Houghton Mifflin Harcourt

GO MATH!

COMMON CORE EDITION © 2012:

A Research-Based Approach

GO MATH!

Contents

Overview

Houghton Mifflin Harcourt's *GO Math!* is a comprehensive Kindergarten through Grade 6 mathematics program written to align with the Common Core and provide students with a solid foundation in mathematics. With its unique write-in student text, the program encourages active learning and enables students to represent, solve, and explain—all in one place.

The purpose of this document is to demonstrate clearly and explicitly the research upon which ***GO Math!*** is based. This research report is organized by the major instructional strands that underpin the program:

- Alignment with the Common Core State Standards;
- Effective approaches to mathematics instruction;
- Data-driven instruction;
- Instruction that meets the needs of all learners; and
- Use of technology to teach mathematics.

Each strand is supported by research in mathematics education, and by research on teaching and learning across the content areas. The content, activities, and strategies presented in ***GO Math!*** align with what we know about teaching for mathematical understanding and align to the Common Core State Standards for Mathematics.

To help readers make the connections between the research strands and the ***GO Math!*** program, the following sections are used within each strand:

- Defining the Strand. This section summarizes the terminology and provides an overview of the research related to the strand.
- Research that Guided the Development of ***GO Math!*** This section identifies subtopics within each strand and provides excerpts from and summaries of relevant research on each subtopic.
- From Research to Practice. This section explains how the research data is exemplified in ***GO Math!***

The combination of the major research recommendations and the related features of ***GO Math!*** will help readers better understand how the program incorporates research into its instructional design.

A list of references is provided at the end of this document.

Introduction to *GO Math!* *Common Core Edition*

We live in a mathematical world. Never before has the workplace demanded such complex levels of mathematical thinking and problem solving (National Council of Teachers of Mathematics, 2009). Clearly, those who understand and can do mathematics will have opportunities that others do not—and building students' early foundational skills is essential. An analysis of the results of the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) led researchers to conclude that "countries that want to improve their mathematics performance should start by building a strong mathematics foundation in the early grades" (American Institutes for Research, 2005, p. v).

The Common Core State Standards at the elementary grade levels were written to provide such a foundation for young students. The standards describe the content and skills needed for young students to "build the foundation to successfully apply more demanding math concepts and procedures, and move into applications" (Common Core State Standards Initiative, 2011).

The Common Core State Standards (CCSS) were systematically developed to:

- Be research-based;
- Focus on the critical skills at each grade level;
- Encourage conceptual mastery of key ideas;
- Develop students' mathematical understanding and procedural skills;
- Prepare students for the demands of the future—in school and work.

The standards detail the knowledge—content and processes—students need at each grade level, but the standards do not describe the instructional approaches needed to meet the standards. Thus, an effective instructional program is needed to bridge between the expectations set out by the standards and the desired student learning and achievement. This alignment between standards, curriculum, instruction, and assessments is critical. Researchers looking at effective educational practices identified nine characteristics of high-performing schools, and reported that several of these relate to standards and standards alignment. High-performing schools have a clear, shared focus; high standards and expectations for all students; and curriculum, instruction, and assessments aligned to the standards (Shannon & Bylsma, 2003).

Houghton Mifflin Harcourt's *GO Math!* was developed with the Common Core State Standards for Mathematics as a foundation, and uses research-tested approaches to address the rigors of the Common Core. Throughout ***GO Math!*** alignment with the Common Core is made explicit. At every grade level, the program is organized around the Critical Areas identified in the Common Core. A special color-coded system in the student and **Teacher Edition**s make each Critical Area easy to locate and use. The Mathematical Practices are completely imbedded in the lessons. Teachers who use ***GO Math!*** can be assured of meeting the expectations of the Common Core.

Beyond this alignment with the content and practices of the Common Core, ***GO Math!*** represents a comprehensive system of mathematics instruction that includes multiple instructional approaches, diagnostic and formative assessments linked to differentiated instructional resources and tired interventions, and technology solutions designed to support and motivate students.

Strand 1: Teaching Mathematics to the Common Core Standards

A standards-based curriculum combined with the creative use of classroom strategies can provide a learning environment that both honors the mathematical strengths of all learners and nurtures students where they are most challenged.

(McREL, 2010, p. 7)

Educational standards help teachers ensure their students have the skills and the knowledge they need to be successful by providing clear goals for student learning.

(Common Core State Standards Initiative [CCSSI], 2011, online)

Defining the Strand

The Common Core Standards for Mathematics were written with the goal of providing greater focus and coherence to Kindergarten through Grade 12 mathematics instruction in the United States. As the writers of the Common Core document point out in their introduction to the Standards for Mathematics, "For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is 'a mile wide and an inch deep.' These Standards are a substantial answer to that challenge" (CCSSI, 2010, p. 3).

In addition, the Standards serve the purpose of helping to ensure equity for all American students. Inconsistent standards, curriculum, and assessments across states have raised equity issues in the past (Reed, 2009) and wide disparities in performance on the National Assessment of Educational Progress (NAEP) (Schneider, 2007).

The Common Core State Standards for Mathematics accomplish the goals of focus, coherence, and equity by citing specific areas of focus for each grade level—the Critical Areas—and by articulating a progression that builds from grade to grade, helping all students at each level develop the building blocks that will serve as foundations for future learning in mathematics.

GO Math! Common Core Edition is a comprehensive Kindergarten through Grade 6 mathematics program designed to support teachers in effectively building students' mathematical knowledge—content and processes—to meet the expectations of the Common Core State Standards. ***GO Math!*** aligns with the Common Core State Standards through its focus on:

- Critical areas
- Content standards
- Mathematical practices

Research that Guided the Development of the *GO Math!* Program

Critical Areas

As stated previously, a primary goal of the Common Core State Standards for Mathematics was to focus mathematics instruction on the most essential skills and practices needed for deep and lasting learning in mathematics. This emphasis on focused learning resulted from the study of various research findings—such as the National Research Council's Early Math Panel report—and on the data on American students' performance in mathematics, particularly as compared to international students on assessments such as the TIMSS and PISA programs.

Some casual observers of these research findings may have concluded that what American mathematics programs need to do is focus on higher-level critical thinking and reasoning. This may in part be true, but fundamentally, research suggests that what is needed is a deeper study of fewer topics to truly build students' conceptual understandings—the kinds of deep understandings that allow for higher-level problem solving. An analysis of TIMSS and PISA results led researchers to conclude that "the evidence does not support proposals to reduce attention to learning computational and simpler mathematical skills in order to focus on strengthening students' ability to handle more complicated mathematics reasoning" (American Institutes for Research, 2005, p. v). Instead, students need to focus each year on developing the skills that will allow them to perform well in low- and high-level problem-solving situations.

Reviews of the mathematics curriculum in top-performing countries find that they "present fewer topics at each grade level but in greater depth" (National Mathematics Advisory Panel, 2008, p. 20). Accordingly, the Common Core State Standards for Mathematics focus on Critical Areas at each grade level to ensure deep and focused learning. The Common Core State Standards, "promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible." (Achieve, 2010, p. 1)

Content Standards

Mathematical learning involves learning content and processes. Mathematical *content* relates to the subject of math—what students know and do—while mathematical *practices* relate to the vehicles for doing math—how students acquire and use knowledge (NCTM, 2000). According to the National Research Council report *Adding It Up: Helping Children Learn Mathematics*, linking content and practice—and reflecting both in the mathematics classroom—is essential to student understanding (National Research Council, 2001). The Common Core State Standards for Mathematics focus on both content and processes through a balanced approach in which "mathematical understanding and procedural skills are equally important" (Common Core State Standards Initiative, 2010, p. 4).

In Grades K through 5, the Standards emphasize a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions, and decimals. Together, this content provides a strong foundation for students to move on to more demanding math concepts, procedures, and applications.

International comparisons have shown that American students do not perform as well as students from other countries on assessments of math achievement (see TIMSS study by Gonzales, Williams, Jocelyn, Roey, Katsberg, & Brenwald, 2008, and PISA study by Baldi, Jin, Skemer, Green, & Herget, 2007). An analysis of the country-by-country results of TIMSS and PISA led researchers to conclude that "the distribution of that [instructional] time across mathematics content areas differs in ways consistent with our findings about relative performance across content areas" (American Institutes for Research [AIR], 2005, p. v). For example, in comparing time spent on specific content areas, researchers found that "the United States devotes about half the time to its study of geometry—its weakest subject—that other countries spend" (AIR, 2005, p. 22). In other words, if teachers want to improve students' performance across mathematical content areas, they would benefit from focusing instruction accordingly. The Common Core offers a thoughtful perspective on the progression across grade levels and the balance of content.

Content in the Common Core State Standards for Mathematics, Grades K to 6

Domain	Grade Level
Counting and Cardinality	K
Operations and Algebraic Thinking	K, 1, 2, 3, 4, 5
Number and Number Operations in Base Ten	K, 1, 2, 3, 4, 5
Measurement and Data	K, 1, 2, 3, 4, 5
Geometry	K, 1, 2, 3, 4, 5, 6
Number and Number Operations—Fractions	3, 4, 5
Ratios and Proportional Relationships	6
The Number System	6
Expressions and Equations	6
Statistics and Probability	6

Mathematical Practices

"The integrated and balanced development of all five strands of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) should guide the teaching and learning of school mathematics. Instruction should not be based on extreme positions that students learn, on one hand, solely by internalizing what a teacher or book says, or, on the other hand, solely by inventing mathematics on their own" (National Research Council, 2001).

Developing children's mathematical ways of thinking is an essential element of effective mathematics instruction. "[C]ompetence in a domain requires knowledge of both concepts and procedures. Developing children's procedural knowledge in a domain is an important avenue for improving children's conceptual knowledge in the domain, just as developing conceptual knowledge is essential for generation and selection of procedures" (Rittle-Johnson, Siegler, & Alibali, 2001, p. 359-360). Research by Franke, Kazemi, and Battey (2007) suggests that students need an environment to develop both concepts and skills in order to become flexible when engaging with mathematical ideas, and to develop as critical thinkers.

In the Common Core State Standards for Mathematics, the Standards for Mathematical Practice, "describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years" (Common Core State Standards Initiative, 2011, p. 8). These Standards are extensions of earlier efforts to define the processes and proficiencies of mathematics, namely the National Council of Teachers of Mathematics (NCTM) process standards—problem solving, reasoning and proof, communication, representation, and connections—and the strands of mathematical proficiency named in *Adding It Up*, a publication of the National Research Council—adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and a productive disposition. Students meet the Standards for Mathematical Practice by demonstrating the ability to:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

From Research to Practice

Critical Areas in GO Math!

Houghton Mifflin Harcourt's *GO Math! Common Core Edition* focuses instructional and learning time on the Critical Areas identified by the Common Core State Standards. Because ***GO Math!*** is organized around the Critical Areas of each grade level of the Common Core State Standards, students and teachers focus on the important content needed to ensure deep understanding and prepare students for the next level.

The program provides multi-day projects for each Critical Area. And, throughout ***GO Math!*** a special color-coded system makes each Critical Area easy to locate and use.

Common Core State Standards / Critical Areas in *GO Math!*

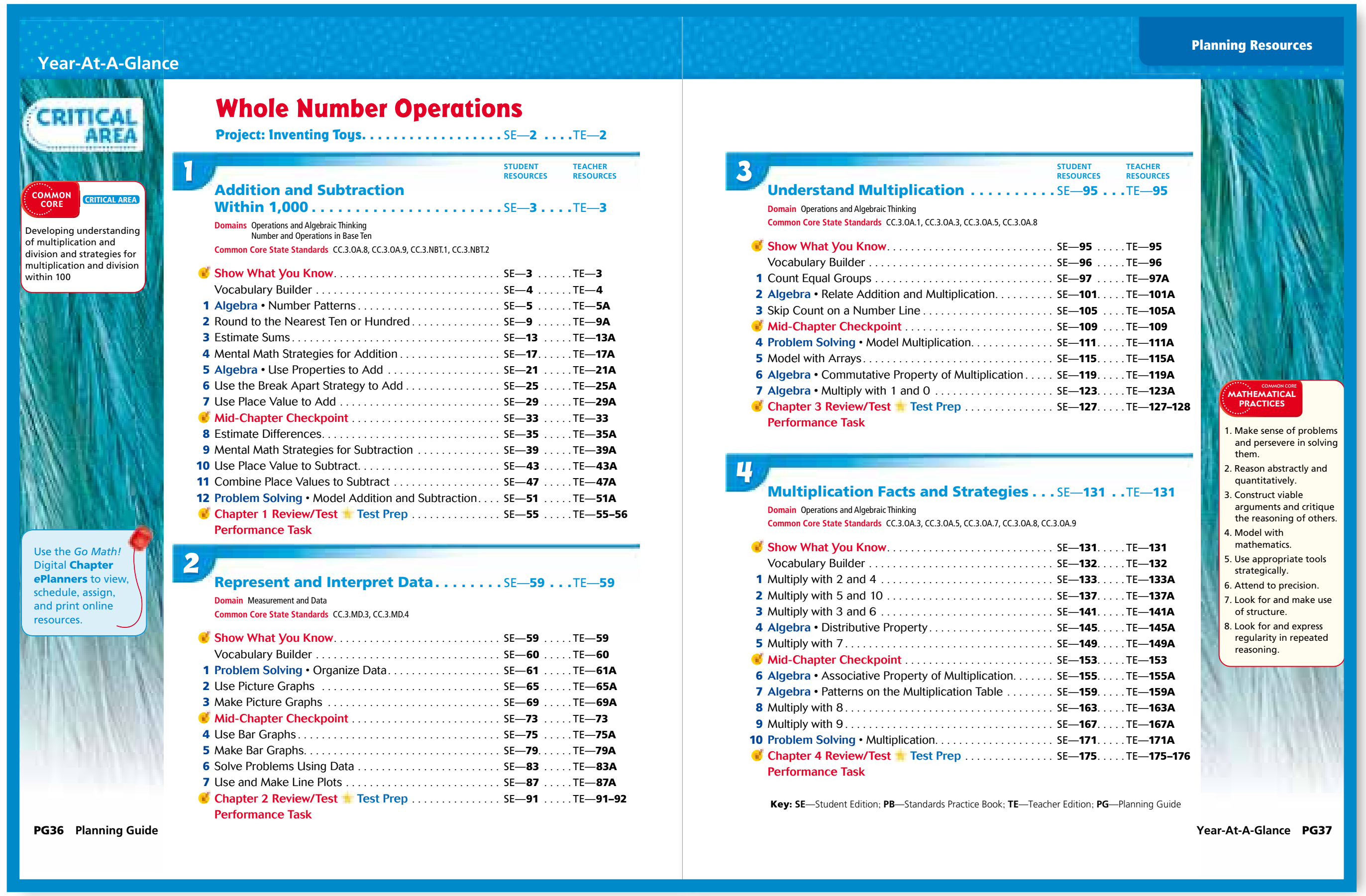
Critical Areas	Critical Area in *GO Math!*
Kindergarten: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space.	**Critical Area** **Student Edition** Critical Area 1: Chapters 1, 2, 3, 4, 5, 6, 7, 8,11 Critical Area 2: Chapters 9, 10
Grade 1: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.	**Critical Area** **Student Edition** Critical Area 1: Chapters 1, 2, 3, 4, 5 Critical Area 2: Chapters 6, 7, 8 Critical Area 3: Chapters 9, 10 Critical Area 4: Chapter 11, 12 **Critical Area: At a Glance** **Teacher Edition** 1A-1B, 229A-229B, 357A-357B, 445A-445B
Grade 2: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.	**Critical Area** **Student Edition** Critical Area 1: Chapters 1, 2 Critical Area 2: Chapters 3, 4, 5, 6 Critical Area 3: Chapters 7, 8, 9, 10 Critical Area 4: Chapter 11 **Critical Area: At a Glance** **Teacher Edition** 1A-1B, 109A-109B, 325A-325B, 497A-497B
Grade 3: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.	**Critical Area** **Student Edition** Critical Area 1: Chapters 1, 2, 3, 4, 5, 6, 7 Critical Area 2: Chapters 8, 9 Critical Area 3: Chapters 10, 11 Critical Area 4: Chapter 12 **Critical Area: At a Glance** **Teacher Edition** 1A-1B, 303A-303B, 385A-385B, 479A-479B
Grade 4: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.	**Critical Area** **Student Edition** Critical Area 1: Chapters 1, 2, 3, 4, 5 Critical Area 2: Chapters 6, 7, 8, 9 Critical Area 3: Chapter 10, 11, 12, 13 **Critical Area: At a Glance** **Teacher Edition** 1A-1B, 223A-223B, 377A-377B
Grade 5: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to the hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.	**Critical Area** **Student Edition** Critical Area 1: Chapters 6, 7, 8 Critical Area 2: Chapters 1, 2, 3, 4, 5 Critical Area 3: Chapter 9, 10, 11 **Critical Area: At a Glance** **Teacher Edition** 1A-1B, 239A-239B, 365A-365B
Grade 6: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) [solving real-world and mathematical problems involving area, surface area, and volume; and developing understanding of statistical thinking.	**Critical Area** **Student Edition** Critical Area 1: Chapters 4, 5, 6 Critical Area 2: Chapters 1, 2, 3 Critical Area 3: Chapters 7, 8, 9 Critical Area 4: Chapter 10, 11, 12, 13 **Critical Area: At a Glance** **Teacher Edition** 1A-1B, 143A-143B, 245A-245B, 367A-367B

