective teacher provides students with appropriate challenges, encourages perseverance in solving problems, and supports productive struggle in learning mathematics" (NCTM, 2014, p.11). Through productive perseverance, students grapple with the issues and are able to find solutions on their own, allowing them to persist and build resilience as they pursue learning and understanding (Jackson & Lambert, 2010). Further, as noted by Matt Larson, success in any area depends on at least two common components: practice and perseverance (Larson, 2016). "Effective mathematics teaching uses students' struggles to deepen their understanding of mathematics. Students come to realize that they are capable of doing well in mathematics with effort and perseverance in reasoning, sense making, and problem solving" (NCTM, 2014, p. 52).

When providing mathematics instruction, it is beneficial for teachers to allow students to explore what they know—and what they don't know yet. Allowing students the opportunity to practice and wrestle with mathematical problems and ideas encourages them to think about their own thinking and to discover that learning can happen without rushing simply to find the correct answer. Teachers who guide students through "productive struggle" are supporting the development of student learning and understanding (Hiebert & Grouws, 2007). Students receive reassurance as they grapple with ideas, and teachers support them through the process rather than give them the answers (Hiebert et al., 1996).

While fostering an environment in which students are free to work through difficulties, teachers, as instructional guides, need to keep students connected to the learning process. Without positive guidance and reinforcement, a "productive struggle" could become an "unproductive struggle" in which students "are unable to progress toward sense-making, explaining, or tackling a problem or task at hand" (Warshauer, 2014, p. 21). "As students engage with a task, they must be mindful about the strategy they employ and assess whether it is productive. When they find they are at a dead end, they must be willing to abandon one strategy for another. When students labor and struggle, but continue to try to make sense of a problem, they are engaging in productive struggle" (Pasquale, 2015). Being proactive with student learning requires that teachers observe student learning closely. "Instead of providing scaffolding just in case students might need it, scaffolding should be offered just in time when there is evidence that the student's struggle is no longer productive" (Dixon, in press-b).

Math is not a talent that someone has or doesn't have. All students can learn mathematics. Students simply need teachers who support them and instruction that is appropriate and effective. Students need to be encouraged to try over and over again while using the strategies they have learned. This creates a meaningful mathematics environment (Larson, 2016).

Problem solving is at the heart of mathematics, and wrestling with it is critical to developing students as problem solvers. Teachers can support students through these efforts by providing them the opportunity to struggle—to persevere. Teachers can use these moments to have conversations with students about their thinking and processing. Students will begin to understand that through grappling with mathematics, they are actually expending effort to stick with it, try new ideas, and figure it out (Larson, 2015).

HOW *INTO AGA* DELIVERS

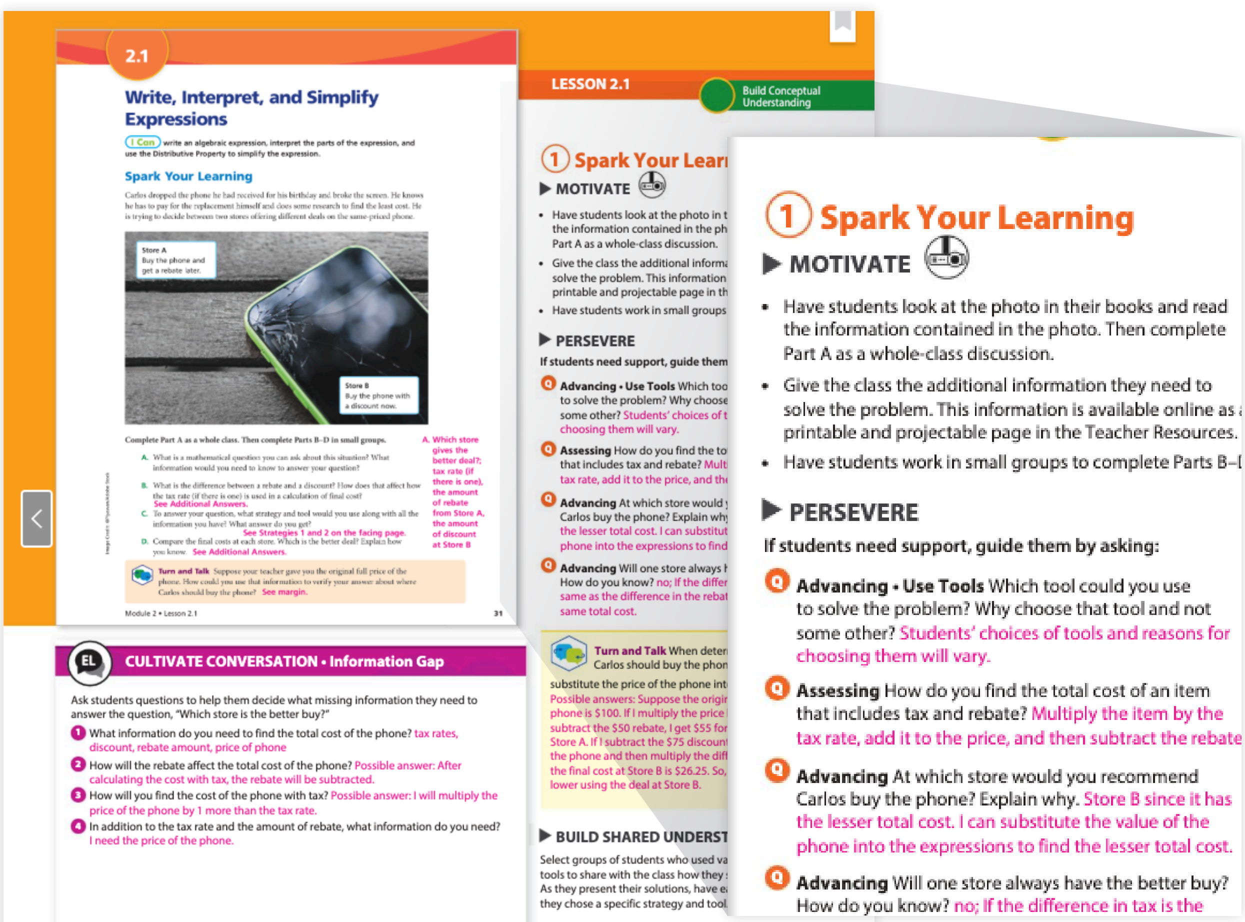
The solution provides a set of motivating exercises for each lesson. It engages students and pushes them to apply what they know while motivating students to learn the new concepts. Teachers are supported with various questions and possible answers that lead the students to think through the problem.

Each lesson of each module of *Into AGA* begins with **Warm-Up Options**. Here teachers **Activate Prior Knowledge** by allowing students to work independently through a short math **Problem of the Day** where they can **Make Connections** as they work through the problem. When students have finished, teachers can then group students heterogeneously or by "on track," "almost there," or "ready for more" and can plan instruction accordingly. Some students can move on to the **Math Center**, where there are additional self-directed activities at centers or stations.

Additionally, to engage students in productive perseverance, each lesson of each module opens with a **Spark Your Learning** section. Here, teachers follow a routine of supporting students through productive perseverance:

- **Motivate**—Set the stage.
- **Persevere**—Ask assessing and advancing questions.
- **Turn and Talk**—Communicate mathematically.
- **Build Shared Understanding**—Share various strategies.
- **Support Sense-Making**—Promote language routine.

From here, teachers will move on to **Learn Together** with whole-class instruction to continue to build understanding. *Into AGA* also offers **videos** that have specific segments in which teachers can learn about students engaging in **productive perseverance**. Teachers can provide these opportunities for students to develop grit.



ROUTINES FOR REASONING

Routines help classrooms run smoothly and efficiently and help students and teachers know what to expect. Instructional routines that focus on developing mathematical thinking and learning mathematical practices open doors for students. Routines are **"designs for interaction that organize classroom activities"** (Lampert, 2015).

"Like the management routines, these 'mathematical thinking routines' also have a predictable set of actions that students learn and then practice repeatedly until they are second nature" (Kelemanik, Lucenta, & Creighton, 2016, p.18). The predictable structure lets students pay less attention to those questions and more attention to the way in which they and their classmates are thinking about a particular math task (Kelemanik et al., 2016).

Students need opportunities to articulate complex mathematical situations (Mondada & Doehler, 2004)—to provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014). The principles presented by Zwiers et al. (2017) motivate the use of mathematical language routines (MLR), which help both teachers and students remain attentive to language as much as possible:

- **MLR1: Stronger and Clearer Each Time**—to provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014).
- **MLR2: Collect and Display**—to capture students' oral words and phrases into a stable, collective reference containing illustrations connected to mathematical concepts and terms.
- **MLR3: Critique, Correct, and Clarify**—to give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop.
- **MLR4: Information Gap**—to create a need for students to communicate [in math] (Gibbons, 2002).
- **MLR5: Co-Craft Questions and Problems**—to allow students to get inside a context before feeling pressure to produce answers, to create space for students to produce the language of mathematical questions themselves, and to provide opportunities for students to analyze how different mathematical forms can represent different situations.
- **MLR6: Three Reads**—to ensure that students know what they are being asked to do, create opportunities for students to reflect on the ways mathematical questions are presented, and equip students with tools used to negotiate meaning (Kelemanik et al., 2016).
- **MLR7: Compare and Connect**—to foster students' meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.
- **MLR8: Discussion Supports**—to support rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies (Chapin, O'Connor, & Anderson, 2009).

English language learners (ELL) represent a large portion of the U.S. public school population, with data from the National Center for Education Statistics placing the number at 10 percent of the student population (U.S. Department of Education, National Center for Education Statistics [NCES], 2017). Nationwide, one out of ten students is an English language learner, and a higher percentage exists in many states, districts, schools, and classrooms. The performance and instructional needs of English language learners is of particular interest to educators.

Research suggests that we could do more to meet the needs of this population. In 2011, the NAEP reading assessment showed a 36-point gap at fourth grade and a 44-point gap at eighth grade between ELL and non-ELL students performance (NCES, 2017). While some may think of English language learners as a homogenous population, in fact, English language learners come to the classroom with different primary languages, English language proficiency levels, abilities, backgrounds, and needs. Beyond these differences, however, English language learners do share some common learning needs. Research suggests some specific strategies that can be of benefit.

Teachers can work with English language learners to internalize and apply these routines independently—whenever students encounter challenging mathematical language. These routines can be adapted and incorporated across lessons and modules—wherever there are productive opportunities to support students in using and improving their English and mathematics languages through many opportunities for reading, writing, listening, and speaking about mathematics.

HOW *INTO AGA* DELIVERS

LANGUAGE DEVELOPMENT • Planning for Instruction

EL By giving all students regular exposure to language routines in context, you will provide opportunities for students to **listen for, and speak, read,** and **write** about mathematical situations. You will also give students the opportunity to develop understanding of both mathematical language and concepts.

Using Language Routines to Develop Understanding

Use the **Professional Learning Cards** for the following routines to plan for effective instruction.

Co-Craft Questions and Problems Lessons 2.2–2.5

Students think of questions to ask about a given situation or problems similar to a given task. Then they answer the questions they have developed or solve the problems they have created.

Three Reads Lessons 2.2 and 2.5

Students read a problem three times with a specific focus each time.

1st Read What is the situation about?
2nd Read What are the quantities in the situation?
3rd Read What are the possible mathematical questions that you could ask for the situation?

Information Gap Lesson 2.1

Students recognize when information given in a problem situation is incomplete, and they pose questions and share knowledge with others to discover any missing facts or relationships and work together to solve the problem.

