

# TEACHER GUIDE

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# HMH Guide to **Success** **in Math** for the **SAT**®

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**Sampler**

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# TEACHER GUIDE

# HMH Guide to **Success** **in Math** for the **SAT**<sup>®</sup>

# Contents

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The math section of the SAT<sup>®</sup> test assesses a variety of skills that are organized into four domains. Each domain is divided into several content dimensions, and each content dimension has a number of skills associated with it. The purpose of this publication is to provide practice on these skills using the two question types on the SAT<sup>®</sup> test: multiple choice and grid-in. For each skill, there is a two-page practice test that includes sample questions with worked-out solutions as well as practice questions. An answer sheet for each practice test is provided for students to record their answers. An answer key page, providing answers to the questions on the practice test, follows each answer sheet.

The skills listed in the Table of Contents below and on the practice tests are paraphrases of the wording of the skills identified in the College Board publication *Test Specifications for the Redesigned SAT<sup>®</sup>* (pages 137–145).

## **Heart of Algebra**

### Linear equations in one variable

Use linear equations in one variable to solve problems in a variety of contexts.	1
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# Gridding In Answers

You should record your answers on the answer sheet provided for each practice test. For the grid-in portion of a test, each numerical answer needs to be entered in a grid. Though not required, writing your answer in the boxes provided at the top of the grid is recommended. Each grid includes four boxes, one above each of the four columns of circles. Write just one character in each box. Notice the characters (fraction bar, decimal point, digits) shown along the left side of the grid. Below each box, you should fill in the circle that is to the right of the character you have written in that box. When fewer than four boxes are needed, be sure to leave the unused columns blank.

Things to remember when gridding in numerical answers:

- 1) Credit for a question is given only if the correct circles have been filled in.
- 2) Mark only one circle in each column, completely filling that circle.
- 3) The grid does not support negative numbers, so no question has a negative answer.
- 4) For answers with fewer than four characters, the answer can start in any column. If your answer is 3, use any of the four columns; if your answer is  $\frac{2}{7}$ , use either the first three columns or the last three columns to grid the answer. [See Figures 1 & 2.]
- 5) A mixed number result must be gridded as an improper fraction or as its decimal equivalent. A result of  $2\frac{4}{5}$  should be gridded as either  $\frac{14}{5}$  or 2.8. [See Figure 3.]
- 6) Decimal numbers less than 1 must be gridded without a leading zero. If your result is 0.5, you must grid it as just .5 (without a 0 in the ones place). [See Figure 4.]
- 7) If your result is a decimal with more than 3 digits (and no rounding instructions were given), then fill the grid with four characters by either rounding the final digit or truncating the decimal at this digit. Both answers will be accepted. [See Figure 5; the answer .167 would also be accepted.]

**Figure 1**

3			
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0			
1			
2			
3	●		
4			
5			
6			
7			
8			
9			

**Figure 2**

	2	/	7
/			
.			
0			
1			
2	●		
3			
4			
5			
6			
7			●
8			
9			

**Figure 3**

1	4	/	5
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1	●		
2			
3			
4		●	
5			●
6			
7			
8			
9			

**Figure 4**

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**Figure 5**

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3			
4			
5			
6		●	●
7			
8			
9			

# Interpret a linear equation in one variable in context, and define the conditions affecting the number of solutions.

## SAMPLE MULTIPLE CHOICE

$$-15 + 9(10 - 2x) = -3(6x) + \underline{\hspace{2cm}}$$

Fill in the missing term so that the equation has infinitely many solutions.

- A) 90
- B) 75
- C)  $9x$
- D)  $-18x$

For there to be infinitely many solutions, the two sides of the equation must be identical.

Simplify both sides of the equation.

$$-15 + 9(10 - 2x) = -3(6x) + \underline{\hspace{2cm}}$$

$$-15 + 90 - 18x = -18x + \underline{\hspace{2cm}}$$

$$75 - 18x = -18x + \underline{\hspace{2cm}}$$

There is no need to simplify any further; it can be seen that the missing term is 75.

The correct answer is B.     A   B   C   D  
☐   ☒   ☐   ☐

1

$$75 + 3(\underline{\hspace{1cm}} + 18) = 14x + 7(4x + 9)$$

Which term, when inserted at the blank line, would result in an equation that has no solutions?

- A)  $-43$                       C)  $14x$
- B)  $14$                         D)  $42x$

2

$$(x + 4) - 4(-5 - x) = \frac{1}{15}(75x) - 2$$

How many solutions does the equation above have?

- A) Infinitely many solutions
- B) A unique solution:  $x = -4.1$
- C) A unique solution:  $x = 45$
- D) No solutions

3

The expression  $45 + 0.05t$  gives the total charges per month, in dollars, for a cellular plan where  $t$  is the total number of text messages sent during the month. Which of the following could be the total amount for a monthly bill?