Content Standards in GO Math!

The Common Core Standards are a balanced combination of procedure and understanding which is reflected in **Houghton Mifflin Harcourt's *GO Math! Common Core Edition*.**

GO Math! focuses instructional and learning time on the important content identified by the Common Core State Standards. In ***GO Math!*** students learn the content deeply so that they are prepared to move on to the next level of study.

Because the program was built around the Common Core Standards, teachers can easily ensure that they are meeting the content of the Common Core. The program's **Planning Guide** shows the Critical Areas, Domains, and Common Core State Standards within each chapter so that alignment is clear and easy to see.



Year-At-A-Glance — Planning Resources

CRITICAL AREA

Developing understanding of multiplication and division and strategies for multiplication and division within 100

Use the *Go Math!* Digital Chapter ePlanners to view, schedule, assign, and print online resources.

Whole Number Operations

Project: Inventing Toys . . . . . SE—2 . . . . TE—2

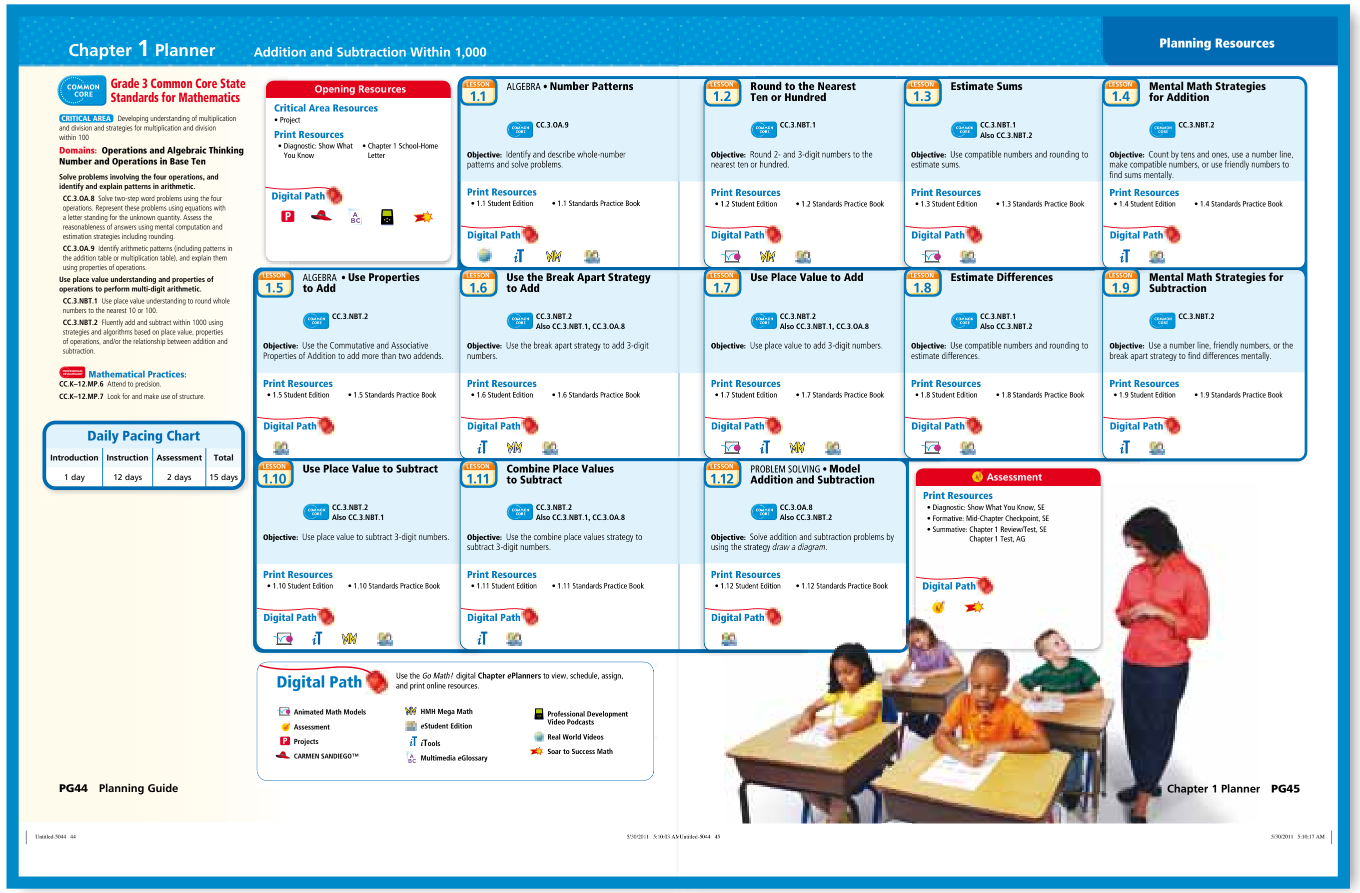
		STUDENT RESOURCES	TEACHER RESOURCES
1	Addition and Subtraction Within 1,000	SE—3	TE—3
	Show What You Know	SE—3	TE—3
	Vocabulary Builder	SE—4	TE—4
1	Algebra • Number Patterns	SE—5	TE—5A
2	Round to the Nearest Ten or Hundred	SE—9	TE—9A
3	Estimate Sums	SE—13	TE—13A
4	Mental Math Strategies for Addition	SE—17	TE—17A
5	Algebra • Use Properties to Add	SE—21	TE—21A
6	Use the Break Apart Strategy to Add	SE—25	TE—25A
7	Use Place Value to Add	SE—29	TE—29A
	Mid-Chapter Checkpoint	SE—33	TE—33
8	Estimate Differences	SE—35	TE—35A
9	Mental Math Strategies for Subtraction	SE—39	TE—39A
10	Use Place Value to Subtract	SE—43	TE—43A
11	Combine Place Values to Subtract	SE—47	TE—47A
12	Problem Solving • Model Addition and Subtraction	SE—51	TE—51A
	Chapter 1 Review/Test Test Prep	SE—55	TE—55–56
	Performance Task		
2	Represent and Interpret Data	SE—59	TE—59
	Show What You Know	SE—59	TE—59
	Vocabulary Builder	SE—60	TE—60
1	Problem Solving • Organize Data	SE—61	TE—61A
2	Use Picture Graphs	SE—65	TE—65A
3	Make Picture Graphs	SE—69	TE—69A
	Mid-Chapter Checkpoint	SE—73	TE—73
4	Use Bar Graphs	SE—75	TE—75A
5	Make Bar Graphs	SE—79	TE—79A
6	Solve Problems Using Data	SE—83	TE—83A
7	Use and Make Line Plots	SE—87	TE—87A
	Chapter 2 Review/Test Test Prep	SE—91	TE—91–92
	Performance Task		
3	Understand Multiplication	SE—95	TE—95
	Show What You Know	SE—95	TE—95
	Vocabulary Builder	SE—96	TE—96
1	Count Equal Groups	SE—97	TE—97A
2	Algebra • Relate Addition and Multiplication	SE—101	TE—101A
3	Skip Count on a Number Line	SE—105	TE—105A
	Mid-Chapter Checkpoint	SE—109	TE—109
4	Problem Solving • Model Multiplication	SE—111	TE—111A
5	Model with Arrays	SE—115	TE—115A
6	Algebra • Commutative Property of Multiplication	SE—119	TE—119A
7	Algebra • Multiply with 1 and 0	SE—123	TE—123A
	Chapter 3 Review/Test Test Prep	SE—127	TE—127–128
	Performance Task		
4	Multiplication Facts and Strategies	SE—131	TE—131
	Show What You Know	SE—131	TE—131
	Vocabulary Builder	SE—132	TE—132
1	Multiply with 2 and 4	SE—133	TE—133A
2	Multiply with 5 and 10	SE—137	TE—137A
3	Multiply with 3 and 6	SE—141	TE—141A
4	Algebra • Distributive Property	SE—145	TE—145A
5	Multiply with 7	SE—149	TE—149A
	Mid-Chapter Checkpoint	SE—153	TE—153
6	Algebra • Associative Property of Multiplication	SE—155	TE—155A
7	Algebra • Patterns on the Multiplication Table	SE—159	TE—159A
8	Multiply with 8	SE—163	TE—163A
9	Multiply with 9	SE—167	TE—167A
10	Problem Solving • Multiplication	SE—171	TE—171A
	Chapter 4 Review/Test Test Prep	SE—175	TE—175–176
	Performance Task		

Mathematical Practices:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Key: SE—Student Edition; PB—Standards Practice Book; TE—Teacher Edition; PG—Planning Guide

PG36 Planning Guide — Year-At-A-Glance PG37

The **Chapter Planners** go into more detail—showing the full text of the relevant Common Core State Standards for Mathematics, and alignment between lessons and specific Standards.



In addition, the **Planning Guide** provides correlations for each of the grade-level Standards with specific **Student Edition** and **Teacher Edition** pages. For these correlations, go to the **Planning Guide** and see:

- Kindergarten, page PG127 through PG131
- Grade 1, page PG129 through PG133
- Grade 2, page PG127 through PG131
- Grade 3, page PG129 through PG133
- Grade 4, page PG131 through PG 135
- Grade 5, page PG127 through PG131
- Grade 6, page PG132 through PG137

The program's **Common Core Standards Practice Books** provide students with additional, daily practice to ensure that they achieve fluency, speed, and confidence with the grade-level Common Core Standards. With the comprehensive ***GO Math!*** program, students learn content and concepts deeply—and retain that information as they move ahead. These **Practice Books** offer:

- A full page of practice for each lesson
- A full page of spiral review every day
- Special Getting Ready lessons that reinforce critical prerequisites for the following grade level
- Multi-day projects for each Critical Area
- The ***GO Math!*** assessment system allows teachers to track their students' progress towards meeting the Common Core.

Assessing students against the Common Core State Standards for Mathematics in *GO Math!*		
Beginning of the Year	**During the Year**	**End of the Year**
The Beginning-of-the-Year Test determines how many of this year's Common Core State Standards students already understand. Teachers can adjust lesson pacing accordingly for skills that need light coverage and to allow more time for skills students find challenging.	Chapter Tests, Performance Assessments, and the Middle-of-the-Year Test monitor students' progress throughout the year. Teachers can plan time to reinforce skills that students have not mastered.	The End-of-Year Test assesses students' mastery of this year's Common Core State Standards. Teachers can reinforce skills that students find challenging in order to provide the greatest possible success.

And, the program's **Getting Ready for Grade**… lessons and resources ensure that students are ready to progress to the next grade level.

Mathematical Practices in GO Math!

The ***GO Math!*** program provides balanced instruction on mathematical content and practices. In ***GO Math!*** instructional time is devoted to developing both students content skills as well as their mathematical practices.

GO Math! supports the Standards for Mathematical Practice through several specific features including:

- Lessons focused on depth of content knowledge;
- **Unlock the Problems** sections to begin lessons;
- **Math Talk** questions prompting students to use varied strategies and to explain their reasoning;
- Support for manipulative use and drawings directly on the student pages;
- Prompts that lead students to write their own problems or to determine if the reasoning of others is accurate, and
- Real-world problems that encourage students to develop productive dispositions.

In addition, the ***GO Math!* Planning Guide** provides teachers with professional development on the Standards for Mathematical Practice to help them understand the intent of the Standards—and how they might look in an elementary school classroom. In addition, suggestions are provided for "Supporting Mathematical Practices Through Questioning" (See pages PG24 through 27 in the Grade 1 **Planning Guide**, PG26-29 in the Grade 3 **Planning Guide**, or PG26 through 29 in the Grade 6 **Planning Guide** for examples.)