Critique, Correct, and Clarify Lesson 2.5

Students correct the work in a flawed explanation, argument, or solution method; share with a partner; and refine the sample work.

EL CULTIVATE CONVERSATION • Co-Craft Questions

Tell students to read the information in the photo three times and prompt them with a different question each time.

1. What is the situation about? The situation is about a cheetah chasing a gazelle. The gazelle has a head start, but the cheetah runs faster.
2. What are the quantities in this situation? How are those quantities related? The quantities are the animals' speeds, running times, distances traveled, and the initial distance separating the animals; Each animal's distance traveled is the product of its speed and running time. The cheetah's distance traveled needs to exceed the gazelle's distance traveled by the initial distance separating the animals if the cheetah is to catch the gazelle.
3. What are possible questions you could ask about this situation? Possible answer: Does the cheetah get tired before it can catch the gazelle? If the cheetah catches the gazelle, how much time does it take to do so?

EL CULTIVATE CONVERSATION • Information Gap

Ask students questions to help them decide what missing information they need to answer the question, "Which store is the better buy?"

1. What information do you need to find the total cost of the phone? tax rates, discount, rebate amount, price of phone
2. How will the rebate affect the total cost of the phone? Possible answer: After calculating the cost with tax, the rebate will be subtracted.
3. How will you find the cost of the phone with tax? Possible answer: I will multiply the price of the phone by 1 more than the tax rate.
4. In addition to the tax rate and the amount of rebate, what information do you need? I need the price of the phone.

Information on using **language routines** to develop understanding appears at the start of each module in the Teacher's Edition of the *Into AGA* solution. Teachers can use the plan provided to strengthen instruction by fortifying language practice for students.

Key Academic Vocabulary

Prior Learning and Current Development • Review and New Vocabulary	
equation a mathematical statement indicating that two expressions are equal by using the symbol =	**like terms** variable terms whose variables and their exponents are the same
equivalent equations equations that have the same solution	**solution of an equation (or inequality) in one variable** a number that, when substituted for the variable in the equation (or inequality), produces a true statement
expression a mathematical phrase that combines numbers and/or variables using mathematical operations	
inequality a mathematical statement indicating that two expressions are unequal by using one of the symbols <, >, ≤, or ≥	**compound inequality** two or more inequalities joined by *and* or *or*
	literal equation an equation in which constants have been replaced by letters

Students use **sense-making activities** to understand their learning, while prompts are embedded for teachers as they support students' mathematical understanding.

COMMUNICATING MATHEMATICALLY

Through classroom discourse, which includes listening and speaking, all aspects of mathematical thinking can be discussed, dissected, and understood. **Dialogue in the classroom provides access to ideas, relationships among those ideas, strategies, procedures, facts, mathematical history, and more** (Chapin et al., 2009).

Programs that engage students in discussions about how people learn, how to overcome obstacles to learning, and how to create a community of learners have shown an increase in students' confidence, motivation, and persistence and a strengthening in students' beliefs that they have control over their intelligence (NCTM, 2018). This approach provides the teacher with better information to use when diagnosing student difficulties, and it makes more than one "teacher" available to help students make connections (Donovan & Bransford, 2005).

When students are using language in ways that are purposeful and meaningful for themselves, in their efforts to understand—and be understood by—each other, they are motivated to attend to ways in which language can be both clarified and clarifying (Mondada & Doehler, 2004). **Conversations act as scaffolds for students developing mathematical language because discussion provides opportunities to simultaneously make meaning and communicate that meaning** (Mercer & Howe, 2012; Zwiers, 2014).

To foster a communicative classroom culture, *Into AGA* provides instruction in mathematical vocabulary and language support, including a Spanish transadaptation of all mathematical terms as well as a chance to record one's own interpretation of new academic language. Mathematical instruction is also available online, adding to the ways students can interact with new knowledge. The Standards for Mathematical Practice are varieties of expertise employed by mathematically proficient students. These standards are emphasized in all aspects of the solution, and teachers are prompted throughout to discuss them.

HOW *INTO AGA* DELIVERS

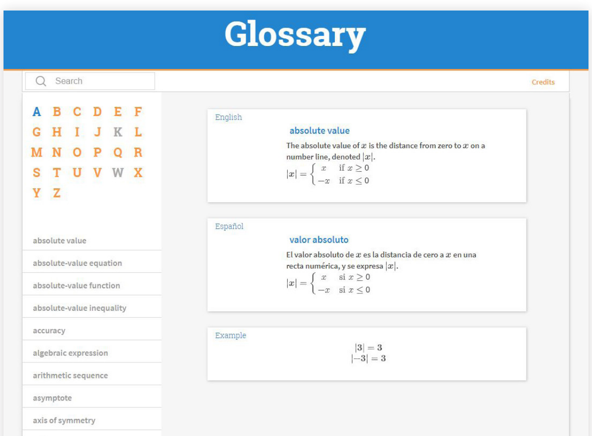
Opportunities that encourage speaking, listening, and writing about mathematics help students learn, reflect on, and refine mathematical ideas. *Into AGA* reinforces the importance of mathematical discourse through communicating mathematically by providing math-specific vocabulary activities that encourage talking and writing mathematically and by allowing vocabulary to emerge naturally rather than front- loading it. The solution reflects and incorporates the guiding principles of mathematical language, a multilingual eGlossary (10 languages), and numerous opportunities for writing and speaking mathematically.

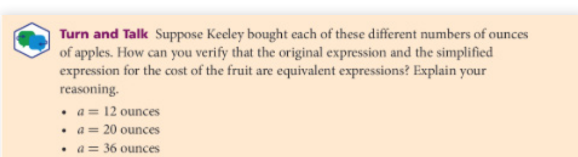
In addition, through *Into AGA*, students learn the language of mathematics through vocabulary-specific activities and guided instruction:

- **Connect to Vocabulary**—Students work alone or with partners to complete a vocabulary activity that introduces them to mathematical language essential to learning in the unit.
- **Interactive Glossary**—Students are able to record their understanding of what words mean to them.

Specific solution features that encourage oral and written communication include the following:

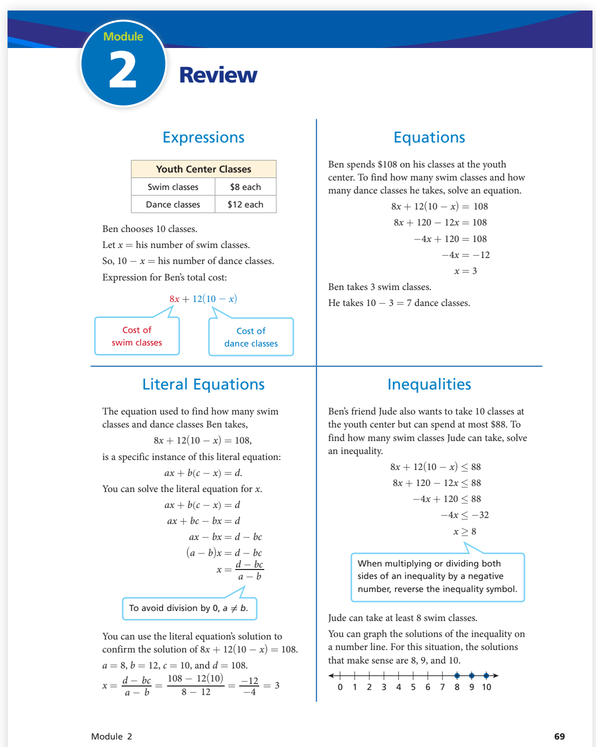
- **Turn and Talk** offers a chance for students to talk about math—and an opportunity for teachers to monitor and assess student progress.
- **Put It in Writing** provides a chance for students to write about what they know.
- **Module Review** includes a review of vocabulary learned throughout each module.





Turn and Talk Suppose Keeley bought each of these different numbers of ounces of apples. How can you verify that the original expression and the simplified expression for the cost of the fruit are equivalent expressions? Explain your reasoning.

- $a = 12$ ounces
- $a = 20$ ounces
- $a = 36$ ounces



Module 2 Review

Expressions

Youth Center Classes	
Swim classes	$8 each
Dance classes	$12 each

Ben chooses 10 classes.
Let x = his number of swim classes.
So, $10 - x$ = his number of dance classes.
Expression for Ben's total cost:

$8x + 12(10 - x)$

Cost of swim classes — Cost of dance classes

Equations

Ben spends $108 on his classes at the youth center. To find how many swim classes and how many dance classes he takes, solve an equation.

$$8x + 12(10 - x) = 108$$
$$8x + 120 - 12x = 108$$
$$-4x + 120 = 108$$
$$-4x = -12$$
$$x = 3$$

Ben takes 3 swim classes.
He takes $10 - 3 = 7$ dance classes.

Literal Equations

The equation used to find how many swim classes and dance classes Ben takes,
$8x + 12(10 - x) = 108$,
is a specific instance of this literal equation:
$ax + b(c - x) = d$.
You can solve the literal equation for x.

$$ax + b(c - x) = d$$
$$ax + bc - bx = d$$
$$ax - bx = d - bc$$
$$(a - b)x = d - bc$$
$$x = \frac{d - bc}{a - b}$$

To avoid division by 0, $a \neq b$.

You can use the literal equation's solution to confirm the solution of $8x + 12(10 - x) = 108$.
$a = 8$, $b = 12$, $c = 10$, and $d = 108$.

$$x = \frac{d - bc}{a - b} = \frac{108 - 12(10)}{8 - 12} = \frac{-12}{-4} = 3$$

Inequalities

Ben's friend Jude also wants to take 10 classes at the youth center but can spend at most $88. To find how many swim classes Jude can take, solve an inequality.

$$8x + 12(10 - x) \leq 88$$
$$8x + 120 - 12x \leq 88$$
$$-4x + 120 \leq 88$$
$$-4x \leq -32$$
$$x \geq 8$$

When multiplying or dividing both sides of an inequality by a negative number, reverse the inequality symbol.

Jude can take at least 8 swim classes.
You can graph the solutions of the inequality on a number line. For this situation, the solutions that make sense are 8, 9, and 10.

0 1 2 3 4 5 6 7 8 9 10

Module 2 69

MATHEMATICAL LANGUAGE

MATHEMATICAL DISCOURSE

Mathematical discourse, or talking about mathematics, is seen as an **increasingly important way for students to learn and make sense of mathematics**. Janzen's 2008 review of research on teaching English language learners in mathematics shares that while many perceive that math is easier for ELLs to learn because it involves numbers, mathematics actually presents specific language challenges. To ensure that all students can gain access to, interpret, and share information fluently, teachers must address multiple dimensions of instruction (Kersaint, Thompson, & Petkova, 2013; Ontario Ministry of Education, 2013). When introducing academic words to English language learners' expressive vocabularies, students respond best to classrooms that are routine and offer frequent and comfortable opportunities to express what they have learned (Feldman & Kinsella, 2008).

Giving English language learners a voice in the classroom increases teachers' opportunities to get to know their students and better understand their readiness to learn. This supports the National Council of Teachers of Mathematics' Communication Standard (2000), which notes that "the communication process also helps build meaning and permanence for ideas and makes them public" (p. 60). Classroom discussions can be organized in ways that support the acquisition of mathematics concepts and language development (Smith & Stein, 2011). Students need to learn (and practice) how to use language to clearly communicate what they know (Zwiers, 2014).

MATHEMATICAL WRITING

"Reflection and communication are intertwined processes in mathematics learning. . . . **Writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas**" (NCTM, 2000, p. 61). While they are writing, students can put concepts and procedures in their own words and reflect on learning that confuses them or makes sense to them. Students need opportunities to engage in academic discourse—and writing is an effective way to achieve this goal.