- A) \$43                        C) \$60.24
- B) \$52.60                  D) \$77.61

4

$$26.2 = 6.5h$$

The equation above gives the number of hours  $h$  it takes for a runner to finish a 26.2-mile marathon. What is the runner's speed, in miles per hour?

- A) 0.25                      C) 6.5
- B) 4.03                     D) 8.06

5

$$16 - h - 4^2 = -9(h + 4) - 2(-18 - 4h)$$

How many solutions does the equation above have?

- A) No solutions
- B) Infinitely many solutions
- C) A unique solution:  $h = 18$
- D) A unique solution:  $h = -2$

6

To install new recessed lighting in a home, an electrician charges a trip fee plus a set amount per light. She uses the expression  $25 + 275x$  to calculate the cost to install  $x$  new lights. Which of the following could be the total amount she charges a homeowner for a job?

- A) \$3,050                    C) \$1,730
- B) \$2,335                   D) \$275

**SAMPLE GRID-IN**

$$P = 2.4t + 314$$

The population  $P$ , in millions, of the United States is modeled by the equation shown above where  $t$  is the number of years since 2012. In what year does the model indicate that the population of the United States reached 326 million people?

For  $P = 326$  (million), the equation becomes  $326 = 2.4t + 314$ . Solve this equation for  $t$ :

$$326 = 2.4t + 314$$

$$12 = 2.4t$$

$$5 = t$$

Add 5 years to the year 2012:

$$2012 + 5 = 2017$$

So, the model predicts that the U.S. population reached 326 million in 2017.

Grid in 2017.

**Time-Saving Tip:** The coefficient of the variable  $t$  means that every year the population increased by 2.4 million. Recognizing that the increase is  $326 - 314 = 12$  (million), dividing 12 by 2.4 gives  $t = 5$  years:  $2012 + 5$ , or 2017.

	2	0	1	7
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.	○	○	○	○
0	○	●	○	○
1	○	○	●	○
2	●	○	○	○
3	○	○	○	○
4	○	○	○	○
5	○	○	○	○
6	○	○	○	○
7	○	○	○	●
8	○	○	○	○
9	○	○	○	○

**7**

$$11y - 5(4 - y) = 3(y - 17) + \underline{\hspace{2cm}}$$

For the equation above, give the value needed for the equation to have the unique solution  $y = -2$ .

**8**

$$12(8c - 1) + \underline{\hspace{2cm}} = 7(9c) + 11(3c + 2)$$

What constant is needed on the left side for the equation to have infinitely many solutions?

**9**

A chimney-cleaning company charges a trip fee plus an hourly rate for their service. The equation  $30(2x + 1) = 420$  models the number of hours  $x$  spent on a recent job. What is the hourly rate, in dollars, charged by the company?

**10**

$$6(4 - z) + 2(3z + 10) = 7(z + \underline{\hspace{2cm}}) + z$$

For the equation above, give the value needed for the equation to have the unique solution  $z = -5$ .

**11**

$$60x + 738 = 5,238$$

The monthly cost to produce  $x$  chairs at a factory is modeled by the equation above. What are the fixed costs for operating the factory?

**12**

The equation  $84 - 2.8t = 42$  models the amount of time  $t$ , in hours, it takes to pump half of the oil (in millions of gallons) from an oil tanker. What is the rate, in millions of gallons per hour, at which the pump removes the oil?

**13**

$$5(9w - 2) - (15w + 2) = 8$$

What value of  $w$  is a unique solution to this equation?

**14**

$$8v - (-5v - 6) = 9v + 1 + 4v + \underline{\hspace{2cm}}$$

In the equation above, what value added to the right side will create an equation with infinitely many solutions?

# Interpret a linear equation in one variable in context, and define the conditions affecting the number of solutions.

## ANSWER SHEET

1. A B C D  
○ ○ ○ ○

2. A B C D  
○ ○ ○ ○

3. A B C D  
○ ○ ○ ○

4. A B C D  
○ ○ ○ ○

5. A B C D  
○ ○ ○ ○

6. A B C D  
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7 ○ ○ ○ ○ ○  
8 ○ ○ ○ ○ ○  
9 ○ ○ ○ ○ ○

**Interpret a linear equation in one variable in context, and define the conditions affecting the number of solutions.**

**ANSWER KEY**

1. C
2. D
3. B
4. C
5. B
6. A
7. 5
8. 34
9. 60
10. 12
11. 738
12. 2.8
13.  $\frac{2}{3}$  or 0.666 or 0.667
14. 5

## \ NOTES \

[illegible]



## TEACHER GUIDE

# HMH Guide to Success in Math for the SAT®

A two-page practice test for each skill  
in these four domains:

- Heart of Algebra (26 skills)
- Problem Solving and Data Analysis (28 skills)
- Passport to Advanced Math (9 skills)
- Additional Topics in Math (18 skills)

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