The **Planning Guide** provides correlations for the Standards for Mathematical Practices with specific **Student Edition** and **Teacher Edition** pages. For these correlations, go to the **Planning Guide** and see:

- Kindergarten, page PG126
- Grade 1, page PG128
- Grade 2, page PG126
- Grade 3, page PG128
- Grade 4, page PG130
- Grade 5, page PG126
- Grade 6, page PG132

Common Core State Standards / Standards for Mathematical Practice in *GO Math!*

Standards for Mathematical Practice	Examples from *GO Math!*
1. Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	**Problem Solving Lessons** Grade K, Lesson 1.9, pp. 45-48 Grade 1, Lesson 8.8, pp. 345-348 Grade 2, Lesson 1.7, pp. 37-40 Grade 3, Lesson 7.10, pp. 291-294 Grade 4, Lesson 13.5, pp. 515-518 Grade 5, Lesson 9.6, pp. 391-394 Grade 6, Lesson 6.5, pp. 237-240 **Try Another Problem** Grade K, Lesson 8.4, p. 322 Grade 1, Lesson 6.8, p. 270 Grade 2, Lesson 1.7, p. 38 **About the Math** Grade 3, Lesson 2.1, p. 61A Grade 4, Lesson 2.3, p. 53A Grade 5, Lesson 2.7, p, 87A Grade 6, Lesson 11.7, p. 441A **Mathematical Practices in Your Classroom/Building Mathematical** Practices Grade 3, Lesson 2.6, p. 83A Grade 4, Lesson 11.2, p. 423 Grade 5, Lesson 10.4, 417A Grade 6, Lesson 12.8, p. 483 For additional examples, and explanations for how each meets the Standard, see: Grade 1, Planning Guide, PG28 Grade 3, Planning Guide, PG30 Grade 6, Planning Guide, PG30

Common Core State Standards / Standards for Mathematical Practice in *GO Math!*

Standards for Mathematical Practice	Examples from *GO Math!*
2. Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	**Unlock the Problem** Grade 3, Lesson 11.3, pp. 441-445 Grade 4, Lesson 2.2, p. 49 Grade 5, Lesson 5.8, pp. 231-234 Grade 6, Lesson 8.2, pp. 297-300 **Measurement and Geometry Lessons** Grade K, Lesson 12.6, pp. 513-516 Grade 1, Lesson 12.3, pp. 493-496 Grade 2, Lesson 8.1, pp. 389-392 Grade 3, Lesson 11.6, pp. 453-456 Grade 4, Lesson 12.8, pp. 475-478 Grade 5, Lessons 11.6-11.10, pp. 463-482 Grade 6, Lessons 11.2, 11.5, pp. 419-422, 433-436 **Algebra Lessons** Grade 4, Lesson 7.9, pp. 301-304 Grade 5, Lesson 1.12, pp. 51-54 Grade 6, Lesson 7.3, pp. 257-260 For additional examples, and explanations for how each meets the Standard, see: Grade 1, Planning Guide, PG29 Grade 3, Planning Guide, PG31 Grade 6, Planning Guide, PG31

Common Core State Standards / Standards for Mathematical Practice in *GO Math!*	
Standards for Mathematical Practice	**Examples from *GO Math!***
3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	**Math Talk** Grade 1, Lesson 8.2, p. 321 Grade 2, Lesson 11.2, p. 513 Grade 3, Lesson 4.8, p. 164 Grade 4, Lesson 9.6, p. 368 Grade 5, Lesson 11.9, p. 477 Grade 6, Lesson 12.1, p. 453 **Vocabulary Builder** Grade K, p. 59 Grade 1, p. 455 Grade 2, p. 387 Grade 3, p. 482 Grade 4, p. 186 Grade 5, p. 290 Grade 6, p. 482 **Sense or Nonsense** Grade 3, Lesson 4.7, p. 162 Grade 4, Lesson 4.9, p. 174 Grade 5, Lesson 8.1, p. 342 Grade 6, Lesson 7.7, p. 278 For additional examples, and explanations for how each meets the Standard, see: Grade 1, Planning Guide, PG29 Grade 3, Planning Guide, PG32 Grade 6, Planning Guide, PG32

Common Core State Standards / Standards for Mathematical Practice in *GO Math!*	
Standards for Mathematical Practice	**Examples from *GO Math!***
4. Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	**Unlock the Problem • Real World** Grade K, Lesson 6.3, pp. 233-236 Grade 1, Lesson 10.7, pp. 437-440 Grade 2, Lesson 4.9, p. 205 Grade 3, Lesson 11.5, pp. 449-452 Grade 4, Lesson 2.12, p. 91 Grade 5, Lesson 7.8, p. 321 Grade 6, Lesson 12.4, p. 463 **Investigate Lessons** Grade 3, Lesson 12.9, pp. 517-520 Grade 4, Lesson 1.5, pp. 23-26 Grade 5, Lesson 8.1, pp. 339-342 Grade 6, Lesson 13.3, pp. 499-502 **Project** Grade 3, pp. 386-387 Grade 4, pp. 377-378 Grade 5, pp. 365-366 Grade 6, pp. 143-144 For additional examples, and explanations for how each meets the Standard, see: Grade 1, Planning Guide, PG31 Grade 3, Planning Guide, PG33 Grade 6, Planning Guide, PG33

Common Core State Standards / Standards for Mathematical Practice in *GO Math!*	
Standards for Mathematical Practice	**Examples from *GO Math!***
5. Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	**Hands-On Lessons** Grade K, Lesson 3.1, pp. 89-92 Grade 1, Lesson 9.3, pp. 377-380 Grade 2, Lesson 9.7, pp. 457-460 **Investigate Lessons with manipulatives** Grade 3, Lesson 6.6, pp. 231-234 Grade 4, Lesson 12.6, p. 467 Grade 5, Lesson 5.5, pp. 219-222 Grade 6, Lesson 4.1, p. 147 **Geometry and Measurement Lessons** Grade K, Lesson 11.5, pp. 481-484 Grade 1, Lesson 9.4, pp. 381-384 Grade 2, Lesson 8.8, pp. 417-420 Grade 3, Lesson 10.7, pp. 415-418 Grade 4, Lesson 11.3, pp. 425-428 Grade 5, Lesson 11.6, pp. 463-466 Grade 6, Lesson 10.2, pp. 375-378 ***i*Tools Animated Math Models** **HMH Mega Math** All student lessons For additional examples, and explanations for how each meets the Standard, see: Grade 1, Planning Guide, PG32 Grade 3, Planning Guide, PG34 Grade 6, Planning Guide, PG34

Common Core State Standards / Standards for Mathematical Practice in *GO Math!*	
Standards for Mathematical Practice	**Examples from *GO Math!***
6. Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	**Math Talk** Grade 1, Lesson 11.3, p. 465 Grade 2, Lesson 5.5, p. 245 Grade 3, Lesson 12.5, p. 501 Grade 4, Lesson 5.5, p. 212 Grade 5, Lesson 10.2, p. 410 Grade 6, Lesson 3.2, p. 101 **Measurement Lessons** Grade 2, Lesson 8.6, p. 409-412 **Skill Lessons on equations and comparisons (<, >, and =)** Grade 3, Lesson 9.4, pp. 363-366 Grade 4, Lesson 6.1, p. 228 Grade 5, Lesson 3.3, pp. 113-116 Grade 6, Lesson 8.9, pp. 327-330 For additional examples, and explanations for how each meets the Standard, see: Grade 1, Planning Guide, PG32 Grade 3, Planning Guide, PG34 Grade 6, Planning Guide, PG34

Common Core State Standards / Standards for Mathematical Practice in *GO Math!*	
Standards for Mathematical Practice	**Examples from *GO Math!***
7. Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	**Algebra Lessons** Grade 3, Lesson 4.4, pp. 145-148 Grade 4, Lesson 5.6, pp. 215-218 Grade 5, Lesson 1.3, pp. 13-16 Grade 6, Lesson 7.8, p. 280 **Lessons with patterns** Grade K, Lesson 9.11, pp. 397-400 Grade 1, Lesson 6.1, pp. 241-244 Grade 2, Lesson 1.8, pp. 41-44 **Lessons with patterns and sequencing** Grade 3, Lesson 1.1, pp. 5-8 Grade 4, Lesson 2.3, pp. 54-56 Grade 5, Lesson 9.6, pp. 391-394 Grade 6, Lesson 7.7, p. 275A For additional examples, and explanations for how each meets the Standard, see: Grade 1, Planning Guide, PG33 Grade 3, Planning Guide, PG35 Grade 6, Planning Guide, PG35

Common Core State Standards / Standards for Mathematical Practice in *GO Math!*	
Standards for Mathematical Practice	**Examples from *GO Math!***
8. Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	**Lessons with basic facts** Grade K, Lesson 6.7, pp. 249-252 Grade 1, Lesson 5.2, pp. 189-192 Grade 2, Lesson 3.3, pp. 129-132 **Computation Lessons** Grade 1, Lesson 8.7, pp. 341-344 Grade 2, Lesson 6.7, pp. 305-308 Grade 3, Lesson 6.7, pp. 235-238 Grade 4, Lesson 8.1, pp. 315-318 Grade 5, Lesson 4.1, pp. 161-164 Grade 6, Lesson 2.1, pp. 49-52 **Connect To…** Grade 3, Lesson 11.8, pp. 463-464 Grade 4, Lesson 4.11, p. 180 Grade 5, Lesson 4.7, p. 187 Grade 6, Lesson 2.1, p. 39 **Mathematical Practice in Your Classroom/ Building Mathematical Practices** Grade 3, Lesson 11.6, p. 455 Grade 4, Lesson 3.1, p. 303 Grade 5, Lesson 9.5, p. 389 Grade 6, Lesson 2.1, p. 49A For additional examples, and explanations for how each meets the Standard, see: Grade 1, Planning Guide, PG33 Grade 3, Planning Guide, PG35 Grade 6, Planning Guide, PG35

Strand 2: Effective Instructional Approaches

But what exactly do highly effective teachers do in their classrooms to help students learn at higher levels? Research tells us that one key trait of effective teachers is their use of instructional strategies that work.

(McREL, 2010)

Defining the Strand

Extensive research has demonstrated the benefits of effective instructional strategies on student learning and achievement. Teaching mathematics is not easy, but employing research-based techniques helps teachers to teach—and to ensure that all students learn.

A number of instructional approaches have been shown to be particularly effective in reaching students across content areas and specifically in the mathematics classroom. Encouraging communication is one. Classrooms in which students write about and discuss their mathematical thinking and reasoning foster increased learning and critical thinking. In addition, teachers who involve students in problem solving encourage learning. Using visuals, models, and representations deepens students' understanding of the content, as does making connections between mathematics and other content areas and real-world contexts. Supporting students through providing scaffolds—instructional supports that assist new learners in becoming independent users of new skills and strategies—is another research-supported strategy. Finally, engaging students in learning has also been shown to be essential to motivating students to persist in learning.

GO Math! is a program designed to support teachers in effectively building students' mathematical skills and understandings. In its design, the program incorporates research-based strategies for effective teaching and learning. ***GO Math!*** supports teaching and learning by incorporating the following instructional approaches:

- Writing to learn
- Math talk
- Problem-based instruction
- Using models and representations
- Making connections
- Scaffolding
- Engagement

Research that Guided the Development of the *GO Math!* Program

Writing to Learn

Numerous studies have emphasized the importance of writing in content-area learning and in the mathematics classroom. Bosse and Faulconer (2008) report that writing in the mathematics classroom results in deeper student learning. A review of studies conducted by the National Council of Teachers of Mathematics revealed that "the process of encouraging students to verbalize their thinking—by talking, writing, or drawing the steps they used in solving a problem—was consistently effective… Results of these students were quite impressive, with an average effect size of 0.98…" (Gersten & Clarke, 2007, p. 2). Other researchers have found that students' conceptual understanding and problem-solving skills improve when they are encouraged to write about their mathematical thinking (Burns, 2004; Putnam, 2003; Russek, 1998; Williams, 2003). Most importantly, writing appears to benefit all students, with researchers finding benefits for low-achieving students (Baxter, Woodward, & Olson, 2005) and for high-achieving students (Brandenburg, 2002).

What makes writing to learn so effective? According to Vygotsky (1978), writing gives students the chance to explore their own thinking. When students communicate in written or verbal formats about mathematics, they make new understandings and organize their learning. In this way, "encouraging students to verbalize their current understandings and providing feedback to the student increases learning" (Gersten & Chard, 2001, online). According to Burns (2004), "Writing in math class supports learning because it requires all students to organize, clarify, and reflect on their ideas—all useful processes for making sense of mathematics" (p. 30). In addition, researchers theorize that verbalizing forces students to slow their thinking down so that they are more careful in their processes—and less likely to make careless errors (Gersten & Clarke, 2007). Other researchers have found that writing during math instruction gives students more confidence in their math abilities, creates more positive attitudes toward math, and makes it easier for students to understand complex math concepts (Furner & Duffy, 2002; Taylor & McDonald, 2007). Pugalee (2005) argues that writing builds students' mathematical reasoning and problem-solving skills.

Writing can be incorporated into the mathematics classroom in numerous ways, including free writing; biography; learning logs, blogs, and journals; summaries; word problems; and formal writing (Urquhart, 2009). Students can engage in more structured or more informal journaling or notetaking. In a study with Grade 9 algebra students, Pugalee (2004) found that journal writing positively impacted students' problem solving. Albert and Antos (2000) examined the impact of journal writing, and found that using the journals, "gives students practice in communicating their ideas clearly and allows for each student to make a personal connection that strengthens his or her learning and understanding of mathematical concepts and ideas" (p. 530-531). Notetaking has also been shown to be an effective instructional and learning strategy in the classroom (Marzano, Pickering, & Pollock, 2001).

Math Talk

Communicating mathematically is a frequent and consistent thread throughout research on effective instructional strategies for teaching mathematics—and is one of the strategies highlighted in NCTM's *Principles and Standards for School Mathematics* and a recurrent thread throughout the Common Core Standards for Mathematical Practice. When students discuss math concepts, they have the chance to think through, defend, and support their ideas.

Providing opportunities for students to talk about mathematics and mathematical concepts can enhance their understanding of mathematics. Instructional practices—such as restating, prompting students, and engaging in whole-class discussion, small-group discussion, and paired conversations—have been shown to be effective in improving student understanding (Chapin, O'Connor, & Canavan Anderson, 2003). Lovitt and Curtis (1968) found that encouraging a student to verbalize problems before giving a written response increased the rate of correct answers. Hatano and Inagaki (1991) found that students who discussed and justified their solutions with peers demonstrated greater mathematical understanding than students who did not engage in such discussions.

Talking about math has also been found to benefit students at different levels of learning and in different contexts. In their study, Hufferd-Ackles, Fuson, and Sherin (2004) found a math-talk community to be beneficial with students who were English language learners in an urban setting.

In addition to promoting greater learning, communication in the mathematics classroom can facilitate teachers in assessing students' performance—and students in engaging in self-assessment; "Classroom communication about students' mathematical thinking greatly facilitates both teacher and student assessment of learning" (Donovan & Bransford, 2005, p. 239).

Problem-Based Instruction

Problem-based instruction is an essential characteristic of effective mathematics instruction in the 21st century. Numerous researchers and organizations have pointed to the importance of problem solving. The Partnership for 21st Century Skills highlights problem solving within "Critical Thinking and Problem Solving" in the *Framework for 21st Century Learning* (2009). Problem solving is central to the NCTM *Principles and Standards for School Mathematics* (2009). And, problem solving is a focus of the Common Core Standards for Mathematical Practice, as evidenced by Standard 1: Make sense of problems and persevere in solving them.

Problem-based learning offers a way for teachers to teach students to solve problems—and to learn more about mathematical concepts and skills through solving problems (MacMath, Wallace, & Chi, 2009). Essentially, problem-based learning is a student-centered approach to mathematics instruction—in which teachers guide students through solving problems with real-world contexts (Roh, 2003). Research suggests that problem-based learning is effective as an approach to instruction—as well as an assessment of student understanding at the end of a lesson or unit (MacMath, Wallace, & Chi, 2009). Problem-based learning offers students the opportunity to engage in critical thinking, communicate about math, and develop strategies to solve problems and evaluate solutions (Roh, 2003). Problem solving is closely related to the development of students' metacognitive skills. Students who are successful at mathematical problem solving "are efficient at keeping track of what they know and of how well or poorly their attempt to solve a problem is proceeding. They continuously ask, 'What am I doing?' 'Why am I doing it?' 'How will it help me?'" (Reys, Suydam, Lindquist, & Smith, 1998, p. 27).