Numerous studies have emphasized the importance of writing in the mathematics classroom. David Pugalee (2005), who researches the relationship between language and mathematics learning, asserts that **writing supports mathematical reasoning and problem solving and helps students internalize the characteristics of effective communication**. He suggests that teachers read student writing for evidence of logical conclusions, justification of answers and processes, and the use of facts to explain their thinking. Bosse and Faulconer (2008) report that writing in the mathematics classroom results in deeper student learning.

Writing during math instruction has been found to give students more confidence in their math abilities, create more positive attitudes toward math, and help students understand complex math concepts (Furner & Duffy, 2002; Taylor & McDonald, 2007). Importantly, writing appears to benefit all students, with researchers finding benefits for both low-achieving students (Baxter, Woodward, & Olson, 2005) and high-achieving students (Brandenburg, 2002).

HOW *INTO AGA* DELIVERS

Each lesson of each module includes **vocabulary** learning as well as numerous prompts to engage in mathematical discourse. Students are encouraged to Turn and Talk with each lesson, and teachers are assisted with call-out boxes to guide students to **connect ideas, reasoning, and language.**

Connect to Vocabulary

When you transform something, you change it in some way. In mathematics, a **transformation** is a change in the position, size, or shape of a figure or graph.

Throughout the solution, students are guided to put their thoughts in writing. Teachers can review students' responses to see what they know or still need to learn. Writing also helps teachers understand what English language learners are thinking so they can best support them.

Put It in Writing

Describe some strategies you can use to solve a linear equation in one variable.

Build Understanding

Investigate Properties of Equality

An **equation** is a mathematical statement comparing two expressions using the symbol =. When you solve an equation, the **solution** produces a true statement when substituted for the variable in the equation. Solving an equation involves using properties of equality to write simpler **equivalent equations**, which all have the same solution as the original equation. In the Symbols column of the table below, *a*, *b*, and *c* are real numbers.

Properties of Equality

Property	Words	Symbols
Addition Property of Equality	Adding the same number to both sides of an equation produces an equivalent equation.	If $a = b$, then $a + c = b + c$. **Example:** If $x - 2 = 3$, then $x - 2 + 2 = 3 + 2$.
Subtraction Property of Equality	Subtracting the same number from both sides of an equation produces an equivalent equation.	If $a = b$, then $a - c = b - c$. **Example:** If $x + 4 = -1$, then $x + 4 - 4 = -1 - 4$.
Multiplication Property of Equality	Multiplying both sides of an equation by the same nonzero number produces an equivalent equation.	If $a = b$ and $c \neq 0$, then $ac = bc$. **Example:** If $\frac{x}{3} = 2$, then $\frac{x}{3} \cdot 3 = 2 \cdot 3$.
Division Property of Equality	Dividing both sides of an equation by the same nonzero number produces an equivalent equation.	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. **Example:** If $-2.5x = 10$, then $\frac{-2.5x}{-2.5} = \frac{10}{-2.5}$.

LEARNING MINDSET

Mathematics can leave students feeling frustrated and overwhelmed. Every educator has heard many students exclaim, "I'm just not good at math." **All students must have the opportunity to develop their mindset**. In her 2006 book *Mindset: The New Psychology of Success*, Carol Dweck observed that some learners assume the critical issue is whether they are inherently "smart" or not. She writes, "What puts an end to this exuberant learning? The fixed mindset. As soon as children become able to evaluate themselves, some of them become afraid of challenges. They become afraid of not being smart" (p. 16). Students with a fixed mindset about math do not see the value in putting forth the effort to learn. Students who struggle, in particular, are more likely to have a negative, fixed mindset toward their own mathematical abilities (Boaler, 2016).

Standing in contrast to the fixed mindset is growth mindset. **Growth mindset** is the belief that through effort and perseverance one can become better at something. Engagement, motivation, choice, ownership, and a growth mindset are positive learning behaviors that are deeply related (Dweck, 2006; Glei, 2013). In support of a growth mindset, teachers should focus on the effort that it takes to master a task or discipline—**effort is the key currency, not initial aptitude**. Teachers can reward students for effort rather than for being smart. As Dweck (2012) notes in *Mindset:*
How You Can Fulfill Your Potential, being smart does not always mean you will end up the smartest.

Dockterman and Blackwell (2014) liken this approach to the way video games work. They observe that learners need to begin by **building confidence** so they stick with it as the challenges increase. The authors further describe the benefits that carrying a growth mindset into instruction can bring. Their work underscores the idea that being challenged and succeeding can bring positive learning behaviors (Dockterman & Blackwell, 2014).

Jo Boaler (2016) is more specific about the effects that the growth mindset can have on learning. She notes that many studies have shown that "when students are given opportunities to pose mathematics problems, to consider a situation and think of a mathematics question to ask of it—which is the essence of real mathematics—they become more **deeply engaged and perform at higher levels**." The ability to consider the application of mathematics—to consider a real problem and grapple with how to represent it mathematically—is critical to internalizing the core concepts (Boaler, 2016).

Students with a growth mindset believe that their **intelligence and abilities can be developed and improved over time through effort and dedication**. Clearly, students' mindsets play an important role in their learning. *Into AGA* embraces this ideology by providing many checkpoints for teacher feedback, ongoing communication, and countless opportunities to practice independence, apply themselves, and make progress. This progress, and the positive feedback it brings, underpin the learning mindset.

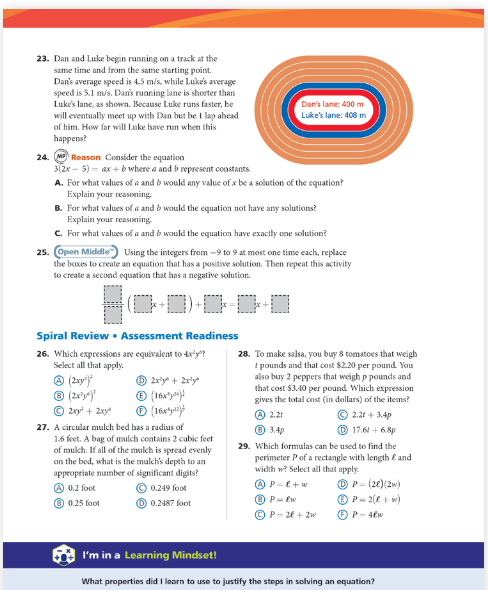
HOW *INTO AGA* DELIVERS

Into AGA develops a growth mindset and encourages learning strategies that are critical to student success and confidence in math. The learning mindset is integrated into the solution by building on techniques that foster a sense of purpose, belonging, and self-efficacy in students. This integration encourages students to **spark their learning** and then **think it through** to build mastery. Students are encouraged to develop awareness of their mindsets in order to continue to grow and achieve—and to believe they can do mathematics.

Into AGA fosters a growth mindset by encouraging students to seek challenges. The materials for students and the supplementary materials for teachers push students toward the sort of reflection that is critical to obtaining a growth mindset in mathematics. The exercises provide problems that allow students to apply their understanding, gathering positive feedback by demonstrating their knowledge. The teacher materials give many suggestions for sparking student discussion and further engaging learners in the process of exploration.

The student exercises and the instructor guidance create an ideal pathway for enabling students to wrestle with smaller problems and have short-term successes that build their confidence. This confidence is critical to building a growth mindset. Students need to enjoy a series of victories that underscore the mindset that grappling with problems, and tackling them, is the pathway to future successes. This solution provides the scaffolding to make that a reality.

As students develop their mindset, teachers are also building their confidence and knowledge of mathematics teaching and learning. As noted later in this report, teachers are provided with ongoing and embedded professional learning throughout *Into AGA*.



23. Dan and Luke begin running on a track at the same time and from the same starting point. Dan's average speed is 4.5 m/s, while Luke's average speed is 5.1 m/s. Dan's running lane is shorter than Luke's lane, as shown. Because Luke runs faster, he will eventually meet up with Dan but be 1 lap ahead of him. How far will Luke have run when this happens?

Dan's lane: 400 m
Luke's lane: 408 m

24. MP **Reason** Consider the equation $3(2x - 5) = ax + b$ where a and b represent constants.

A. For what values of a and b would any value of x be a solution of the equation? Explain your reasoning.

B. For what values of a and b would the equation not have any solutions? Explain your reasoning.

C. For what values of a and b would the equation have exactly one solution?

25. Open Middle™ Using the integers from −9 to 9 at most one time each, replace the boxes to create an equation that has a positive solution. Then repeat this activity to create a second equation that has a negative solution.

$$\frac{\square}{\square}(\square x + \square) + \square x = \square x + \square$$

Spiral Review • Assessment Readiness

26. Which expressions are equivalent to $4x^2y^6$? Select all that apply.

Ⓐ $(2xy^3)^2$
Ⓑ $(2x^2y^6)^2$
Ⓒ $2xy^2 + 2xy^4$
Ⓓ $2x^2y^6 + 2x^2y^6$
Ⓔ $(16x^8y^{24})^{\frac{1}{4}}$
Ⓕ $(16x^4y^{12})^{\frac{1}{2}}$

27. A circular mulch bed has a radius of 1.6 feet. A bag of mulch contains 2 cubic feet of mulch. If all of the mulch is spread evenly on the bed, what is the mulch's depth to an appropriate number of significant digits?

Ⓐ 0.2 foot
Ⓑ 0.25 foot
Ⓒ 0.249 foot
Ⓓ 0.2487 foot

28. To make salsa, you buy 8 tomatoes that weigh t pounds and that cost $2.20 per pound. You also buy 2 peppers that weigh p pounds and that cost $3.40 per pound. Which expression gives the total cost (in dollars) of the items?

Ⓐ $2.2t$
Ⓑ $3.4p$
Ⓒ $2.2t + 3.4p$
Ⓓ $17.6t + 6.8p$

29. Which formulas can be used to find the perimeter P of a rectangle with length ℓ and width w? Select all that apply.

Ⓐ $P = \ell + w$
Ⓑ $P = \ell w$
Ⓒ $P = 2\ell + 2w$
Ⓓ $P = (2\ell)(2w)$
Ⓔ $P = 2(\ell + w)$
Ⓕ $P = 4\ell w$

I'm in a **Learning Mindset!**

What properties did I learn to use to justify the steps in solving an equation?

Student Edition

I Can

The scale below can help you and your students understand their progress on a learning goal.

4	I can use the properties of equality and the Distributive Property to solve an equation with the variable on both sides, and I can explain my solution steps to others.
3	I can use the properties of equality and the Distributive Property to solve an equation with the variable on both sides.
2	I can use the properties of equality and the Distributive Property to solve an equation with the variable and grouping symbols on one side.
1	I can recognize which property of equality is being used to produce an equivalent equation in a stepped-out solution.

I Can scales available in both the Student and Teacher Editions

Learning Mindset mindset works

Resilience **Monitors Knowledge and Skills**

Point out that making sure a given step in solving a problem follows logically from the previous step is essential in all areas of mathematics, not just when solving equations. Encourage students to develop the habit of justifying the steps of their solutions when solving any mathematical problem. Also let students know that although they used the Distributive Property and the properties of equality to solve only linear equations in this lesson, they will use these properties to solve other types of equations later in the course. *How does justifying each step when solving a mathematical problem help you have a more confident mindset? When you can't think of a good justification for one of your solution steps, what might that suggest about your approach to solving the problem?*

Teacher Edition

ASSESSMENT, DATA, AND REPORTS

Into AGA provides ongoing, balanced assessment and reporting and harnesses digital technologies to empower teachers with data-driven decision making and tools for effective instructional planning. *Into AGA* provides grouping and resource recommendations. This data-driven solution additionally provides critical feedback loops that encourage students' self-assessment and reflection while freeing teachers from guesswork and time-consuming assessment reporting and subsequent material selections and planning. These approaches to evaluation of learning support optimal instructional practices and drive positive outcomes for each and every student.