Problem-based instruction has been shown to be effective with diverse populations of students—English language learners, below-level learners, advanced learners, girls as well as boys (Boaler, 1998; Brenner et al., 1997; Erickson, 1999; Mevarech & Kramarski, 1997).

Using Models and Representations

Representations and models include a wide variety of images or likenesses that can be used to support understanding in the mathematics classroom. Representations and models might include those that students create themselves to better understand concepts, or those that teachers provide to better illustrate concepts for students. They might include pictures, manipulatives, symbols, or diagrams. Visual representations include graphic organizers—illustrations used organize and highlight content. Technology also provides new means of illustrating concepts with models and representations.

In *Principles and Standards for School Mathematics* (2000), the importance of representations in mathematics instruction is highlighted: "Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one's self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling" (p. 67).

Students' use of mathematical models and representations—images, likenesses, or depictions—can help to make mathematical concepts more concrete for students. Representations help students to understand and communicate mathematical ideas and to model and interpret concepts. Research by Clarke and Clarke (2004) suggests that the effective use of models and representations is an important element in successful mathematics instruction. Marzano included non-linguistic representations as one of the nine most effective instructional strategies teachers can use in the classroom (Marzano, Pickering, & Pollock, 2001). A recent review indicated there was strong evidence supporting the use of visual representations to improve student performance in general mathematics, prealgebra, word problems, and operations (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). Graphic organizers have been shown to be effective in helping students organize and remember content-area information (Horton, Lovitt, & Bergerud, 1990). In mathematics, graphic organizers can offer students guidance and assistance in approaching new problems (Maccini & Gagnon, 2005).

Making Connections

Connections—among mathematical ideas, with other content areas, and in real-world contexts—are an essential part of successful mathematics learning. Making connections between new information and students' existing knowledge—knowledge of other content areas and of the real world—has proved to be more effective than learning facts in isolation (Beane, 1997; Bransford, Brown, & Cocking, 1999; Caine & Caine, 1997; Kovalik, 1994). Students learn best when they can make connections between ideas.

By its nature, mathematics lends itself to interdisciplinary learning; mathematics can be applied to situations in social studies, technology, engineering, science, and real-world contexts, and knowledge of mathematics is essential to study in many disciplines. Connecting mathematics to science, social science, and business topics can increase students' understanding of and ability with mathematics (Russo, Hecht, Burghardt, Hacker, & Saxman, 2011). These interdisciplinary connections build students' knowledge and increase their perceptions of mathematics as useful and interesting—thereby increasing their motivation to learn (Czerniak, Weber, Sandmann, & Ahem, 1999; Reed, 1995). Research shows that making inter-disciplinary connections can increase students' achievement (Russo, Hecht, Burghardt, Hacker, & Saxman, 2011).

Making connections makes learning more relevant to students. Students see the purpose of learning when they can apply it to real-world contexts; "When instruction is anchored in the context of each learner's world, students are more likely to take ownership for…their own learning" (McREL, 2010, p. 7). These connections build students' knowledge and increase their perception of the content as interesting and useful—thereby increasing their motivation to learn (Czerniak, Weber, Sandmann, & Ahem, 1999). In his examination of U.S. and Korean students' performance on the 2003 TIMSS, House (2009) found a strong correlation between connections linking mathematical study with students' daily lives and students' enjoyment of mathematics. In its review of studies on using real-world problems in the mathematics classroom, the National Mathematics Advisory Panel (2008) found that for "certain populations (upper elementary and middle grade students, and remedial ninth-graders) and for specific domains of mathematics (fraction computation, basic equation solving, and function representation), instruction that features the use of 'real-world' contexts has a positive impact on certain types of problem solving" (p. 50). Thus, the use of real-world problems can be recommended for this population of students.

Scaffolding

Learning mathematics is a sequential process, in which students at each stage must build on and expand on their previous understandings. At each level, students learn to think about mathematics in increasingly sophisticated ways. Scaffolding—an instructional technique in which teachers provide support to students as they learn, and then gradually decrease support until students work independently—can support this kind of deeper learning. Vygotsky defined scaffolding as the "role of teachers and others in supporting the learner's development and providing support structures to get to that next stage or level" (Raymond, 2000, p. 176).

According to Rosenshine and Meister (1992), "Although scaffolds (forms of support to help students bridge the gap between their current abilities and intended goals) can be applied to teaching all skills, they are almost indispensable for teaching higher-level cognitive strategies" (p. 26). Scaffolding, though, is not only effective with higher-level learning. Larkin (2001) interviewed and observed teachers who scaffolded instruction. He concluded that "scaffolding principles and techniques can guide teachers to assist students in any grade level to become more independent learners" (p. 34).

Scaffolding can take many forms in the classroom. Scaffolds may include activating prior knowledge, modeling, questions, or using tools. Structuring students' hands-on, active learning is another scaffold. In a series of experiments, Scruggs and colleagues looked at the impact of highly structured inquiry methods and concluded that students who were coached to derive their own explanations and elaborate on their own reasoning recalled information more fully and consistently than did students who were in an explicitly taught, direct instruction group (Scruggs, Mastropieri, & Sullivan, 1994; Sullivan, Mastropieri, & Scruggs, 1995). Kilpatrick and colleagues (2001) suggest other forms of scaffolds: "By offering a subtle hint, posing a similar problem, or asking for ideas from other students, [the teachers] provide some scaffolding…without reducing the complexity of the task at hand or specifying exactly how to proceed" (p. 336). Review can serve as a scaffold; review helps "to refocus [students'] attention and give them further opportunity to develop their own understanding rather than relying on that of the teacher" (Anghileri, 2006, p. 41).

Research has long documented the connection between a student's sense of confidence and self-efficacy for learning and his or her learning and achievement. Students who believe they can learn persist in learning, are engaged in learning, and subsequently learn more than peers who are less confident in their abilities. As a result, building students' confidence in learning is an important element of effective instruction. Scaffolding is one way to accomplish this goal (Baker, Schirner, & Hoffman, 2006). As Hyde (2006) states, "Scaffolding does not necessarily make the problem easier, and the teacher does not do the work for students or show them how to do it. It enables the person to do it" (p. 28). This empowerment gives students confidence in their ability and allows them to take on increasingly more challenging material and assignments as they demonstrate success completing previous tasks. Williams (2008) found that "scaffolding tasks allowed students to work independently at appropriately challenging levels…and develop a sense of self-confidence in their mathematics knowledge and skills" (p. 329).

Engagement

The experience of teachers and the findings of research suggest that when students are engaged they are motivated to persist in learning and are better able to learn in the classroom. Research links engagement with student achievement and development (Finn, 1993; Newmann, 1992). In a study conducted by Park (2005) student engagement had positive effects on student academic growth—even after taking into account variables such as gender, minority status, SES, and interaction effects.

As discussed earlier in this report, using real-world problems and making interdisciplinary connections engages students in the mathematics classroom. Also discussed previously, communicating about mathematics engages students. Research has shown that learners become more engaged in the learning process when they are asked to explain and reflect on their thinking processes (Good & Whang, 1999; Hettich, 1976; Surbeck, 1994). Continually requiring that students explain how they solved problems is another research-based strategy for maintaining student engagement (National Research Council, 2001). Teachers who press students to explain their answers help to keep their students engaged in a dynamic way. Choosing tasks that use students' prior knowledge as a foundation is another way that teachers can keep student engagement high (National Research Council, 2001). Thus, the sequence of instruction, and the reinforcement of prerequisite learning, is important to engagement. Multimedia learning, too, engages students. Students who often used computers in their mathematics classrooms reported greater enjoyment of mathematics than did students who did not frequently use computers for mathematics study (House & Telese, 2011). Reinking (2001) attributes the greater student engagement in multimedia learning environments to the interactive nature of the technology, the availability of scaffolds, the game-like nature of computer-based instructional materials, and the social learning environment that can be created with technological tools.

When students are motivated they are more engaged. They are motivated when they expect that they will be able to perform mathematical tasks successfully. For this reason, teachers can keep students engaged by supporting students' expectations that they can succeed in solving problems (National Research Council, 2001).

From Research to Practice

Writing to Learn in GO Math!

Opportunities that encourage writing to learn—for students to reflect on and refine their mathematical ideas—are incorporated throughout ***GO Math!***

The program is uses a unique **Write-In Student Edition** that provides students with numerous opportunities to write about and reflect on new mathematical concepts and the processes used to solve problems. Throughout, students are asked to write in responses to prompts that ask them to engage in thinking and reflection. Students explain their approaches to problem solving, and describe the steps they take to arrive at solutions. Opportunities to write about mathematics appear in every exercise set. In addition, numerous portfolio activities engage students in writing to learn, such as the **Performance Task** and **Portfolio Suggestions** in the **Teacher Edition.**

In addition, the following examples show some of the write-to-learn activities in ***GO Math!***

Writing to Learn in *GO Math!*

Kindergarten	Grade 1
Math Journal, in every **Teacher Edition** lesson. Some examples are: 16, 44, 116, 171, 196, 244, 420, 516 **Summarize**, in every **Teacher Edition** lesson. Some examples are: 20, 64, 180, 200, 300, 480 **Write Math**, in most **Student Edition** lessons. Some examples are: 16, 100, 176, 236, 300, 388, 448	**Math Journal**, in every **Teacher Edition** lesson. Some examples are: 16, 88, 144, 208, 272, 340, 404, 488 **Summarize**, in every **Teacher Edition** lesson. Some examples are: 20, 56, 188, 204, 496, 524 **Write Math**, in most **Student Edition** lessons. Some examples are: 16, 108, 204, 292, 376, 436, 516
Grade 2	**Grade 3**
Math Journal, in every **Teacher Edition** lesson. Some examples are: 16, 60, 83, 152, 208, 264, 380, 524 **Summarize**, in every **Teacher Edition** lesson. Some examples are: 20, 64, 284, 340, 476, 512 **Write Math**, in most **Student Edition** lessons. Some examples are: 16, 100, 180, 240, 304, 372, 440, 520	**Math Journal**, in every **Teacher Edition** lesson. Some examples are: 8, 64, 136, 212, 310, 392, 486 **Summarize**, in every **Teacher Edition** lesson. Some examples are: 8, 64, 136, 212, 310, 392, 486 **Write Math**, in most **Student Edition** lessons. Some examples are: 7, 38, 104, 260, 286, 508, 519
Grade 4	**Grade 5**
Math Journal, in every **Teacher Edition** lesson. Some examples are: 8, 48, 230, 270, 448, 500 **Summarize**, in every **Teacher Edition** lesson. Some examples are: 8, 52, 230, 270, 448, 500 **Write Math**, in most **Student Edition** lessons. Some examples are: 8, 30, 277, 308, 489, 518	**Math Journal**, in every **Teacher Edition** lesson. Some examples are: 16, 64, 204, 246, 408, 444 **Summarize**, in every **Teacher Edition** lesson. Some examples are: 16, 64, 204, 246, 408, 444 **Write Math**, in most **Student Edition** lessons. Some examples are: 7, 12, 262, 280, 465, 490
Grade 6	
Math Journal, in every **Teacher Edition** lesson. Some examples are: 8, 52, 194, 230, 502, 516 **Summarize**, in every **Teacher Edition** lesson. Some examples are: 12, 52, 194, 230, 458, 502 **Write Math**, in most **Student Edition** lessons. Some examples are: 12, 52, 264, 286, 502, 524	

Math Talk in GO Math!

Students learn by engaging in conversations about mathematics in the ***GO Math!*** program.

The program's special **Write-On/Wipe-Off MathBoards** help students organize their thinking—and support the program's emphasis on Math Talk.

Math Talk in *GO Math!*

Kindergarten	Grade 1
Math Talk in Action, see the **Teacher Edition**: 43, 51, 67, 95, 115, 151, 159, 191, 211, 231, 235, 287, 295, 311, 331, 367, 383, 423, 443, 471, 495, 515	**Math Talk**, in every **Student Edition** lesson. Some examples are: 13, 21, 265, 277, 509, 521 **Math Talk in Action**, see the **Teacher Edition**: 15, 31, 83, 103, 115, 159, 207, 211, 243, 255, 275, 279, 307, 331, 335, 379, 395, 415, 423, 459, 487, 499, 507, 511
Grade 2	**Grade 3**
Math Talk, in every **Student Edition** lesson. Some examples are: 13, 121, 145, 397, 473, 525 **Math Talk in Action**, see the **Teacher Edition**: 23, 35, 39, 43, 71, 99, 147, 159, 179, 203, 211, 247, 259, 295, 319, 343, 367, 391, 395, 455, 475, 511, 515, 547	**Math Talk**, in every **Student Edition** lesson. Some examples are: 5, 31, 261, 283, 495, 518 **Math Talk in Action**, see the **Teacher Edition**: 11, 15, 41, 49, 67, 77, 85, 117, 135, 157, 165, 201, 219, 241, 255, 267, 285, 309, 313, 331, 357, 395, 413, 425, 435, 451, 461, 473, 503, 511
Grade 4	**Grade 5**
Math Talk, in every **Student Edition** lesson. Some examples are: 5, 24, 268, 289, 499, 515 **Math Talk in Action**, see the **Teacher Edition**: 25, 55, 59, 71, 77, 107, 129, 169, 185, 199, 251, 259, 327, 361, 409, 427, 469, 489, 503	**Math Talk**, in every **Student Edition** lesson. Some examples are: 5, 22, 253, 271, 457, 484 **Math Talk in Action**, see the **Teacher Edition**: 7, 15, 37, 53, 63, 85, 89, 115, 133, 149, 167, 193, 203, 229, 275, 283, 309, 319, 331, 345, 415, 419, 451, 455, 477, 489
Grade 6	
Math Talk, in every **Student Edition** lesson. Some examples are: 7, 28, 265, 283, 503, 521 **Math Talk in Action**, see the **Teacher Edition**: 11, 37, 41, 51, 73, 81, 99, 117, 129, 149, 171, 179, 203, 229, 273, 281, 303, 315, 347, 381, 385, 417, 425, 457, 479, 493, 505, 513	

Problem-Based Instruction in GO Math!

In ***GO Math!*** problem-based instruction provides the foundation for learning. The program is designed with lessons that begin with context-based situations and then build to more abstract problems. Throughout ***GO Math!,*** students are asked to think about the steps they need to go through in order to solve problems. Students are prompted to solve problems in different ways to develop a more thorough understanding of mathematical concepts. Students are taught problem-solving strategies.

Various program features provide opportunities for problem-solving applications, including:

- About the Math
- Building Mathematical Practices
- Go Deeper, Mathematical Practices
- H.O.T. (Higher Order Thinking) Problems
- Independent Practice
- Mental Math Problems
- Pose a Problem
- Problem of the Day
- Real-World Problem Solving
- Real World Unlock the Problem
- Sense or Nonsense?
- Try This!
- What's the Error?
- What's the Question?

For specific examples of problem solving in ***GO Math!*** see the following pages.

Problem Solving in *GO Math!*

Kindergarten	Grade 1
H.O.T. (Higher Order Thinking) Problems, in every Teacher Edition lesson. Some examples are: 19, 43, 207, 275, 467, 515 **Problem of the Day**, in every **Teacher Edition** lesson. Some examples are: 61A, 65A, 69A	**H.O.T. (Higher Order Thinking) Problems**, in every **Student Edition** lesson. Some examples are: 15-16, 19, 24, 107-108, 188, 387, 419-420, 523-524 **Problem of the Day**, in every **Teacher Edition** lesson. Some examples are: 13A, 17A, 21A, 41A
Grade 2	**Grade 3**
H.O.T. (Higher Order Thinking) Problems, in every **Student Edition** lesson. Some examples are: 16, 88, 139, 184, 231 **Problem of the Day**, in every **Teacher Edition** lesson. Some examples are: 13A, 17A, 77A, 145A	**H.O.T. (Higher Order Thinking) Problems**, in every **Student Edition** lesson. Some examples are: 7, 31, 260, 285, 508, 520 **Problem of the Day**, in every **Teacher Edition** lesson. Some examples are: 5A, 61A, 133A, 209A, 307A, 389A, 483A
Grade 4	**Grade 5**
H.O.T. (Higher Order Thinking) Problems, in most **Student Edition** lessons. Some examples are: 8, 29, 251, 273, 500, 517 **Problem of the Day**, in every **Teacher Edition** lesson. Some examples are: 5A, 45A, 227A, 267A, 445A, 497A	**H.O.T. (Higher Order Thinking) Problems**, in most **Student Edition** lessons. Some examples are: 7, 8, 23, 262, 284, 470, 490 **Problem of the Day**, in every **Teacher Edition** lesson. Some examples are: 5A, 69A, 201A, 255A, 405A, 441A
Grade 6	
H.O.T. (Higher Order Thinking) Problems in most **Student Edition** lessons. Some examples are: 8, 29, 264, 286, 506, 524 **Problem of the Day**, in every **Teacher Edition** lesson. Some examples are: 5A, 49A, 191A, 227A, 455A, 499A	

Using Models and Representations in GO Math!

The creators of ***GO Math!*** have followed the research on the benefits of learning visually and through models and representations in designing the program. As the Common Core State Standards note, translating ideas visually is critical in mathematics. As a result, the ***GO Math!*** program uses non-linguistic representations to convey mathematical ideas throughout.

Throughout, students are asked to use pictorial representations to solve problems. Students are provided with graphic organizers to solve problems and make sense of new mathematical concepts. The write-in text design allows students to fully utilize the structure and organization of visuals.

The program's **MathBoards**, special write-on, wipe-off boards, help students to organize their thinking with visual models and graphic organizers.

Throughout, students use models, manipulatives, quick pictures, and symbols to build their mathematical understanding. The program's **Grab-and-Go® Classroom Manipulatives Kits** are prepackaged kits in handy zipper bags that allow for easy classroom distribution so that students can be quickly engaged in learning through hands-on, concrete manipulatives.

Using the program's **Digital Path**, students can use the *i*Tools to solve problems with interactive digital manipulatives and model and explore lesson concepts. The **Animated Math Models** model and reinforce mathematical concepts for students.

Animated Math Models in *GO Math!*

Kindergarten	Grade 1
In most Teacher Edition lessons. Some examples are: 21A, 73A, 213A, 225A, 465A, 481A	In most Teacher Edition lessons. Some examples are: 13A, 57A, 257A, 293A, 325A, 501A
Grade 2	**Grade 3**
In most **Teacher Edition** lessons. Some examples are: 17A, 61A, 281A, 337A, 473A, 509A	In most Teacher Edition lessons. Some examples are: 5A, 97A, 123A, 133A, 307A, 483A
Grade 4	**Grade 5**
In most Teacher Edition lessons. Some examples are: 5A, 45A, 227A, 267A, 445A, 511A	In most Teacher Edition lessons. Some examples are: 9A, 83A, 201A, 259A, 409A, 441A
Grade 6	
In most Teacher Edition lessons. Some examples are: 13A, 49A, 187A, 233A, 499A, 517A	

Making Connections in GO Math!

Throughout the ***GO Math!*** program, the relevance of learning and the usefulness of mathematics is highlighted. Students are supported in making connections to other disciplines and to the world around them.

For example, the program's regular feature, the **Secret Millionaires Club** Grades 3–6, makes interdisciplinary connections with economics and builds students' economic literacy. With the **Secret Millionaires Club** Grades 3–6:

- Online lessons are presented to build economic literacy in the classroom.
- Students watch engaging webisodes and complete activities to reinforce educational concepts.
- With the help of Warren Buffet, students learn good financial habits and use mathematics to solve economic problems.
- Online games further reinforce important economic concepts.

But economics is just one content area that benefits from cross-curricular instruction with mathematics. Throughout all grade levels of ***GO Math!*** students make connections with real-world contexts and situations and with other content areas, including reading, science, and social studies.

Real-Life Connections in *GO Math!*

Kindergarten	Grade 1
Listen and Draw, 13, 17, 25, 29, 33, 37, 41, 49, 61, 77, 89, 93, and so on. **Real World Problem Solving**, 16, 24, 32, 52, 64, 68, 80, 92, 96, 100, 108, 112, 116 **Real World Unlock the Problem**, 45, 73, 121, 177, 233, 281, 433, 473, 513	**Real World Videos** **Real World Problem Solving**, in most **Student Edition** lessons. Some examples are: 16, 104, 220, 320, 400, 496 **Real World Unlock the Problem**, in most Student Edition lessons. Some examples are: 25, 65, 141, 173, 185, 269, 437, 469
Grade 2	**Grade 3**
Real World Videos **Real World Problem Solving**, in most **Student Edition** lessons. Some examples are: 20, 128, 196, 268, 348, 412, 476 **Real World Unlock the Problem**, in most **Student Edition** lessons. Some examples are: 37, 97, 205, 301, 405, 545	**Real World Videos** **Real World Problem Solving**, in most **Student Edition** lessons. Some examples are: 12, 38, 260, 286, 508, 520 **Real World Unlock the Problem**, in most **Student Edition** lessons. Some examples are: 9, 29, 261, 283, 463, 513
Grade 4	**Grade 5**
Real World Videos **Real World Problem Solving**, in most **Student Edition** lessons. Some examples are: 8, 30, 270, 292, 490, 514 **Real World Unlock the Problem**, in most **Student Edition** lessons. Some examples are: 9, 26, 271, 289, 497, 515	**Real World Videos** **Real World Problem Solving**, in most **Student Edition** lessons. Some examples are: 12, 23, 254, 284, 444, 482 **Real World Unlock the Problem**, in most **Student Edition** lessons. Some examples are: 9, 21, 250, 277, 441, 483
Grade 6	
Real World Videos **Real World Problem Solving**, in most **Student Edition** lessons. Some examples are: 8, 38, 264, 282, 502, 520 **Real World Unlock the Problem**, in most **Student Edition** lessons. Some examples are: 5, 21, 265, 283, 503, 521	

Cross-Curricular Activities and Connections in *GO Math!*

Kindergarten	Grade 1
Cross-Curricular Center Activities Art, 9F, 85F, 165F, 305F, 353F, 409F, 461D, 489D **Dramatic Play**, 57D **Music**, 57D, 489D **Science**, 9F, 57D, 85F, 165F, 221F, 305F, 409F, 461D, 489D **Social Studies**, 129F, 257F, 305F, 353F, 409F, 461D **Technology**, 129F, 165F, 221F, 257F, 353F	**Connections to Science**, 8A, 364A In the **Teacher Edition**: 19, 39, 59, 63, 99, 107, 127, 167, 175, 191, 215, 223, 247, 267, 271, 295, 323, 339, 371, 391, 419, 435, 475, 495, 515 **Connections to Social Studies**, 236A In the **Teacher Edition**: 19, 39, 59, 63, 99, 107, 127, 167, 175, 191, 215, 223, 247, 267, 271, 295, 323, 339, 371, 391, 419, 435, 475, 495, 515
Grade 2	**Grade 3**
Connections to Science, 116A, 332A In the **Teacher Edition**: 15, 47, 75, 79, 123, 127, 135, 183, 187, 191, 239, 267, 287, 291, 339, 347, 411, 419, 459, 471, 487, 523 **Connections to Social Studies**, 8A, 504A In the **Teacher Edition**: 15, 47, 75, 79, 123, 127, 135, 183, 187, 191, 239, 267, 287, 291, 339, 347, 411, 419, 459, 471, 487, 523	**Connect to Reading**, 42, 90, 152, 246, 376, 470, 504 **Connect to Science**, 166, 404 In the **Teacher Edition**: 37, 45, 63, 71, 99, 139, 143, 169, 211, 223, 227, 245, 263, 275, 289, 317, 335, 343, 353, 399, 443, 493, 497 **Connect to Social Studies**, 298 In the **Teacher Edition**: 37, 45, 63, 71, 99, 139, 143, 169, 211, 223, 227, 245, 263, 275, 289, 317, 335, 343, 353, 399, 443, 493, 497 Economics, see Secret Millionaires Club activities and resources
Grade 4	**Grade 5**
Connect to Reading, 60, 156 **Connect to Science**, 20, 152, 354, 384, 428, 478, 514 In the **Teacher Edition**: 11, 121, 245, 285, 455, 513 **Social Studies**, in the **Teacher Edition**: 11, 121, 245, 285, 455, 513 Economics, see Secret Millionaires Club activities and resources	**Connect to Reading**, 272, 408, 460 **Connect to Science**, 134, 230, 384, 448 In the **Teacher Edition**: 11, 23, 29, 67, 81, 111, 137, 141, 171, 211, 253, 267, 271, 279, 293, 313, 341, 349, 383, 407, 425, 433, 447, 459 **Connect to Social Studies**, 72 In the **Teacher Edition**: 11, 23, 29, 67, 81, 111, 137, 141, 171, 211, 253, 267, 271, 279, 293, 313, 341, 349, 383, 407, 425, 433, 447, 459 Economics, see Secret Millionaires Club activities and resources
Grade 6	
Connect to Reading, 122, 150, 236, 330, 348, 458 **Connect to Science**, 30, 42, 254, 396, 442, 494 In the **Teacher Edition**: 29, 33, 59, 77, 89, 111, 121, 133, 153, 165, 193, 207, 225, 255, 267, 319, 325, 351, 373, 389, 407, 439, 443, 475, 497, 511 **Connect to Social Studies**, 298 In the **Teacher Edition**: 29, 33, 59, 77, 89, 111, 121, 133, 153, 165, 193, 207, 225, 255, 267, 319, 325, 351, 373, 389, 407, 439, 443, 475, 497, 511 Economics, see Secret Millionaires Club activities and resources	

Scaffolding in GO Math!

Learners are supported through the design of ***GO Math!*** with scaffolds that help students solidify what they know in order to build on it.

Scaffolds in ***GO Math!*** include:

- **Reviewing**—At the opening of each chapter, students **Review Prerequisite Skills**. Before moving on, students are given a chance to review the concepts and vocabulary they have learned previously in ***GO Math!***
- **Activating students' prior knowledge—Show What You Know** questions encourage students to connect previous learning to the current lesson. The **Teacher Edition** offers ideas on how to engage students and **Access Prior Knowledge**.
- **Cues and tools**—Throughout ***GO Math!*** students encounter problems that use manipulatives, graphics, pictures, line graphs, models, and other techniques to help support their thinking.
- **Multi-step problem solving—The Write-In Student Edition** allows students to complete problems in a graduated way. Students are often asked how to solve the problem before they solve the problem, or are given a less difficult problem to solve before the more difficult one. With the write-in design, students can easily refer to previous concepts or steps while problem solving.
- **Questioning**—In ***GO Math!*** graduated questioning is used as a scaffold. **Teach and Talk** sections in the **Teacher Edition** offer suggested questions for each activity.

Engagement in GO Math!

In ***GO Math!*** students are the focus of the program and student engagement is promoted throughout.

The program provides students with new ways to interact through the Write-In **Student Edition**, which engages students in recording their strategies, explanations, solutions, and practice in their books.

The program's **Digital Path** and engaging write-in design create multimedia learning opportunities that engage students in the study of mathematics. (See the final section of this report for further detail on the program's **Digital Path**.)

The ***GO Math!*** **Real World Videos** are motivating videos of real-world settings that teachers can use to introduce lessons and engage students.

The program's **Carmen Sandiego™ Math Detective Activities** Grades 3–6 offer engaging activities for each Critical Area and provide students with the chance to solve math problems with real-world themes. To further engage students, **Carmen Sandiego** characters introduce lesson activities with audio and animation.

Carmen Sandiego* Math Detective Activities in *GO Math!

Grade 3	Grade 4
1C, 3, 59, 95, 131, 179, 207, 303C, 305, 349, 385C, 387, 431, 479C, 481	3, 43, 99, 135, 191, 225, 265, 313, 341, 379, 415, 443, 495
Grade 5	**Grade 6**
3, 59, 103, 159, 199, 241, 289, 337, 403, 439	3, 47, 95, 145, 185, 217, 247, 291, 339, 369, 413, 449, 489

Strand 3: Data-Driven Instruction

Assessment…refers to all those activities undertaken by teachers—and by their students in assessing themselves—that provide information to be used as feedback to modify teaching and learning activities…

(Black & Wiliam, 1998, p. 140)

Defining the Strand

Data-driven instruction refers to a comprehensive system of instruction in which educators use effective tools to collect data about what is working—and what is not—so that they can take precise, swift, and effective action in meeting the specific needs of students. In a data-driven system, clear and shared standards are important so that all students and teachers know the intended outcomes of instruction. Assessments aligned to these standards are essential so that teachers can analyze how well students are meeting the goals for learning. And aligned instruction is essential so that teachers adjust accordingly to tailor teaching to student needs.

When teachers have detailed data from assessment they can make necessary adjustments to instruction to meet diverse needs. As noted by numerous research studies, the regular use of assessment to monitor student progress can mitigate and prevent mathematical weaknesses and improve student learning (Clarke & Shinn, 2004; Fuchs, 2004; Lembke & Foegen, 2005; Skiba, Magnusson, Marston, & Erickson, 1986). Formative assessment has a positive effect on learning (Black & Wiliam, 1998b; Cotton, 1995; Jerald, 2001). In their research, Baker, Gersten, and Lee (2002) concluded that "providing teachers and students with information on how each student is performing seems to enhance…achievement consistently" (p. 67). There is agreement that "assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions" (National Council of Teachers of Mathematics, 2000, p. 1).

Research also points to the importance of using varied item types and tasks in order to get the best reflection of student understanding. As noted by McREL (2010) "Using multiple types of assessments provides more insight into students' learning because students have more than one way to demonstrate their knowledge and skills" (p. 44).