ASSESSMENT, DATA, AND REPORTS

DIAGNOSTIC ASSESSMENT

Far more than a vehicle for grades, **effective assessment is a powerful source of evidence** regarding readiness to learn (diagnostic), for learning (formative), and of learning (summative); yet the strength of this information depends upon **alignment** of assessments with standards and learning goals and a representative **balance** among the skills, concepts, applications, and depth of knowledge assessed (Leinwand, 2020).

Assessment is an essential part of instruction and a process by which teachers can continuously gauge student understanding. Teachers can collect a variety of evidence from students before, during, and after instruction to best meet students' needs. Assessment includes "all those activities undertaken by teachers—and by their students in assessing themselves—that provide information to be used as feedback to modify teaching and learning activities" (Black & Wiliam, 1998, p. 140). **Diagnostic assessment**, which takes place at the beginning of the school year or opening of a unit of instruction, **is undertaken in order to provide information about individual student needs**. This form of assessment guides teachers as they plan and make instructional decisions. Diagnosing students' current knowledge and skill is essential because "what students can learn at any particular grade level depends upon what they have learned before" (National Governors Association [NGA] Center for Best Practices and Council of Chief State School Officers [CCSSO], 2010, Introduction).

Diagnostic assessments are beneficial to teachers because they "provide information about students' mastery of relevant prior knowledge and skills within the domain as well as preconceptions or misconceptions about the material" (Ketterlin-Geller & Yovanoff, 2009, p. 1). Assessing students' skills and knowledge at the beginning of the year, course, or unit enables educators to identify at-risk students or those who need additional support (Fuchs & Fuchs, 2006). Research points to the advantages of using diagnostic assessments—and tailoring instruction and supplemental practice according to the results of the diagnostics (Mayes, Chase, & Walker, 2008). In classrooms using a Response to Intervention model to support mathematics learning, effective use of screening tools and diagnostic data are essential elements for success (Lembke, Hampton, & Beyers, 2012).

Certain features make diagnostic assessment useful to educators. Lemke and colleagues (2012) argue in their review of the research that diagnostic assessments—including computer-based diagnostic assessments—offer the possibility of analyzing patterns of student errors and identifying students' specific deficits. Effective assessment tied closely to learning provides evidence of proficiency with important mathematical concepts and includes a variety of strategies and data sources (NCTM, 2014).

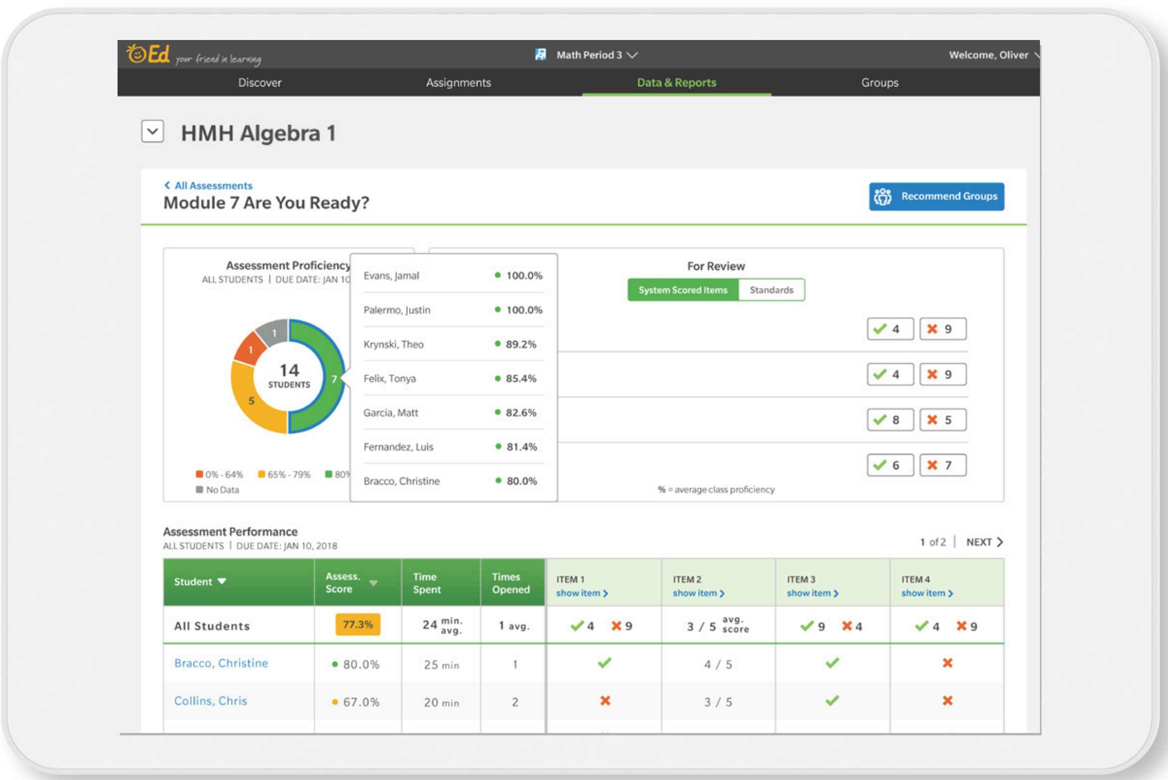
Effective assessment tools allow teachers to collect data about what is working—and what is not—so they can take precise, swift, and effective action in meeting the specific needs of students. As noted by numerous research studies, the regular use of assessment to monitor student progress can mitigate and prevent mathematical weaknesses and improve student learning (Clarke & Shinn, 2004; Fuchs & Wößmann, 2004; Lembke & Foegen, 2005). In their research, Baker, Gersten, & Lee (2002) concluded that "providing teachers and students with information on how each student is performing seems to enhance . . . achievement consistently" (p. 67).

We know that effective assessments are far more than just tests we use to help us grade students. Instead, effective assessments are powerful vehicles for gathering evidence of readiness to learn (diagnostic), for learning (formative), and of learning (summative). We also know that the strength and usefulness of the evidence we gather depends upon the alignment of these assessments with our standards and our learning goals, as well as upon the balance among skills, concepts, applications, and depth of knowledge found in our assessments.

HOW *INTO AGA* DELIVERS

Into AGA provides **a balanced assessment system, including diagnostic, formative, and summative measures—as well as a plan for growth**. This comprehensive system provides teachers and students **actionable feedback** they can use throughout the year to learn more about student learning and progress. A comprehensive assessment is available from every perspective to best understand, monitor, predict, and accelerate students' mathematical learning growth.

Into AGA provides specific features and tools that teachers can use for diagnostic assessment to more effectively assess students' need for instruction. **Each module begins with Are You Ready?, a brief diagnostic that focuses on strategic concepts of the module, prior knowledge, and foundational prerequisites**. Teachers gain a better understanding of what students know and can do, allowing for better informed instruction. From the diagnostic assessment, the lessons build upon **conceptual and procedural understanding** and integrate applications to build understanding and formative checkpoints to gauge understanding where it is needed.



Reports on ***Ed: Your Friend in Learning***® are interactive. Selecting a part of the report will reveal a drill down into the data.

FORMATIVE ASSESSMENT

Formative assessment, according to James Popham, is a carefully planned process in which teachers use evidence from assessments of student progress to adjust ongoing instruction or in which students use assessment-based information to adjust their learning strategies. Assessment should be ongoing, "more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions" (NCTM, 2000, p. 1).

A body of research demonstrates the **positive impacts that ongoing, formative assessment has on student learning** (Black & Wiliam, 1998; Cotton, 1995; Jerald, 2001); "providing teachers and students with information on how each student is performing seems to enhance . . . achievement consistently" (Baker, Gersten, & Lee, 2002, p. 67). Regular monitoring of students' progress in mathematics has been shown to mitigate and prevent problems and improve learning (Clarke & Shinn, 2004; Fuchs & Wößmann, 2004; Lembke & Foegen, 2005; Skiba, Magnusson, & Erikson, 1986).

Effective teachers monitor student learning through informal, low-stakes assessments—such as short questions and homework assignments—and informal measures—such as discussion and observation (Cotton, 1995; Christenson, Ysseldyke, & Thurlow, 1989). This kind of ongoing assessment must be curriculum based; because curriculum-based assessment rationally relates to the instructional objectives and can inform instructional decisions related to student difficulties, it can lead to higher student achievement (Jones, Wilson, & Bhojwani, 1997).

Numerous studies have shown that this use of **regular, formative assessment positively impacts student learning and achievement**. Baker et al. (2002) conducted a meta-analysis on the subject of formative assessment in the math classroom and found that achievement increased as a result of regular assessment use: "One consistent finding is that providing teachers and students with specific information on how each student is performing seems to enhance mathematics achievement consistently. The effect of such practice is substantial" (p. 67).

A review of high-quality studies on formative assessment led the National Mathematics Advisory Panel (2008) to conclude that "use of formative assessments benefited students at all ability levels" (p. 46). But a specific benefit of formative assessment may be that it has been shown to be particularly helpful to lower-performing students. Gersten et al. (2007) found that "the use of ongoing formative assessment data invariably improved mathematics achievement of students with mathematics disability" (p. 2). For this reason, the use of formative assessments can serve to minimize achievement gaps while raising overall achievement (Black & Wiliam, 1998).

HOW *INTO AGA* DELIVERS

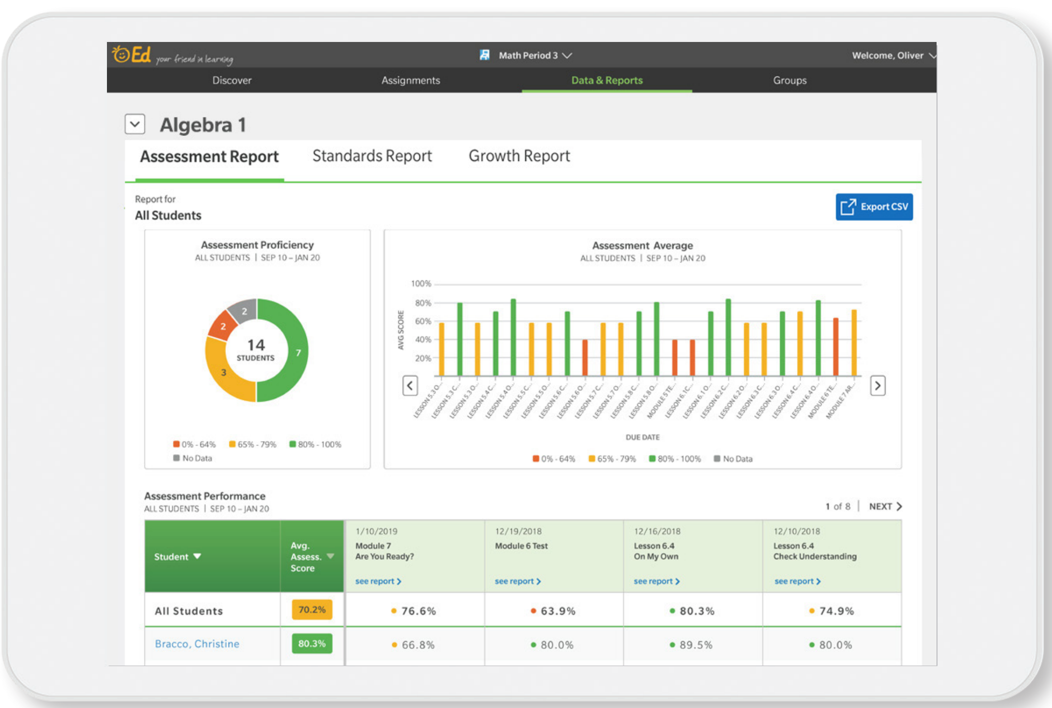
The *Into AGA* assessment system also includes the following informal classroom measures, which are embedded into the solution to help teachers gauge student understanding before and during instruction:

- Activate Prior Knowledge
- Warm-Up Options
- Problem of the Day
- Make Connections
- Math Center Options
- Spark Your Learning
- Motivate
- Persevere
- Turn and Talk
- Build Shared Understanding
- Support Sense-Making

As the lessons build upon conceptual and procedural understanding and integrate applications to build understanding and to differentiate instruction, further formative measures are embedded throughout to support teachers' efforts to differentiate instruction. **Formative Assessment built into each lesson gives teachers the ability to quickly assess understanding**. The solution's assessment system includes these measures:

- Build Understanding, which includes Turn and Talk, Leveled Questions, Vocabulary, Mathematical Language Routines, Language Proficiency Level Supports, and Check Understanding
- Step It Out, which includes Turn and Talk, Leveled Questions, Mathematical Language Routines, and Check Understanding
- Independent Practice, which includes On Your Own and Learning Mindset
- Homework/More Practice and Wrap-Up Options (Exit Ticket, Put It in Writing, and I Can scale)

Into AGA provides opportunities for formative assessment, which are embedded throughout instruction. Teachers can use what they learn to best group students and provide targeted and specific intervention or enrichment where it is needed, when it is needed. In addition, Test Prep is available within the lesson as part of the homework, and the **Getting Ready for High-Stakes Assessment** provides test prep.



Class-level scores on assessments (module/unit and lesson-level assessments) and overall class proficiency can be viewed.

SUMMATIVE ASSESSMENT

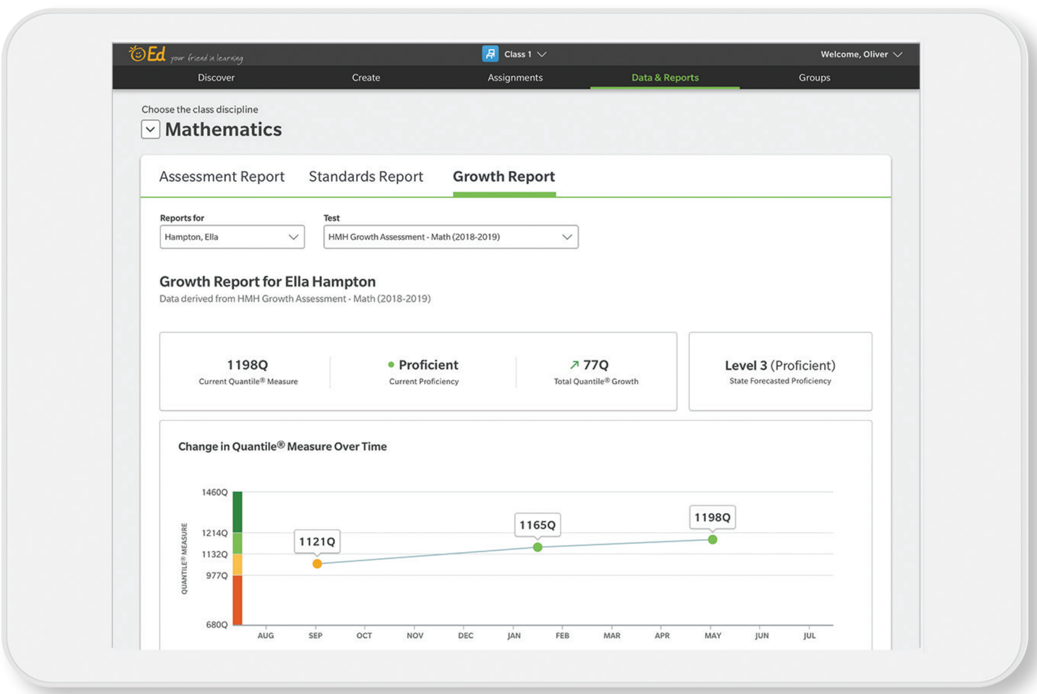
Many times, the term *summative assessment* is associated with high-stakes testing, but there is a role for summative assessments in the classroom because they act as an additional constructive measure. "Tests given in class . . . are also an important means of promoting feedback. A good test can be an occasion for learning" (Black & Wiliam, 1998, p. 8). When summative results are elicited and interpreted—and teachers take action on that interpretation—that "action will then (directly or indirectly) generate further evidence leading to subsequent interpretation and action, and so on" (Wiliam, 2000, Presentation). Teachers can use summative assessments as another measure, another point in time, and another means by which to best evaluate student understanding. As part of an integrated assessment system, summative measures can also help teachers shape instruction and differentiate to personalize learning.

Summative assessments are also useful as accountability measures for grading and gauging student learning against a set of standards or expectations. Summative assessments provide evaluative information to teachers about the effectiveness of their instructional program. The program also offers opportunities for summative assessment through unit and module tests. These evaluations, in combination with the Math Growth Measure, provide a cumulative system of assessment to understand students' mathematical development. Classroom summative assessments also appear to have the potential to positively impact learning (Moss, 2013). When students have a handle on what they can do, they can be encouraged and motivated to move forward in their learning.

Houghton Mifflin Harcourt's Into AGA offers robust summative assessments in print and digital formats. **Measuring student growth is possible through detailed reports connected to the learning goals, which offer insight to student growth over time.** The program measures formative and summative assessment through predictions based on standardized scales to measure student growth. In combination with a teacher's understanding of student achievement from the interim benchmark assessment data, summative assessment data can show growth over time for students as they progress through their learning. By adding data analytics reports to classroom practice, teachers can easily use assessments to provide specific feedback to students about their progress toward their learning goals and to advance in their learning of mathematics

HOW *INTO AGA* DELIVERS

The *Into AGA* assessment system includes continuous growth monitoring through regular check-ins. Growth reports are available digitally on ***Ed*** and provide detailed analyses of student performance. *Into AGA* includes the HMH Math Growth Measure, which is an adaptive math assessment intended to be administered three times a year to measure student growth, forecast to state summative results and track longitudinal progress as well as, providing the ability to forecast to state-level summative assessments. In addition, HMH has partnered with Renaissance Learning® so *Into AGA* can provide an integrated approach to differentiation and predictive measures for student growth. Within a school year and across school years, growth monitoring ensures that students have the skills to meet the mathematics standards and advance to higher-level mathematical thinking.



Data are displayed in a variety of representations, and users can drill down into data for thorough insights into performance.

EQUITY

Mathematics instruction must meet the needs of each and every student. To accomplish this, teachers must employ effective instructional strategies and follow an intentional sequence, scaffolding instruction so that all students can access the information. *Into AGA* centers on helping students gain a deeper understanding of mathematics by incorporating tactics to meet their varying needs through strong differentiation supports. In addition to providing equitable access to learning, the solution aims to promote learning about students and using their experiences to facilitate learning. Teachers are given the tools to help create supportive classroom environments to facilitate the deep learning of mathematics for all.

EQUITY

ACCESS TO LEARNING

"What is good mathematics instruction? If we expect that our instruction is going to be strong, then we need to ensure that our instruction reaches all levels of learners; that students are engaged in daily, rich mathematical discussions; and that we allow ample time for students to grapple with mathematical ideas" (Lempp, 2017, p. 4).

All students must have opportunities to study—and support to learn—mathematics. Equity does not mean students should receive identical instruction; instead, it requires that **reasonable and appropriate accommodations be made as needed to promote access and attainment for all students** (NCTM, 2014).

In the classroom, teachers are challenged to support the diverse needs of students. As Vygotsky (1978) noted in his seminal research on learning, "Optimal learning takes place within students' 'zones of proximal development'—when teachers assess students' current understanding and teach new concepts, skills, and strategies at an according level" (p. 86). Research continues to support the notion that for learning to take place, activities must be at the right level for the learner (Tomlinson & Allan, 2000; Valencia, 2007). Teachers must personalize students' needs for instruction and additional support, which includes examining levels that are just outside or beyond what the student has already mastered—encouraging and motivating students forward.

Most important, instruction must meet the needs of each and every student. To achieve this universal access, teachers must employ effective assessment practices, diagnose student needs, and assess progress regularly. Teachers must teach intentionally, scaffolding instruction so all students can access the information. Universal design and differentiation are terms used to describe two related and complementary approaches to meeting the needs of each and every student in the classroom.

In universal design, the needs of all students, including those with disabilities, are considered at the point of instructional design; methods, materials, and assessments (diagnostic, formative, summative) are created to recognize and address the wide range of student needs. In differentiation, modifications and WCAG-related accommodations take place at the point of instruction; in differentiating instruction, teachers are responsive to what happens in the classroom and are "flexible in their approach to teaching and adjust the curriculum and presentation of information to learners rather than expecting students to modify themselves for the curriculum" (Strangman, Hall, & Meyer, 2004).

LEARNING ABOUT STUDENTS

Students in today's classrooms come from increasingly diverse backgrounds in regard to culture and language as well as in their background knowledge, abilities, motivations, interests, and modes of learning (Tomlinson, 2005). Mathematical learning is important to each of these different students: "All young Americans must learn to think mathematically, and they must think mathematically to learn" (NRC, 2001, p. 1). To effectively teach mathematics skills and concepts, **teachers of mathematics must be knowledgeable of, and sensitive to, the needs of all learners in the mathematics classroom**.

To create an environment in which the barriers that limit comprehensive student access to learning are removed, teaching allows for flexible methods of presentation, expression, and engagement by offering multiple examples, employing multiple media and formats, engaging in supported practices, and allowing flexible opportunities for demonstrating skill (Strangman, Hall, & Meyer, 2004; Shaw, 2011). Technology can be particularly beneficial to allow for such flexibility (Hitchcock, Meyer, Rose, & Jackson, 2002).

An important component within effective instruction is leveraging students' experiences and background (Lawrence-Brown, 2004). Children whose experiences are devalued or unrecognized become alienated and disengaged from the learning process whereas **incorporating the experiences of students enhances learning.** "We argue that students need to learn mathematics in light of who they are and the diverse gifts that they bring to their experiences every day" (Aguirre, Mayfield-Ingram, & Martin, 2013, p. 10).

CREATING CLASSROOM ENVIRONMENTS

Students must be socially, emotionally, and academically safe. Given the acknowledged diversity of student preparation and background, personal and familial conditions, and cultural environment, this is more than challenging. For some students, the classroom is naturally safe, but this is certainly not the case for all. Research suggests that some students do better or participate more frequently in classrooms with cooperative learning projects and open discussions; for instance, for students from supportive and interdependent ethnic minority communities, competitive learning environments can cause feelings of isolation or alienation (Wlodkowski & Ginsberg, 1995).