GO Math! supports data-driven instruction. Throughout the program, varied assessments provide valuable information about student learning that can help teachers plan and modify instruction. ***GO Math!*** integrates effective assessment practices by supporting teachers in using:

- Diagnostic Assessment
- Formative Assessment; and
- Varied Assessment Types and Options.

Research that Guided the Development of the *GO Math!* Program

Diagnostic Assessment

Effective instruction depends upon teachers who make good decisions about how best to meet their students' needs. To make these kinds of decisions, teachers need information that they can trust about students' strengths and weaknesses, knowledge, and understandings. In an instructional context, a *diagnostic assessment* is one in which "assessment results provide information about students' mastery of relevant prior knowledge and skills within the domain as well as preconceptions or misconceptions about the material" (Ketterlin-Geller & Yovanoff, 2009, p. 1).

Studies attest to the benefits of using valid diagnostic measures—and tailoring instruction and supplemental practice according to the results of the diagnostics (for example, see Mayes, Chase, & Walker, 2008). Today's classrooms have disparity in students' prerequisite skills and knowledge, and preparation and diagnostic assessment can help to identify the best instructional approach for each student at the outset, so that instructional time is not wasted.

Formative Assessment

"Effective instruction depends on sound instructional decision-making, which in turn, depends on reliable data regarding students' strengths, weaknesses, and progress in learning content" (National Institute for Literacy, 2007, p. 27). The phrase *formative assessment* encompasses the wide variety of activities—formal and informal—that teachers employ throughout the learning process to gather this kind of instructional data to assess student understanding and make and adapt instructional decisions. Its purpose is not an end in itself—such as the assignment of a grade—but rather, the purpose is to guide instruction. Formative assessment moves testing from the end into the middle of instruction, to guide teaching and learning as it occurs (Shepard, 2000; Heritage, 2007). Formative assessment shifts the way that students view assessments: "Assessment should not merely be done to students; rather, it should also be done *for* students, to guide and enhance their learning" (NCTM, 2000, p. 22).

Educators agree on the benefits of ongoing assessment in the classroom. "Well-designed assessment can have tremendous impact on students' learning … if conducted regularly and used by teachers to alter and improve instruction" (National Research Council, 2007, p. 344). In its review of high-quality studies on formative assessment, the National Mathematics Advisory Panel (2008) found that "use of formative assessments benefited students at all ability levels" (p. 46). Several reviews of instructional practices used by effective teachers have revealed that effective teachers use formal (such as quizzes or homework assignments) and informal tools (such as discussion and observation) to regularly monitor student learning and check student progress (Cotton, 1995; Christenson, Ysseldyke, & Thurlow, 1989). A meta-analytic study by Baker, Gersten, and Lee (2002) found that achievement increased as a result of regular assessment use: "One consistent finding is that providing teachers and students with specific information on how each student is performing seems to enhance mathematics achievement consistently…The effect of such practice is substantial" (p. 67). In a study of student learning in a multimedia environment, Johnson and Mayer (2009) found that students who took a practice test after studying multimedia material outperformed students who studied the material again (without the assessment). Stecker, Fuchs, and Fuchs (2005) examined research on curriculum-based measurement, in which teachers used outcomes-based assessments regularly to monitor student progress, and found that the use of these assessments produced significant gains—when teachers used the data to make appropriate adjustments to instruction.

Research shows that regularly assessing and providing feedback to students on their performance is a highly effective tool for teachers to produce significant—and often substantial—gains in student learning and performance (Black & Wiliam, 1998a, 1998b; Hattie, 1992). Feedback is essential so that students know how to monitor their own performance and know which steps to take to improve (National Research Council, 2001).

An additional benefit of formative assessment is that it has been shown to be particularly helpful to lower-performing students. Gersten and Clarke (2007) conveyed similar findings for lower-achieving math students, concluding that "the use of ongoing formative assessment data invariably improved mathematics achievement of students with mathematics disability" (p. 2). In this way, use of formative assessments minimizes achievement gaps while raising overall achievement (Black & Wiliam, 1998b).

Varied Assessment Types and Options

One single assessment or type of assessment cannot serve all of the purposes of assessment. Research supports that looking at multiple means of assessment is the best way to capture a whole picture of student learning. As noted by Krebs' (2005) research, using one data point, such as written responses, to evaluate and assess students' learning can be "incomplete and incorrect conclusions might be drawn…" (p. 411). In addition, "using multiple types of assessments provides more insight into students' learning because students have more than one way to demonstrate their knowledge and skills" (McREL, 2010, p. 44). Therefore, variety in assessment item types is an integral part of an effective mathematics program.

Using performance-based assessments or problem-solving tasks in the classroom is one effective way to assess student understanding—and encourage critical thinking. Research indicates that high-quality tasks foster students' abilities to reason, solve problems, and conjecture (Matsumura, Slater, Peterson, Boston, Steele et al., 2006). Students can gain a deeper understanding of mathematics by exploring and reasoning through performance-based tasks.

Items in which students were asked to construct a response—rather than choose among options for answer choices—were shown to involve greater cognitive effort in a study by O'Neil and Brown (1998).

Asking students to respond to open-ended questions—in writing or through classroom discussion—is another useful way to assess what students are learning. As discussed by Moskal (2000) in her guidelines for teachers for analyzing student responses, students' responses to open-ended questions afford them the opportunity to show their approaches in solving problems and expressing mathematically what they know, which in turn allows the teacher to see the students' mathematical knowledge. Research by Aspinwall and Aspinwall (2003) on using open-writing prompts supports the use of open-ended questions in assessment in the mathematics classroom: "Students' responses to open-ended questions offer opportunities for understanding how students view mathematical topics… this type of writing allows teachers to explore the nature of students' understanding and to use this information in planning instruction" (p. 352-353). Similarly, by asking students to respond to open-ended questions verbally, researchers Gersten and Chard (2001) found that "encouraging students to verbalize their current understandings and providing feedback to the student increases learning."

Multiple-choice items can play an important role in an assessment system as well. The National Mathematics Advisory Panel (2008) found that formative assessments based on items sampled from important state standard objectives resulted in "consistently positive and significant" effects on student achievement (p. 47). In addition, the Panel found multiple-choice items to be equally valuable in assessing students' knowledge of mathematics (National Mathematics Advisory Panel, 2008).

From Research to Practice

Diagnostic Assessment in GO Math!

Specific features in ***GO Math!*** support teachers in using diagnostic assessment effectively to assess students' need for instruction.

The program's **Diagnostic Assessments** help teachers in determining if students need intervention for the chapter's prerequisite skills. These **Show What You Know** assessments provide the chance for teachers to regularly diagnose students' readiness for instruction. **Show What You Know** identifies students' levels of preparation for the chapter content—and then is linked to special intervention and challenge resources in the **Teacher Edition** so that every student gets off to the right start. If students are not successful with **Show What You Know**, teachers can intervene with **Strategic Intervention** or **Intensive Intervention** activities. If students are successful, they can challenge themselves with the **Extend Book** or engage in independent activities from the ***Grab-and-Go*** **Differentiated Centers Kit**.

A variety of tools ensure that students will receive the instruction they need from the beginning with ***GO Math!***

Diagnostic Assessment in *GO Math!*
• **Prerequisite Skills Inventory** (in the **Assessment Guide**)
• **Beginning-of-Year Test** (in the **Assessment Guide**)
• **Diagnostic Interview Task** (in the **Assessment Guide**)
• **Show What You Know** (in the **Student Edition**)
• **Diagnostic Inventory Assessment** (in the **Assessment Guide**)
• ***Soar to Success*** **Math**

Formative Assessment in GO Math!

Opportunities for assessing what students know and can do are incorporated throughout ***GO Math!*** The program clearly links assessment results to instruction—so that results can be used formatively to inform and guide instructional planning and delivery.

The program's **Online Assessment System** allows teachers to:

- Receive instant results, including prescriptions for intervention;
- Track student progress with a variety of reports;
- Create customized tests.

Formative Assessment in *GO Math!*
• **Lesson Quick Check** (in the **Teacher Edition**)
• **Test Prep** (in the **Standards Practice Book**)
• **Mid-Chapter** Checkpoint (in the **Student Edition**)
• **Portfolio** (in the **Assessment Guide**)
• **Middle-of-Year Test** (in the **Assessment Guide**)

Examples of Formative Assessment in *GO Math!*

Kindergarten	Grade 1
Formative Assessment, Quick Check, in every **Teacher Edition** lesson. Some examples are: 18, 30, 198, 234, 470, 482 **Mid-Chapter Checkpoint**, 28, 72, 104, 148, 184, 240, 284, 324, 380, 432, 476, 504	**Formative Assessment, Quick Check**, in every **Teacher Edition** lesson. Some examples are: 27, 62, 294, 330, 462, 518 **Mid-Chapter Checkpoint**, 28, 76, 120, 164, 200, 260, 300, 328, 388, 428, 468, 504
Grade 2	**Grade 3**
Formative Assessment, Quick Check, in every **Teacher Edition** lesson. Some examples are: 18, 62, 282, 338, 474, 510 **Mid-Chapter Checkpoint**, 32, 84, 144, 200, 252, 300, 356, 408, 448, 480, 532	**Formative Assessment, Quick Check**, in every **Teacher Edition** lesson. Some examples are: 7, 63, 135, 211, 308, 391, 485 **Mid-Chapter Checkpoint**, 33-34, 73-74, 109-110, 153-154, 189-190, 229-230, 277-278, 327-328, 367-368, 409-410, 457-458, 499-500
Grade 4	**Grade 5**
Formative Assessment, Quick Check, in every **Teacher Edition** lesson. Some examples are: 7, 47, 229, 269, 447, 499 **Mid-Chapter Checkpoint**, 21-22, 73-74, 117-118, 161-162, 205-206, 247-248, 287-288, 323-324, 363-364, 397-398, 429-430, 465-466, 509-510	**Formative Assessment, Quick Check**, in every **Teacher Edition** lesson. Some examples are: 7, 53, 275, 305, 443, 489 **Mid-Chapter Checkpoint**, 25-26, 77-78, 129-130, 181-182, 217-218, 263-264, 315-316, 351-352, 385-386, 421-422, 461-462
Grade 6	
Formative Assessment, Quick Check, in every **Teacher Edition** lesson. Some examples are: 7, 51, 193, 229, 457, 505 **Mid-Chapter Checkpoint**, 25-26, 65-66, 113-114, 167-168, 199-200, 231-232, 269-270, 321-322, 353-354, 391-392, 431-432, 467-468, 507-508	

Varied Assessment Types and Options in GO Math!

Throughout the ***GO Math!*** program, multiple effective types of assessment appear in an effort to best allow students to demonstrate their knowledge and skills. ***GO Math!*** features strong performance task assessments and extended end-of-year review projects, as well as multiple-choice items, constructed response tasks, and other problem-solving prompts.

Varied Assessment Types and Options in *GO Math!*	
Assessment Type	Description and Examples
Multiple-Choice Assessments	Multiple-choice items allow teachers to quickly get a sense of what students know and do not know. In ***GO Math!*** the **Online Assessment System** is designed with multiple-choice items to allow ease of delivery and scoring. **Chapter Tests** include multiple-choice items to provide a quick summative view of student learning.
Mixed Response Formats	Mixed response format items—such as constructed response items—allow for a deeper look at students' thinking and understanding of concepts and practices. The ***GO Math!* Chapter Tests** include both constructed response and extended constructed response items to provide this deeper look.
Performance Assessments	Performance assessments can reveal thinking strategies that students use to work through problems. ***GO Math!*** has a **Performance Assessment** for each Critical Area. Each assessment has four tasks that target specific math concepts, skills, and strategies. These performance assessments can be used flexibly and diagnostically. They model good instruction and encourage thinking.
Portfolios	The ***GO Math!* Assessment Guide** provides information about how to organize, share, and evaluate portfolios—which can be used to represent the growth, talents, achievements, and reflections of the mathematics learner. Portfolio suggestions are provided throughout each grade level of ***GO Math!*** At the end of each chapter, the **Teacher Edition** provides **Portfolio Suggestions**, along with suggested questions for students.
Online Assessments	The ***GO Math!* Online Assessment System** allows for immediate diagnosis and prescriptions for intervention and follow-up instruction. Because the tests are administered online they can be automatically scored; easily customized from a bank of test items; and used to create reports for both teachers and school administrators.

Strand 4: Meeting the Needs of All Students

All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.

(NCTM Principles & Standards for School Mathematics, 2000, p. 12)

Defining the Strand

Students in today's classrooms come from increasingly diverse backgrounds, in regard to culture and language as well as in their background knowledge, abilities, motivations, interests, and modes of learning (Tomlinson, 2005). Mathematical learning is important to each of these different students; "All young Americans must learn to think mathematically, and they must think mathematically to learn" (National Research Council, 2001, p. 1). To effectively teach mathematics skills and concepts, teachers of mathematics must be knowledgeable of and sensitive to the needs of all learners in the mathematics classroom.

In the classroom, teachers encounter students who are on-grade, above grade, below grade—as well as English language learners, students with special needs, and students with varying learning styles and cultural backgrounds. As Vygotsky (1978) noted in his seminal research on learning, "Optimal learning takes place within students' 'zones of proximal development'—when teachers assess students' current understanding and teach new concepts, skills, and strategies at an according level." Research continues to support the notion that for learning to take place, activities must be at the right level for the learner (Tomlinson & Allan, 2000; Valencia, 2007). Therefore, teachers must correctly identify each child's needs for instruction and additional support. Differentiation offers teachers a means to provide instruction to a range of students in today's classroom (Hall, Strangman, & Meyer, 2009). A Response-to-Intervention (RtI) model uses interventions at different tiers, or levels, to determine students' needs and the intensity of support required.

GO Math! centers on helping all students gain a deeper understanding of mathematics concepts and practices. ***GO Math!*** supports the achievement of diverse learners by incorporating tactics to meet the varying needs of students through effective:

- Differentiation; and
- Response to Intervention.

Research that Guided the Development of the *GO Math!* Program

Differentiation

As Tomlinson (1997) emphasizes in her discussion of differentiation,

> Students are not all alike. They differ in readiness, interest, and learning profile…Shoot-to-the-middle teaching ignores essential learning needs of significant numbers of struggling and advanced learners. To challenge the full range of learners appropriately requires that a teacher modify or "differentiate" instruction in response to the varying needs of varying students in a given classroom. (p. 1)

Differentiating instruction is an organized, but flexible way to alter teaching and learning to help all students maximize their learning (Tomlinson, 1999), and is necessary in order to meet the needs of all learners in today's diverse classrooms (Tomlinson, 2000). In the mathematics classroom, "mathematics instructors must respond to the diverse needs of individual students…using differentiated instruction, a process of proactively modifying instruction based on students' needs" (Chamberlin & Powers, 2010, p. 113). To differentiate instruction, teachers can adjust the content of what is being learned; adjust the process of learning (by providing additional supportive strategies, for example); and tailor the expected outcomes (assessments, products, or tasks) of how learning is assessed (Tomlinson, 2001).