Critically, students must be engaged in the productive perseverance that motivates and underpins the introduction of new concepts in a given module. This process and discourse cannot be derailed by failing to include all students' perspectives. If a student is stymied in this phase of the process, then it is unlikely that he or she would continue to embrace a learning mindset about this particular topic. Thus, **the classroom environment must be safe and conducive to learning for each and every student to motivate a growth mindset**.

HOW *INTO AGA* DELIVERS

Into AGA is built on principles of equity. The solution is designed to engage students based on their experiences and to leverage their diverse backgrounds to promote discussion. Students are motivated by being asked to share about themselves while setting the stage for the lesson.

The **Turn and Talk** portion of the lessons encourages students to reflect and discuss in small groups. This facilitates sharing of opinions. When student opinions and perspectives are heard and valued, students become more engaged in the learning process. This feature stems from *Talk Moves* by Chapin, O'Connor, & Anderson, 2013.

The solution is intentionally designed with specific mechanisms built in to **Activate Prior Knowledge**. Again, engaging students in learning challenges with references to their own experiences and knowledge is critical to motivating and activating their learning.

Finally, the most critical aspect of providing an equitable learning environment is supporting students of various levels through differentiated instruction. The teacher materials provide clear and intentional checkpoints for formative assessment. These checkpoints guide the differentiated aspects of the lesson, which are essential to instruct each and every student. Further, these can indicate potential issues to trigger interventions as necessary.

At the same time, students of higher abilities are not excluded. The differentiated materials provide additional enrichment opportunities for students who have mastered the base material. Equity cannot be achieved by simply teaching to the middle—each and every learner must be supported in their quest for mathematical learning.

With ever-evolving standards and classroom needs, monitoring growth thoroughly, thoughtfully, and easily through a comprehensive assessment system becomes increasingly significant. *Into AGA* Assessment Solutions give teachers what they need to personalize instruction and prepare students with the right skills to keep them growing.

DIFFERENTIATION

Dacey, Lynch, and Salemi (2013) describe differentiation as "a philosophy of teaching based on the belief that **all learners are different and that all students are capable of learning**" (pp. 5–6). In the classroom, teachers encounter students who are on grade, above grade, below grade—as well as English language learners, students with special needs, students who are gifted, and students with varying learning styles and cultural backgrounds. In the mathematics classroom, "mathematics instructors must respond to the diverse needs of individual students . . . using differentiated instruction, a process of proactively modifying instruction based on students' needs" (Chamberlin & Powers, 2010, p. 113).

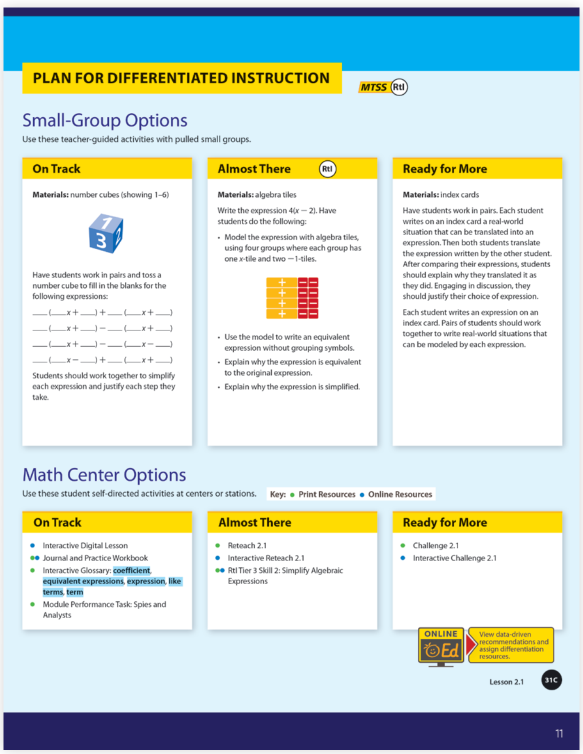
Differentiating instruction is an organized but flexible way to alter teaching and learning to help all students maximize their learning (Tomlinson, 1999) and is necessary in order to meet the needs of all learners in today's diverse classrooms (Tomlinson, 2000). At the core of differentiated is modification of at least one of of four curriculum-related elements: content, process, product, and affect (Lempp, 2017; Tomlinson and Imbeau, 2010). Teachers can have students work in groups to help support the development of their listening and speaking skills while increasing mathematical understanding (Garrison & Mora, 1999). Dacey, Lynch, and Salemi (2013) point out that "like all language skills, learning the language of mathematics is an important goal for all students and can remove barriers to learning mathematical ideas" (p. 149).

Research points to the benefits of differentiation. In a study of numerous teachers using differentiated instruction, researchers found these benefits: students felt learning was more relevant; students were motivated to stay engaged in learning; students experienced greater success; students felt greater ownership of content, products, and performances; and teachers gained new insights (Stetson, Stetson, & Anderson, 2007). Tomlinson and Allan purport that in order for learning to take place, activities must be at the right level for the learner (Tomlinson & Allan, 2000; Valencia, 2007).

While differentiation is a component of effective instruction, it is no small charge. Classroom teachers have students with varying needs; some need more help, and some need to work alone. It is up to the teacher to make these decisions, which are difficult, and some teachers might not have the experience or training to do so—making it more challenging. But teachers want to be successful and support each and every student (Small, 2012). As Marian Small (2012) instruction notes, "Understanding differences and differentiating instruction are important processes for achievement of that goal" (p. 1). Houghton Mifflin Harcourt's *Into AGA* provides a wide range of resources designed to differentiate instruction and meet the needs of each and every student throughout the lesson design.

HOW *INTO AGA* DELIVERS

Into AGA meets the needs of each and every learner. Differentiation in *Into AGA* supports teachers in implementing instruction that meets the varied needs of students in their classrooms. With *Into AGA*, practical, point-of-use support is built into each lesson so all learners can achieve success. The solution's Student Edition allows students to explore concepts, take notes, answer questions, and complete homework, thus encouraging active learning. Additional videos, activities, and learning aids support students at the point of use in the print and online textbook, supporting students' various learning styles. While print and digital are both available to meet schools' needs, robust real-time data and analytics are thorough digital options and are a great value to schools.



Multiple solution features, employing varied and flexible multimedia formats to support differentiated instruction, occur throughout *Into AGA*. These include the Plan for Differentiated Instruction, which contains activities for everyday use in the classroom in tailoring instruction to meet students' needs. There are also Math Center activities that focus on meeting students where they are in their learning, providing opportunities to strengthen and expand their mathematical connections.

In addition, these features help teachers meet the needs of all students:

- **Differentiated Instruction** suggestions provided in the **Teacher's Edition** offer ways for teachers to meet the needs of specific populations.
- **Journal** and **Practice Workbook** exercises provide ongoing practice for students at different levels of understanding.
- **Reteach** suggestions target ways for teachers to develop the skills of students who may not master concepts the first time around.
- **On Track, Almost There,** and **Ready for More** activities in text and online offer challenging extensions for students ready to move ahead.
- ***Math on the Spot*** video tutorials provide step-by-step instruction of the math concepts covered.
- **Change for Module Performance Tasks** Stem or Spies and Analysts Tasks let students collaboratively explore connection to the Module.
- **English Language Learners** are supported specifically by language routines that align to current research on how ELLs learn best. In addition, there are personalized ELL call-outs, vocabulary activities, games, organizers, and more included throughout to support students as they develop language and mathematics communication.
- **Recommendations within *Ed*** give teachers access to all content from grade 6 - Algebra 2 they teach to help with differentiation.

DIGITAL LEARNING

Research shows positive effects on student achievement when digital learning is embedded. *Into AGA* employs digital learning to engage students and increase their learning and achievement and to provide teachers with opportunities for additional options and variation in instructional delivery. Students can watch videos to evaluate their work, and practice in a testing environment similar to high-stakes assessment. There are also interactive tools for Spanish speakers and tools for home to further support instruction and enhance student learning.

DIGITAL LEARNING

TECHNOLOGY IN TEACHING AND LEARNING

Technology has long been part of the K–12 learning environment, yet technology changes rapidly, and new opportunities for mathematical teaching and learning are constantly emerging. **Technology has been shown to be a powerful tool in the classroom** across grade levels, content areas, and student abilities. The use of technology in the classroom has been shown to support teaching and learning. In fact, Weber and Kieschnick (2016) argued that 21st-century learning necessitates that teachers integrate technology strategically, as properly implemented educational technology elevates teacher practices.

A large number of systematically designed research studies have been conducted to answer the question of whether students learn more deeply from words and visuals than from written or verbal messages alone. Weiss, Kramarski, and Talis (2006) examined the impact of multimedia activities on the mathematics learning of young children and found that **students who engaged in digital learning, individually or in groups, significantly outperformed control-group students**. Much research concludes that visual images and/or audio along with text have positive impacts on student learning (Mayer, 2001). Researchers who have subsequently tested Mayer's results have agreed with his findings. Liu (2012) designed a study in which elementary students participated in multimedia mathematics instruction and found that the participants scored significantly higher after the intervention. Further, in a meta-analysis conducted by the U.S. Department of Education, it was determined that on average, students in online learning conditions performed better than those receiving face-to-face instruction (U.S. Department of Education, 2010).

One particular benefit of technology-based approaches to instruction is that they appear to be effective across different types of learners (Means, Toyama, Murphy, Bakia, & Jones, 2009). Technology-based multimedia learning opportunities have been shown to narrow achievement gaps between student groups and to be effective with average and lower-achieving students (Huppert, Lomask, & Lazarowitz, 2002; White & Frederiksen, 1998).

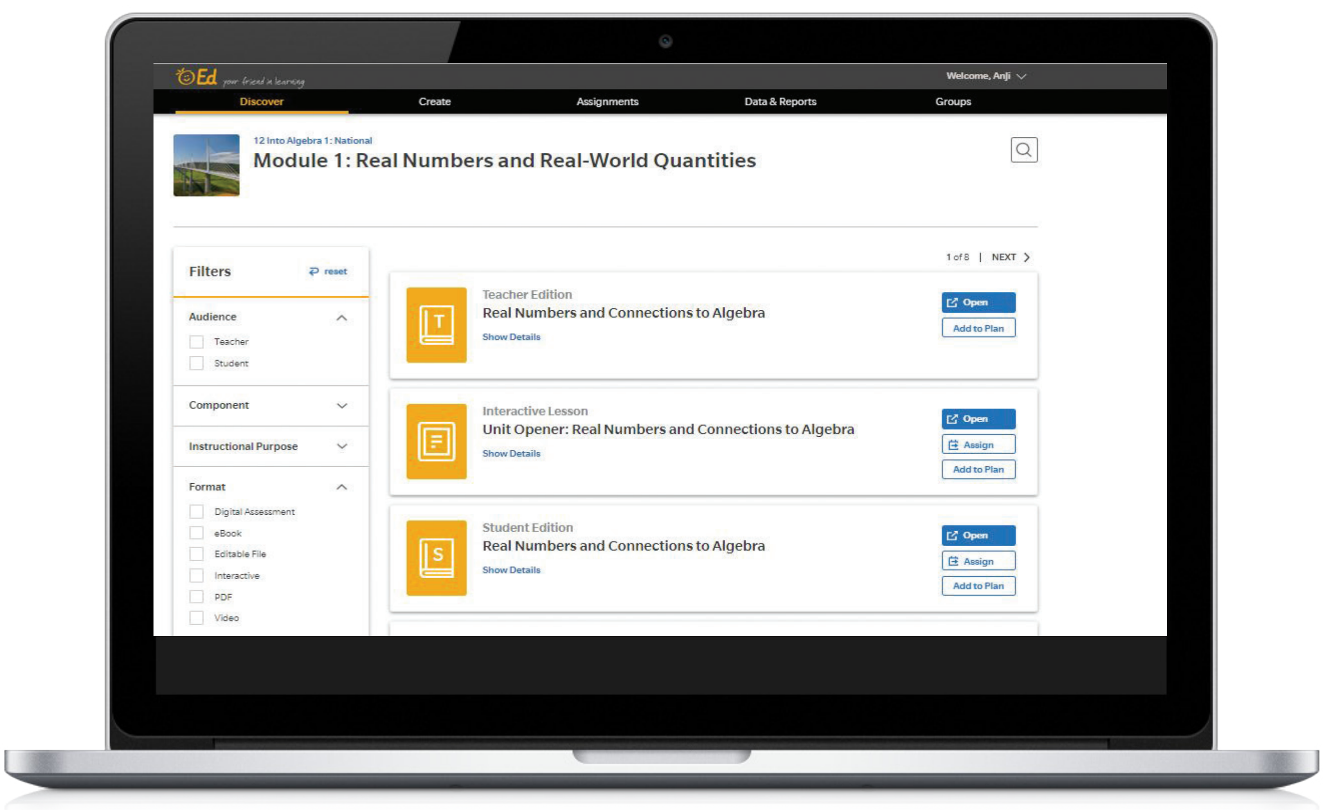
As technology's capabilities have continued to advance, more schools and teachers are turning toward models of instruction that incorporate technology. These models of instruction "entail selecting, designing, and implementing instructional models that leverage technology and best teaching practices effectively" (Digital Convergence, 2018, p. 8). The Modern Curriculum requires "redesigning existing curriculum to incorporate digital content within the instructional framework" (Digital Convergence, p. 9) and requires that teachers have "education on how to use the chosen technology, and in the context of the instructional model and modern curriculum" (Digital Convergence, p. 11). "**Technological tools and instructional strategies offer teachers the ability to transform their teaching, providing students with a plethora of benefits**" (Delgado, Wardlow, McKnight, & O'Malley, 2015, p. 410). Houghton Mifflin Harcourt's *Into AGA* provides digital learning as part of its model and curriculum to support the modern classroom.

In the modern world, it is important to consider the status quo of today's students. As noted by Weston Kieschnick (2017), blending new technologies into instruction is necessary. Effective instruction is that which is fused with useful and purposeful technology and teacher knowledge and wisdom. This solution does not venture to use technology because technology is here and now; rather, this solution encourages teachers to create a purposeful teaching plan with outcomes while integrating technology strategically (Kieschnick, 2017).

HOW *INTO AGA* DELIVERS

Into AGA focuses on innovative technology to personalize instruction for all students. While a wide variety of materials is available online, some key features include:

- **Interactive Student Edition**—Students can view all lesson content and submit their work as assigned by the teacher.
- **Online High-Stakes Assessment Test Prep**—Here students can practice in a testing environment similar to high-stakes assessments.
- **Check for Understanding**—Online item types include traditional multiple-choice as well as technology-enhanced item types similar to those students will see on actual high-stakes assessments.
- **Tools for Home**—Materials support families as they learn about students' math lessons.
- **Interactive Glossary**—Students can record their own definitions of mathematical vocabulary to increase their own understanding.
- ***Math on the Spot***—Students can work on a problem and then review their work at home by watching a video.



BLENDED PROFESSIONAL LEARNING AND SERVICES

Into AGA features effective approaches to professional learning that support teachers in becoming developers of high-impact educational experiences for their students. Comprehensive blended professional learning solutions are data- and evidence-driven, mapped to instructional goals, and centered on students—and they build educators' collective capacity. With a focus on teacher agency, HMH professional learning is purposeful, job-embedded, and ongoing.

BLENDED PROFESSIONAL LEARNING AND SERVICES

RELEVANT INSTRUCTIONAL STRATEGIES AND PRACTICES FOR EVERYDAY TEACHING

Teachers' professional learning should be high quality, ongoing, and accessible to help all students develop mathematical proficiency. Research on teacher learning shows that one core feature of effective professional learning is that it must be ongoing and based on a clear framework. Student learning goals should be clear; teachers should observe what is happening in the classroom, assess how teaching has impacted students, and then modify and improve teaching (Hiebert et al., 2007). Effective professional learning is embedded and ongoing as part of a wider reform effort, rather than as an isolated activity or initiative (Darling-Hammond et al., 2009; Garet et al., 1999). "The duration of professional learning must be significant and ongoing to allow time for teachers to learn a new strategy and grapple with the implementation problem" (Gulamhussein, 2013, p. 3).

Teachers' professional knowledge and capacities develop throughout their careers as they interact with more students, participate in professional learning opportunities, and make use of research-based, educative print and online resources. One way of thinking about this growth is movement from being a novice teacher toward being one who demonstrates mastery (Snow, Griffin, & Burns, 2005). Novices depend almost entirely on declarative knowledge—what they learned in their teacher education program. The process of working toward professional mastery increases stores of what have been called "expert/adaptive" knowledge and "reflective" knowledge. Master teachers have the procedural knowledge—strategies and practices—to deal successfully with a full array of instructional challenges and to then evaluate, analyze, and reflect upon their effectiveness (Snow et al., 2005).

In addition to teachers' own tendencies to evaluate and analyze their practice, many external factors and experiences contribute to their development as professionals. Feedback from principals, colleagues, coaches, parents, and students contribute significantly to an individual teacher's professional growth (Hattie & Timperley, 2007).

As with student learning, teacher learning is part of a long and complex process. In fact, as noted in an article on teacher learning, "Some schools have begun to create new models of induction . . . and ongoing professional learning for teachers and principals" (Garet et al., 1999, p. 921). Providing teachers the opportunity to learn—sustained over time—allows for in-depth discussion and study, as well as the chance to try out new strategies and approaches. In their research, (Garet et al., 1999) looked at the impact of duration on teacher learning and found that it has a substantial positive impact on opportunities for active learning and has a moderately positive influence on content knowledge.

Research indicates that an approach consisting of a single-session workshop independent from job-embedded learning will likely have minimal impact (Ball & Cohen, 1999; Gulamhussein, 2013). In a meta-analysis of research on teacher training, Joyce and Showers (2002) found that when professional learning consisted only of theory and discussion of a targeted practice—such as through a workshop session—gains in knowledge and ability to demonstrate the new skills were modest in the transfer to actual classroom situations. However, demonstration, practice, and feedback—such as through follow-up and coaching—combined with theory and discussion yielded more substantial gains. Joyce and Showers (2002) also emphasized that on average, teachers may make 20 or more attempts to implement a new instructional practice before it becomes effective, and this number is likely higher when the skill is exceptionally complex. This means that teachers must see enough value in the content of their professional learning sessions to put them to use in their classrooms and work toward mastery; this process is the same one students use when learning new, challenging strategies, skills, and concepts. Fortunately, the transfer rate of learning for teachers is much higher when instruction and practice are coupled with coaching. In *Visible Learning*, John Hattie (2009) summarizes and explains the findings of his metastudy and notes that microteaching has a strong influence on student achievement. Hattie (2009) describes microteaching as a practice that typically involves teachers conducting short, or micro, lessons to a small group of students and then engaging in a discussion about the lessons, with the goal of improving teaching methods.

HOW *INTO AGA* DELIVERS

The resources provided throughout *Into AGA* support teachers as they guide all students' learning in mathematics. The Teacher's Editions offer many ongoing strategies that teachers can incorporate into instruction on the spot and provide opportunities for professional learning.

Each lesson provides teachers these techniques:

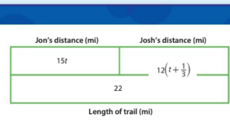
- Lesson Focus and Coherence
- Mathematical Progressions
- Unpacking the Standards
- Warm-Up Options
- Activate Prior Knowledge
- Sharpen Skills
- Spark Knowledge
- Learn Together
- Homework and Test Prep
- Plans for Differentiation
- Small-Group Options
- Math Center Options
- Teaching for Depth
- Language Routines
- Learning Mindsets
- Making Connections
- Professional Learning Cards

PROFESSIONAL LEARNING

Visualizing the Math

Students may find it helpful to draw a bar model to represent real-world situations, such as the problem in Task 5 on page 41. The bar model at the right shows that the sum of Jon's distance and Josh's distance equals the length of the trail.

So an equation that models the problem is $15t + 12\left(t + \frac{1}{3}\right) = 22$.



In addition, there are specific **Professional Learning** reminders embedded in the Teacher's Editions. Teachers gain a wealth of support by having professional learning opportunities built into the solution, which can be accessed at any time. Then teachers are empowered to decide which approaches or resources to use in particular contexts.

Another approach that *Into AGA* offers is providing teachers with **sample scenarios they can model** while working with students. These prompts help teachers look at student work and support students as they decide what action to take.

If students . . . use a verbal description to represent the problem, they understand how the quantities in the problem are related but may not know how to model the problem algebraically.

Activate Prior Knowledge . . . by having students write algebraic expressions for the total cost of the phone with tax and the rebate. **Ask:**

- Q How can you write expressions with a variable for the total cost of the phone with tax at each store?
- Q How can you represent the rebate that will be given?

If students . . . use an expression to model the problem, they are employing an efficient method and demonstrating an exemplary understanding of writing expressions from Grades 6 and 7.

Have these students . . . explain how they determined their expression. **Ask:**

- Q Why did you multiply the cost of the phone by 1.05, the decimal form of 105%?
- Q Why did you subtract the rebate?

COMMON ERROR: Uses Incorrect Expression

The price of the phone is unknown. Let it be p.

Store A:
$1.05(p - 50)$

Store B:
$1.05p - 75$

Similarly, embedded prompts help teachers assess **Common Errors** students might make when working to solve problems.

JOB-EMBEDDED COACHING AND LESSON MODELING

As research has shown for years, traditional forms of professional learning, like one-day workshops, are not effective, usually resulting in minimal impact (Gulamhussein, 2013). However, with continued interest in improving teaching and learning, there is strong awareness surrounding teachers' professional learning. Hattie's work in *Visible Learning* (2009) revealed a core set of instructional methods that have a high impact on student achievement. It is important that teachers are clear about what they want their students to learn; adopt evidence-based teaching strategies; monitor their impact on students' learning and adjust their approaches accordingly; and actively seek to improve their own teaching. The Every Child Succeeds Act (ESSA) redefines professional development as follows: "The term 'professional development' means activities that . . . are sustained (not stand-alone, 1-day, or short-term workshops), intensive, collaborative, job-embedded, data-driven, and classroom focused." (U.S. Department of Education, 2015).

"Educators have known for decades that modeling is an important component of learning, and numerous research studies have demonstrated the power of modeling" (Knight et al., 2015, p. 110). Effective modeling of targeted instructional practices is purposeful and deliberate and is based on research (Taylor & Chanter, 2016). Gulamhussein (2013) reports that "Modeling has been shown to be particularly successful in helping teachers understand and apply a concept and remain open to adopting it" (p.17).

According to a large-scale survey commissioned by the Bill & Melinda Gates Foundation (2014), teachers seek more opportunities to be coached in learning new practices and instructional techniques, believing these professional learning efforts are more valuable. And what does Knight's research reveal about how teachers respond to modeling efforts? "At the Center for Research on Learning, we have completed two studies to capture teachers' perceptions of the value of model lessons. The results of both studies strongly support . . . [that] modeling is where the rubber meets the road" (Knight, 2009, p. 116).