In a study of numerous teachers using differentiated instruction, researchers found these benefits: students felt learning was more relevant; students were motivated to stay engaged in learning; students experienced greater success; students felt greater ownership of content, products and performances; and teachers gained new insights (Stetson, Stetson, & Anderson, 2007).

Differentiation benefits all students—low-achievers in mathematics as well as high-achievers. In its review of studies on teaching mathematically gifted students, the National Mathematics Advisory Panel (2008) found that:

- The studies reviewed provided some support for the value of differentiating the mathematics curriculum for students with sufficient motivation, especially when acceleration is a component (i.e., the pace and level of instruction are adjusted).
- A small number of studies indicated that individualized instruction, in which pace of learning is increased and often managed via computer instruction, produces gains in learning.

The Panel concluded that gifted students, too, benefit from a differentiated curriculum (National Mathematics Advisory Panel, 2008).

Response to Intervention

Both differentiated instruction and Response to Intervention (RtI) "share a central goal: to modify instruction until it meets the needs of all learners" (Demirsky Allan & Goddard, 2010). According to Demirsky Allan and Goddard (2010), these two instructional approaches are complementary and share the premises that all students have different academic needs and that teachers must teach accordingly to meet these needs—and ensure student success. Like differentiation, "At the heart of the RtI model is personalized instruction, during which each student's unique needs are evaluated and appropriate instruction is provided, so that students will succeed" (McREL, 2010, p. 15). While differentiation was conceived as a way to respond to the needs of diverse learners in the classroom, RtI was envisioned as a prevention system with multiple layers—a structured way to help students who were struggling before they fell behind their peers—and so it focuses on early, and ongoing, identification of needs and tiers of responses.

RtI is a model that integrates instruction, intervention, and assessment to create a more cohesive program of instruction that can result in higher student achievement (Mellard & Johnson, 2008). RtI is most commonly depicted as a three-tier model where Tier 1 represents general instruction and constitutes primary prevention. Students at this level respond well to the general curriculum and learn reasonably well without additional support. Tier 2 represents a level of intervention for students who are at risk. Students at Tier 2 receive some supplementary support, in the form of instruction or assessment. Tier 3 typically represents students who need more extensive and specialized intervention or special education services. (Smith & Johnson, 2011)

According to Griffiths, VanDerHeyden, Parson, and Burns (2006), an effective RtI model should include three elements:

1. Systematic assessment and collection of data to identify students' needs;
2. The use of effective interventions in response to the data; and
3. Continued assessment of students to determine the effectiveness of interventions—and the need for any additional intervention.

A growing body of research supports the effectiveness of RtI (for example, see Burns, Appleton, & Stehouwer, 2005). Research from Ketterlin-Geller, Chard, and Fien (2008) found that an integrated system, like that of RtI, can lead to improvement in mathematics performance on various achievement measures when used to intervene with students who are underperforming in mathematics. Fuchs, Fuchs, and Hollenbeck (2007) looked at RtI in mathematics with students in Grade 1 (a comprehensive program) and Grade 3 (a focus on word problems). They found that the data supported RtI at both grade levels, and showed "how two tiers of intervention, designed strategically to work in supplementary and coordinated fashion, may operate synergistically to decrease math problem-solving difficulties for children who are otherwise at risk for poor outcomes" (p. 19).

Research suggests that to integrating RtI successfully into classroom instruction involves a number of elements, described in a 2009 publication from the What Works Clearinghouse of the U.S. Department of Education (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). At the Tier 1 level, all students should be screened to identify those at risk—and interventions for those at risk should be provided. At Tiers 2 and 3, the following are important and proven effective by research:

- The focus of instructional materials in Grades K through 5 should be on whole numbers; in 4 through 8, rational numbers.
- Instruction during intervention should be explicit and systematic, and should include providing models of problem solving, communication about math, practice, feedback, and review.
- Interventions should require students to solve word problems.
- Interventions should provide opportunities for students to work with graphic organizers and visual representations.
- Interventions should devote regular time to arithmetic fact fluency.
- Students' progress should be monitored.
- Interventions should be designed to motivate students. (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009)

From Research to Practice

Differentiation in GO Math!

GO Math! supports teachers in implementing effective differentiation so that they meet the varied needs of students in their classrooms. Throughout ***GO Math!*** practical, point-of-use support is built into each lesson. All students—those who need extra support and intervention and gifted and talented students—can achieve success in learning mathematics with the program.

Multiple program features support differentiated instruction throughout ***GO Math!*** These include:

- ***Grab-and-Go*** **Differentiated Centers Kit**s contain:
  - Readers integrate math skills with cross-curricular content.
  - Games engage students in math skills practice.
  - Math Center Activities focus on computation, mental math, geometry, measurement, and challenge activities.
- Regular program assessments—such as the Quick Check (in every **Teacher Edition** lesson) and **Show What You Know**—provide needed information for differentiated instruction.
- Intervention and challenge resources (such as the Enrich Activities and the Independent Activities) linked to **Show What You Know** support differentiation.

In addition, specific program features support teachers in meeting students, varied learning styles and preferences. In the e**Student Edition**, each lesson includes audio reinforcement, which can particularly support struggling readers and English language learners. **Animated Math Models** and **Real World Videos** support visual learners. **HMH Mega Math** offers the opportunity for additional lesson practice—with engaging activities to motivate all learners, as well as audio and animation to engage auditory and visual learners. The program's **Multimedia eGlossary** includes audio, graphics, and animation to engage all students, and to reach students with varied learning styles and preferences.

Response to Intervention in GO Math!

Through print and digital resources, ***GO Math!*** supports an instructional model.

GO Math! offers **Interactive Interventions for RtI**. These activities and materials engage all levels of RtI learners with focused hands-on activities, print support, and a comprehensive online intervention solution (***Soar to Success*** **Math**). Teachers can select instructional strategies and resources that specifically align with each student's level of understanding and preferred learning style. ***Soar to Success*** **Math** allows teachers to diagnose and prescribe interactive intervention lessons for all RtI Tiers.

The program offers resources specifically for each level of RtI:

- **Tier 1: On-Level Intervention** is a primary prevention. The ***GO Math!*** program uses research-based instructional strategies to ensure quality instruction for all students.

Numerous program features and materials support teachers and their students at the Tier 1 level. Students who need on-level intervention can benefit from the program's **Reteach Book** activities.

- **Tier 2: Strategic Intervention** is for students who are not responding to Tier 1: On-Level Intervention. Tier 2 is met with increased time and focus to support struggling learners and reinforce skills that might not have been previously mastered. Students at Tier 2 can benefit from the ***GO Math!*** **Strategic Intervention** and **Teacher Activity Guide**.
- **Tier 3: Intensive Intervention** is the most intensive and tailored intervention. For those students who need more specialized intervention, ***GO Math!*** offers the **Intensive Intervention Users Guide**.

Because ***GO Math!*** is designed to support data-driven decision making, the program's assessment options are closely linked to suggestions for intervention so that assessment can be used as it should be in an effective classroom—to diagnose and intervene so that instructional time is tailored to students' needs.

Assessment and Intervention in *GO Math!*

Assessment	Intervention
Diagnostic • **Prerequisite Skills Inventory** (in the **Assessment Guide**) • **Beginning-of-Year Test** (in the **Assessment Guide**) • **Show What You Know** (in the **Student Edition**) • **Diagnostic Inventory Assessment** (in the **Assessment Guide**)	• **Intensive Intervention** • **Intensive Intervention User Guide** • **Strategic Intervention** • ***Soar to Success*** **Math**
Formative • **Lesson Quick Check** (in the **Teacher Edition**) • **Test Prep** (in the **Standards Practice Book**) • **Mid-Chapter Checkpoint** (in the **Student Edition**) • **Portfolio (in the Assessment Guide)** • **Middle-of-Year Test** (in the **Assessment Guide**)	Formative • **Lesson Quick Check** (in the **Teacher Edition**) • **Test Prep** (in the **Standards Practice Book**) • **Mid-Chapter Checkpoint** (in the **Student Edition**) • **Portfolio (in the Assessment Guide)** • **Middle-of-Year Test** (in the **Assessment Guide**)
Summative • **Chapter Review/Test** (in the **Student Edition**) • **Chapter Test** (in the **Assessment Guide**) • **Performance Assessment** (in the **Assessment Guide**) • **End-of-Year Test** (in the **Assessment Guide**) • **Getting Ready for Grade X Test** (in the **Assessment Guide**)	• **Reteach Book** (with each lesson) • **RtI: Tier 1 and Tier 2 Activities** (on the B page of every Teacher Edition lesson) • ***Soar to Success*** **Math**

Strand 5: Technology

Research on instructional software has generally shown positive effects on students' achievement in mathematics as compared with instruction that does not incorporate such technologies.

(National Mathematics Advisory Panel, 2008, p. 50)

Defining the Strand

At its most basic, technology can refer to any tools, inventions, or techniques that help us solve problems or perform activities. Technology has always played a role in mathematical learning and study, and can serve as a valuable tool in teaching and learning.

Technology can support students' development of skills, exploration and communication of concepts, and ability to reason and problem-solve. With advances in technology, specifically in graphics technology and information technology, new opportunities for mathematical teaching and learning are constantly emerging. According to Weglinsky (1998), who found benefits particularly for teaching and learning higher-order skills, "…when they are properly used, computers may serve as important tools for improving student proficiency in mathematics, as well as the overall learning environment in the school" (p. 4).

According to the findings of the National Research Council's 2001 review, "research has shown that instruction that makes productive use of computer and calculator technology has beneficial effects on understanding and learning algebraic representation" (p. 420). Numerous studies and meta-analyses support the use of computers in the classroom to improve student learning (see Britt & Aglinskas, 2002; Means, Toyama, Murphy, Bakia, & Jones, 2009; North Central Regional Educational Laboratory, 2003; Teh & Fraser, 1995). The effective use of computers in the mathematics classroom also correlates with higher levels of motivation for and interest in learning mathematics, a finding supported by House's 2009 study of U.S. and Korean students' computer use and TIMSS assessment results.

GO Math! was developed to take advantage of the instructional benefits of technology and support student learning through multimedia. The program offers Student eTextbooks for learning with mobile devices and tablets, as well as Online Editions for interactive instruction. The Online Resource Center provides teachers with materials and support for planning, teaching, and assessing.

Research that Guided the Development of the *GO Math!* Program

Multimedia Learning

In Mayer's second edition (2009) of *Multimedia Learning*, he again lays out the case for multimedia learning, presenting a cognitive theory of multimedia learning and citing the results of numerous, systematically designed studies which demonstrate the ways in which people learn more deeply from words and visuals rather than from verbal messages alone. According to Mayer, "the case for multimedia learning is based on the idea that instructional messages should be designed in light of how the human brain works" (Mayer, 2001, p. 4). Mayer (2001, 2005), a leading researcher in the field of multimedia learning, argues that student learning is increased in multimedia environments because information can be presented in multiple formats—including words, audio, and pictures. Students are able to learn more and retain information when they can access information using these different pathways.

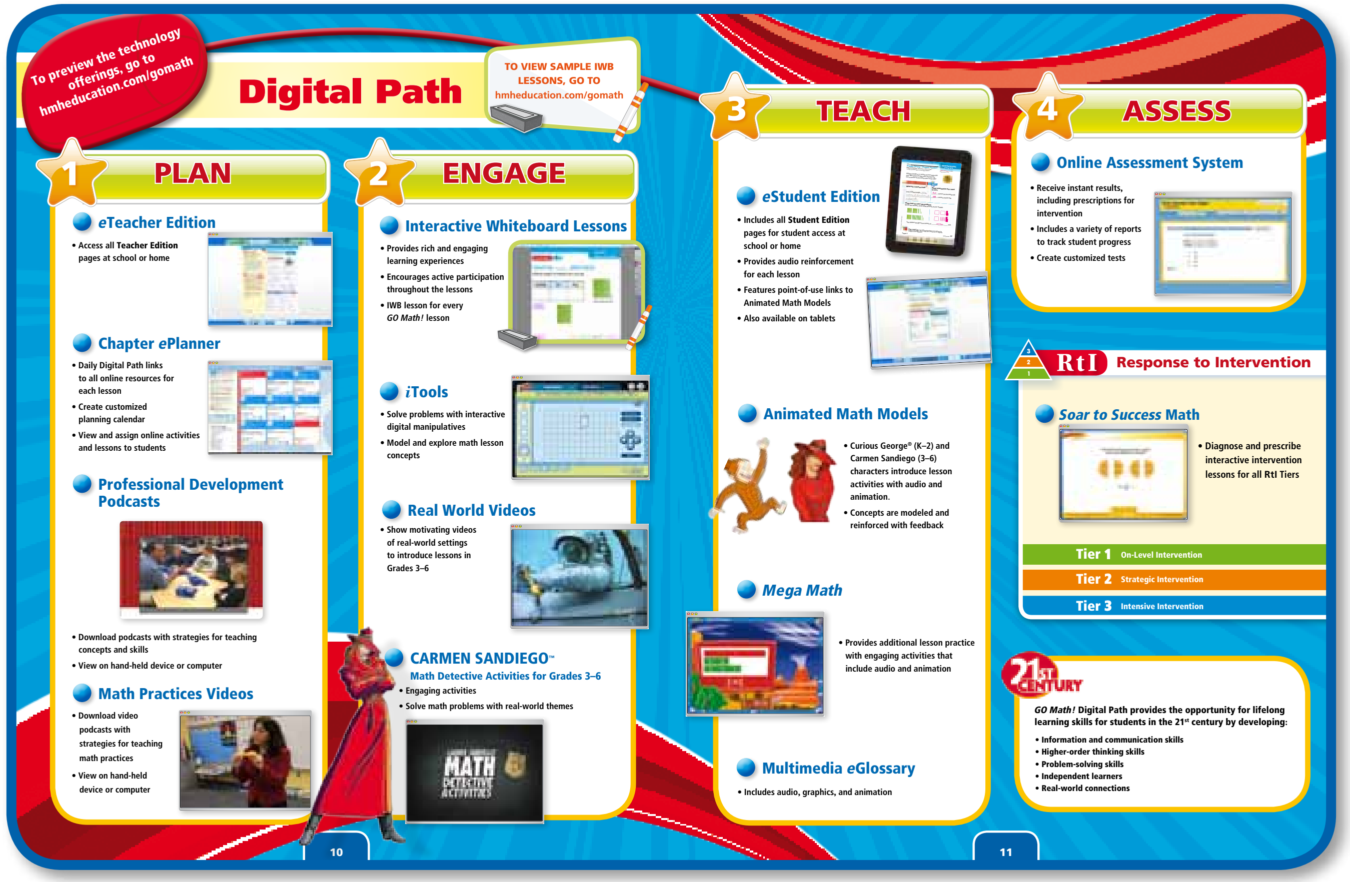
Multimedia learning opportunities can increase student achievement. Large-scale meta-analyses conducted by the North Central Regional Educational Laboratory (2003) and the U.S. Department of Education (Means, Toyama, Murphy, Bakia, & Jones, 2009) have established the connections between computer use and increased student learning. Weiss, Kramarski, and Talis (2006) examined the impact of multimedia activities on the mathematics learning of young children and found that students who engaged in multimedia learning either individually or in cooperative learning groups significantly outperformed control group students. In a study which compared users of classroom computer games with a control group, Kebritchi, Hirumi, and Bai (2010) found that the games had a "significant positive effect on students' mathematics achievement" (p. 435).