The most effective professional learning affirms what the Center for Public Education refers to as the "dual roles teachers play": teachers as technicians and teachers as intellectuals (Gulamhussein, 2013, p. 20). Teachers who are strong technically can draw on reserves of procedural knowledge to tailor instruction to their students' needs. As intellectuals, teachers are empowered to reflect on theory, research, and their practice to innovate and implement new instructional strategies and approaches. This process of reflection can lead to teachers turning to their colleagues for advice and clarification—a process sometimes called "collective sense-making" (Coburn, 2005). Research shows that collective sense-making, often in the form of professional learning communities, can be a powerful motivator for school improvement.

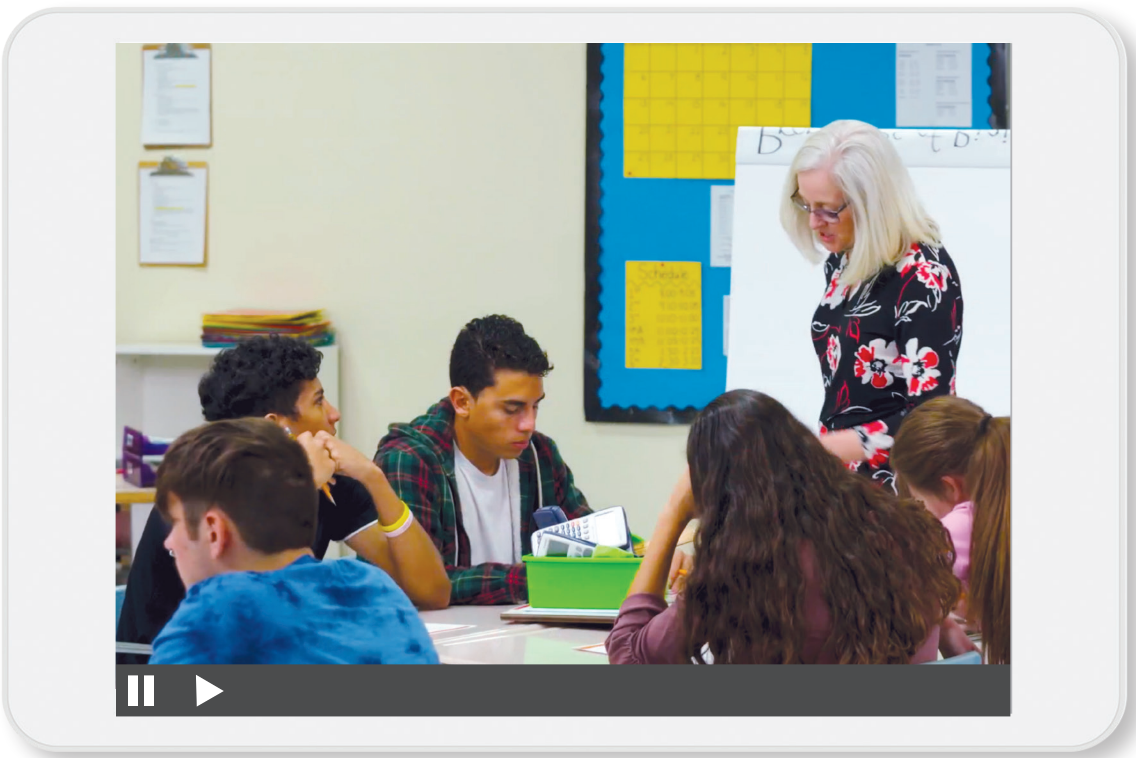
Teachers who seek to improve their practice and student outcomes can also turn to print, online, and in-person resources to help them continue successfully on their path toward professional mastery; this process represents **blended learning**, which has the advantage of allowing teachers to control the place, pace, and path of learning. Individually and collaboratively, they engage in a process sometimes called "self-coaching that addresses the common question: 'The professional development is over, so now what?'" (Wood, Kissel, & Haag, 2014). There are five steps to self-coaching, and they represent high-quality teaching. They include:

1. **Collecting** data to help answer one's questions about instructional improvement. Formative and benchmark data are important, but so, too, is information about students' interests, styles of learning, and work habits.
2. **Reflecting** on the data as a whole and on the data that result from looking back on each days and each week's instruction
3. **Acting on** the reflections, trying things out, and, as appropriate, sharing the results of teachers' actions in a collaborative and mutually supportive group
4. **Evaluating** one's practice, especially through video self-reflection; for example, asking questions about effectiveness of instruction and students' receptivity to the instruction
5. **Extending** one's actions, for example, trying out a successful approach to teaching students to understand complex narrative texts to instruction on reading, social studies, or science textbooks or other informational texts

HOW *INTO AGA* DELIVERS

Throughout *Into AGA* there are opportunities for teachers to collaborate and talk about the many strategies and practices they are learning. However, one key feature that this solution offers is the chance to watch **videos of actual practitioners modeling** these very strategies and practices. Teachers are guided through learning by real-world situations.

Getting Started modules are provided to offer teachers on-demand learning opportunities. The interactive, asynchronous modules focus on key topics to help teachers successfully get started with *Into AGA*.



ONGOING PROFESSIONAL LEARNING

A critical component to teachers' professional services and learning is that teachers have **on-demand access**. Early research has shown that online, sustained professional learning can have a positive impact on teaching and learning (Bahr, Shaha, Farnsworth, Lewis, & Benson, 2004; Benson, Farnsworth, Bahr, Lewis, & Shaha, 2004; Magidin, Masters, O'Dwyer, Dash, & Russell, 2012; Rienties, Brouwer, & Lygo-Baker, 2013; Cho & Rathbun, 2013). The positive impact continues as learners become decision makers about their path, place, and pace. Providing teachers with professional learning opportunities that are readily and easily available allows for flexibility in time and need—again, supporting learners to control path, place, and pace. Teachers can access the resources they need based on their own schedules, and they can determine which resources best suit their needs.

A study of the effects of on-demand, online professional learning conducted by Shaha and Ellsworth (2013) found that higher levels of teacher engagement (e.g., quantity and quality of utilization and participation) correlates with higher student achievement and successes for educators and schools (e.g. teacher retention, student discipline.) [C]onclusions were that higher levels of teacher utilization, engagement, and active use are correlated with higher student achievement and successes for educators and schools" (Shaha & Ellsworth, p. 19).

One specific type of online, on-demand resource that has shown benefits is the use of videos or clips for professional learning. Viewing video clips allows teachers the opportunity to reflect on instructional practices and content (Marzano, Pickering, & Pollock, 2012). As noted by Hiebert et al. (2003), using video clips or recordings can also reinforce the message that teaching mathematics is not just an isolated practice—reflecting on instruction is an opportunity to improve professionally. Further, a study conducted by the Harvard University Center for Education Policy (2012) revealed that the use of video technology can help teachers engage in the process of opening instruction to observation and feedback to improve instruction.

Researchers who study professional learning that supports teachers in effective improvement of practice remind professional learning developers and providers that teachers' active involvement may make them feel vulnerable because they are being asked to take the stance of "learner." As Bryk, Gomez, Grunnow, and LeMahieu (2015) noted in a study of reform efforts that included professional learning, positive changes happen in the presence of teachers' "good will and engagement," which is often rooted in teachers having choice and autonomy in their own learning. These qualities are essential whether teachers meet for large-group professional learning, attend professional learning communities within their schools, or work on their own to find experts to guide them through self-study with print or online resources.

HOW *INTO AGA* DELIVERS

The professional learning components included throughout *Into AGA* provide continued, coherent support for teachers to improve student learning. Teachers are provided with the following:

- Differentiation Guidance
- Leveled Questions Guidance
- Modeling Videos
- Professional Learning Cards
- Professional Learning Modules
- Student Support Suggestions
- Teacher-to-Teacher Tips

SUMMARY

In this paper, we have demonstrated how *HMH Into AGA* © 2020 aligns with research-based principles and practices for high-quality, highly effective mathematics instruction. With its flexible design, including expanded access to rich and varied digital resources, support of productive perseverance and a growth mindset, and engaging and rigorous texts throughout, *Into AGA* provides a cohesive, innovative solution that builds intellectual stamina and tenacity while developing mathematical thinkers, problem solvers, and communicators.

Into AGA features student-centered learning that encompasses and integrates procedural fluency and conceptual understanding, tasks that require high cognitive demand, routines for reasoning, mathematical modeling tasks, and more. The solution is also data driven, providing a comprehensive, balanced assessment system to ensure teachers help students meet targeted learning goals. Finally, the solution is supported by ongoing professional learning for teachers, including modeling and coaching to maximize educator agency and accommodate individual students.

Into AGA addresses the needs of today's classrooms and the requirements of tomorrow's world to better prepare students for college, career, and citizenship.

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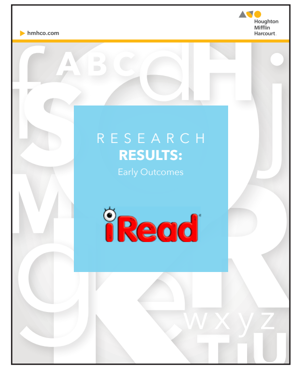
HMH Research Publications

Research Into Practice Into Results

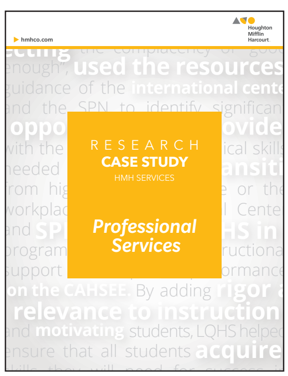
Research Foundations

Research Foundations papers, which include the Evidence and Efficacy papers, provide an in-depth account of the theoretical underpinnings, evidence base, and expert opinions that guide the design and development of new and revised programs. Research Foundations map known research and design principles to practical applications of the program.



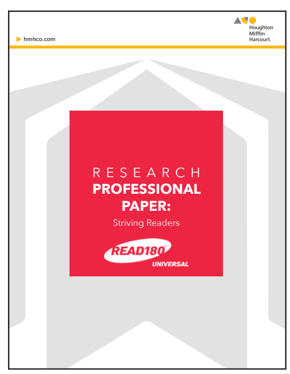
Research Results including Efficacy Compendiums

Research Results papers document the efficacy of a program in terms of Gold level studies (strong evidence), Silver level studies (moderate evidence), and Bronze level studies (promising evidence). At HMH®, program efficacy is monitored closely and continuously in a variety of settings, including varying geographical locations, implementation models, and student populations.



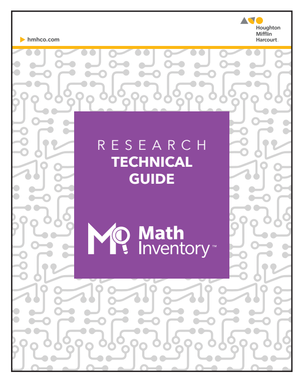
Research Case Studies

Research Case Study papers showcase research that is primarily qualitative and/or anecdotal. Research Case Study papers may profile a particular educator, student, implementation, or special population of students. Research Case Study papers strive to provide more context for understanding programs in practice.



Research Professional Papers

Research Professional papers are typically authored by an expert in the field and highlight an important theoretical construct, practical application, program component, or other topic related to learning in the context of HMH programs.



Research on Assessments

Research Assessments such as the Technical Guide accompany the release of a stand-alone assessment to demonstrate its reliability and validity. Technical Guides and supporting papers are periodically updated as additional reliability and validity evidence is collected in support of an assessment's use and functionality.

HMH Into AGA®

RESEARCH FOUNDATIONS PAPER

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