And, finally, multimedia learning opportunities can help to close achievement gaps between groups of students and can be particularly effective with average and lower-achieving students (see Huppert, Lomask, & Lazarowitz, 2002; Mayer, 2001; White & Frederiksen, 1998). Means, Toyama, Murphy, Bakia, and Jones (2009) found that online learning approaches were effective across types of learners. One reason for this may be because multimedia learning environments are able to reach students who learn in different ways—visual learners, auditory learners, kinesthetic learners. Another reason may be the power of technology to embed scaffolds at the point of use.

From Research to Practice

Multimedia Learning in GO Math!

GO Math! employs technology to support instruction and enhance student learning. Multimedia learning materials engage students through audio and video.



The technology tools and resources available with the ***GO Math!*** program enhance the teaching and learning of mathematics. These multimedia learning tools and resources comprise the program's **Digital Path** which supports teachers as they Plan, Engage, Teach, and Assess.

The Digital Path in *GO Math!*

Plan	Engage	Teach	Assess
eTeacher Edition—Allows teachers to access all of the Teacher Edition pages at school or home. **Chapter ePlanner**—Provides daily digital path links to all online resources for each lesson, and allows teachers to create a customized planning calendar and to view and assign online activities and lessons to students. **Professional Development Video Podcasts**—Teachers can download video podcasts with strategies for teaching concepts and skills. Podcasts can be viewed on a hand-held device or computer.	**Interactive Whiteboard Lessons**—Activities are available for every **Student Edition** lesson. ***iTools***—Students can model and explore lesson math concepts and solve problems with interactive digital manipulatives. **Real World Videos**—Motivating videos of real-world settings introduce lessons in ***GO Math!*** ***Carmen Sandiego* Math Detective Activities**—Students solve math problems with real-world themes with these engaging activities for each Critical Area.	**eStudent Edition**—Students can access all pages of the **Student Edition** at school or home, with audio reinforcement for each lesson and point-of-use links to **Animated Math Models.** **Interactive Whiteboard Lessons**—For every **Student Edition** lesson. **Animated Math Models**—Here, program concepts are modeled and reinforced with feedback. **HMH Mega Math**—Students can engage in additional lesson practice with engaging activities that include audio and animation. **Multimedia eGlossary**—Includes audio, graphics, and animation to support and engage all learners.	**Online Assessment System**—With the Online Assessment System, teachers can receive instant results, track student progress, receive suggestions for intervention, and create customized tests. ***Soar to Success* Math**—Provides teachers with tools to diagnose and prescribe interactive intervention lessons for all RtI Tiers.

References

Achieve. (2010). *Comparing the Common Core State Standards in Mathematics and NCTM's Curriculum Focal Points.* Washington, DC: Author. Retrieved January 29, 2012 from http://www.achieve.org/CCSSandFocalPoints.

American Institutes for Research. (2005). *Reassessing U.S. international mathematics performance: New findings from the 2003 TIMSS and PISA.* U.S. Department of Education, Policy and Program Studies Service (PPSS). Washington, DC: American Institutes for Research. Retrieved April 27, 2012 from http://www.achievethecore.org/you-ve-got-to-read-this

Albert, L.R., & Antos, J. (2000). Daily journals connect mathematics to real life. *Mathematics Teaching in the Middle School*, 5(8), 526-531.

Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education*, 9, 33-52.

Aspinwall L. & Aspinwall J. (2003). Investigating mathematical thinking using open writing prompts. *Mathematics Teaching in the Middle School*, 8(7), 350-353.

Baker, S., Gersten, R., & Lee, D. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students. *The Elementary School Journal*, 103(1), 67.

Baker, A., Schirner, K., & Hoffman, J. (2006). Multiage mathematics: Scaffolding young children's mathematical learning. *Teaching Children Mathematics*, 13(1), 19-21.

Baldi, S., Jin, Y., Skemer, M., Green, P.J., & Herget, D. (2007). *Highlights from PISA 2006: Performance of U.S. 15-year-old students in science and mathematics literacy in an international context* (NCES-2008-016). National Center for Education Statistics, Institute of Education Sciences. Washington, DC: U.S. Department of Education.

Baxter, J. A., Woodward, J., & Olson, D. (2005). Writing in mathematics: An alternative form of communication for academically low-achieving students. *Learning Disabilities Research & Practice*, 20(2), 119-135.

Beane, J. A (1997). *Curriculum integration: Designing the core of democratic education.* Alexandria, VA: Association for Supervision and Curriculum Development.

Black, P., & Wiliam, D. (1998a). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan, 80*(2), 139-148.

Black, P., & Wiliam, D. (1998b). *Assessment and classroom learning. Assessment in Education: Principles, Policy, and Practice, 5*(1), 7-73.

Boaler, . (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education, 29*, 41-62.

Bosse, M.J., & Faulconer, J. (2008). Learning and assessing mathematics through reading and writing. *School Science & Mathematics*, 108(1), 8-19.

Brandenburg, Sr. M. L. (2002). Advanced math? Write! *Educational Leadership*, 60(3), 67-68.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (1999). *How people learn: Brain, mind, experience, and school.* National Research Council. Washington, DC: National Academies Press.

Brenner, M.E., Mayer, R.E., Moseley, B., Brar, T., Duran, R., Smith Reed, B., & Web, D. (1997). Learning by understanding: The role of multiple representations in learning algebra. *American Educational Research Journal*, 34, 663-689.

Britt, M., & Aglinskas, C. (2002). Improving students' ability to identify and use source information. *Cognition and Instruction, 20*(4), 485-522.

Burns, M. (2004). Writing in math. *Educational Leadership, 62*(2), 30-33.

Burns, M.K., Appleton, J.J., & Stehouwer, J.D. (2005). Metaanalytic review of responsiveness-to-intervention research: Examining field-based and research-implemented models. *Journal of Psychoeducational Assessment, 23*, 381-394.

Caine, R. N., & Caine, G. (1991). *Making connections: Teaching and the human brain.* Alexandria, Virginia: Association for Supervision and Curriculum Development.

Chamberlin, M.T., & Powers, R.A. (2010). The promise of differentiated instruction for enhancing the mathematical understandings of college students. *Teaching Mathematics and Its Applications An International Journal of The Institute of Mathematics and Its Applications, 29*(3), 113-139.

Chapin, S.H., O'Connor, C., & Canavan Anderson, N. (2003). *Classroom discussions: Using Math Talk to help students learn, grades 1-6.* Sausalito, CA: Math Solutions Publications.

Christenson, S.L., Ysseldyke, J.E., & Thurlow, M.L. (1989). Critical instructional factors for students with mild handicaps: An integrative review. *Remedial and Special Education*, 10(5), 21-31.

Clarke, D.M., & Clarke, B.A. (2004). Mathematics teaching in grades K-2: Painting a picture of challenging, supportive, and effective classrooms. In R.N. Rubenstein & G.W. Bright (Eds.), *Perspectives on the teaching of mathematics* (66th Yearbook of the National Council of Teachers of Mathematics, pp. 67-81). Reston, VA: NCTM.

Clarke, B., & Shinn, M.R. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review*, 33(2), 234-248.

Clement, L.L. (2004). *A model for understanding, using, and connecting representations.* Reston, VA: National Council of Teachers of Mathematics.

Common Core State Standards Initiative (CCSSI). (2010). *Common Core State Standards for Mathematics.* The Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA). Retrieved May 5, 2012 from http://www.corestandards.org/the-standards/mathematics

Common Core State Standards Initiative. (2011). *Key points in mathematics.* Retrieved April 27, 2012 from http://www.corestandards.org/about-the-standards/key-points-in-mathematics.

Cotton, K. (2000). *The schooling practices that matter most.* Alexandria, VA: Association for Supervision and Curriculum Development.

Cotton, K. (1995). *Effective schooling practices: A research synthesis 1995 update.* Portland, OR: Northwest Regional Educational Laboratory. Retrieved March 10, 2012 from http://home.comcast.net/~reasoned/4410/PDFonCRM/Effective%20School%20Prac.pdf

Czerniak, C.M., Weber, W.B., Jr., Sandmann, A., & Ahem, J. (1999). *A literature review of science and mathematics integration.* School Science & Mathematics, 99 (8), 421-430.

Demirsky Allan, S., & Goddard, Y.L. (2010). Differentiated instruction and RtI: A natural fit. *Interventions that Work*, 68(2).

Donovan, M.S., & Bransford, J.D. (2005). *How students learn: Mathematics in the classroom.* Washington, DC: National Academy of Sciences.

Erickson, D.K. (1999). A problem-based approach to mathematics instruction. *Mathematics Teachers*, 92(6), 516-521.

Finn, J. D. (1993). *School engagement and students at risk.* Washington, DC: National Center for Education Statistics.

Franke, M. L., Kazemi, E., & Battey, D. S. (2007). Mathematics teaching and classroom practices. In F. K. Lester Jr. (Ed.), *The second handbook of research on mathematics teaching and learning* (pp. 225–256). Charlotte, NC: Information Age.

Fuchs, L.S. (2004). The past, present, and future of curriculum-based measurement research. *School Psychology Review*, 33, 188-192.

Fuchs, L.S., Fuchs, D., & Hollenbeck, K.N. (2007). Extending responsiveness to intervention to mathematics at first and third grades. *Learning Disabilities Research & Practice, 22*(1), 13-24.

Furner, J. M., & Duffy, M. L. (2002). Equity for all students in the new millennium: Disabling math anxiety. *Intervention in School and Clinic, 38*(2), 67-74.

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J.R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools.* Institute of Education Sciences What Works Clearinghouse. Washington, DC: U.S. Department of Education. Retrieved February 25, 2012 from http://ies.ed.gov/ncee/wwc/practiceguide.aspx?sid=2.

Gersten, R., & Chard, D. (2001). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *LD OnLine*. Retrieved February 20, 2012 from http://www.readingrockets.org/article/5838/.

Gersten, R., & Clarke, B.S. (2007). *Effective strategies for teaching students with difficulties in mathematics*. Instruction Research Brief. Reston, VA: National Council of Teachers of Mathematics. Retrieved February 21, 2012 from http://www.nctm.org/news/content.aspx?id=8452.

Gonzales, P., Williams, T., Jocelyn, L., Roey, S., Katsberg, D., & Brenwald, S. (2008). *Highlights from TIMSS 2007: Mathematics and science achievement of U.S. fourth- and eighth-grade students in an international context* (NCES 2009-001 Revised). National Center for Education Statistics, Institute of Education Sciences. Washington, DC: U.S. Department of Education.

Good, J. M., & Whang, P. A. (1999). *Making meaning in educational psychology with student response journals*. Paper presented at the annual meeting of the American Educational Research Association, Montreal, Canada.

Griffiths, A., VanDerHeyden, A.M., Parson, L.B., & Burns, M.K. (2006). Practical applications of Response-to-Intervention research. *Assessment for Effective Intervention, 32*(1), 50-57.

Hall, T., Strangman, N., & Meyer, A. (2009). *Differentiated instruction and implications for UDL implementation*. Wakefield, MA: National Center on Accessible Instructional Materials. Retrieved May 8, 2012 from http://aim.cast.org/learn/historyarchive/backgroundpapers/differentiated_instruction_udl

Hatano, G., & Inagaki, K. (1991). Constrained person analogy in young children's biological inference. *Cognitive Development, 6*(2), 219-231.

Hattie, J. (1992). *Self-concept*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Heritage, M. (2007). Formative assessment: What do teachers need to know and do? *Phi Delta Kappan*, 89(2), 140-145.

Hettich, P. (1976). The journal: An autobiographical approach to learning. *Teaching of Psychology*, 3(1), 60-63.

Hill, H.C., Rowan, B., & Loewenberg Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Horton, S.V., Lovitt, T. C., & Bergerud, D. (1990). The effectiveness of graphic organizers for three classifications of secondary students in content area classes. *Journal of Learning Disabilities, 23*(1), 12-22.

House, J.D. (2009). Motivational effects of computers and instructional strategies for mathematics teaching in the United States and Korea: Results from the TIMSS 2003 assessment. *International Journal of Instructional Media, 36*(3), 351-363.

House, J.D., & Telese, J.A. (2011). Effects of computer activities and classroom lesson strategies on motivation for mathematics learning for eighth-grade students in the United States and Korea. *International Journal of Instructional Media, 38*(3), 295-306.

Hufferd-Ackles, K., Fuson, K.C., & Sherin, M.G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81-116.

Huppert, J., Lomask, S. M., & Lazarowitz, R. (2002). Computer simulations in the high school: students' cognitive stages, science process skills and academic achievement in microbiology. *International Journal of Science Education, 24*(8), 803-821.

Hyde, A. (2006). *Comprehending math: Adapting reading strategies to teach mathematics K–6*. Portsmouth, NH: Heinemann.

Jerald, C. D. (2001). *Dispelling the myth revisited*. Washington DC: Education Trust.

Johnson, C.I., & Mayer, R.E. (2009). A testing effect with multimedia learning. *Journal of Educational Psychology*, 101(3), 621-629.

Kebritchi, M., Hirumi, A., & Bai, H. (2010). The effects of modern mathematics computer games on mathematics achievement and class motivation. *Computers & Education, 55*, 427-443.

Ketterlin-Geller, L.R., Chard, D.J., & Fien, H. (2008). Making connections in mathematics: Conceptual mathematics intervention for low-performing students. *Remedial and Special Education, 29*(1), 33-45.

Ketterlin-Geller, L.R., & Yovanoff, P. (2009). Diagnostic Assessments in mathematics to support instructional decision making. *Practical Assessment, Research & Evaluation, 14*(16), 1-11. Retrieved May 6, 2012 from http://pareonline.net/getvn.asp?v=14&n=16.

Kilpatrick, J., Swafford, J., & Findell, B., Eds. (2001). *Mathematics Learning Study Committee, National Research Council: Conclusions and recommendations. Adding It Up: Helping Children Learn Mathematics*. Washington, D.C.: The National Academies Press.

Kovalik, S. (1994). *Integrated thematic instruction: The model*. Kent, WA: Susan Kovalik & Associates.

Krebs, A. (2005). Analyzing student work as a professional development activity. *School Science and Mathematics*, 105(8), 402-411.

Larkin, M. J. (2001). Providing support for student independence through scaffolded instruction. *Teaching Exceptional Children, 34*(1), 30-34.

Lembke, E. & Foegen, A. (2005). *Indentifying indicators of early mathematics proficiency in kindergarten and grade 1*. (Technical Report No. 6). Minneapolis: University of Minnesota. College of Education and Human Development. Retrieved February 20, 2012 from www.progressmonitoring.org/pdf/TREarlymath6.pdf.

Lovitt, T.C., & Curtiss, K.A. (1968). Effects of manipulating an antecedent event on mathematic response rate. *Journal of Applied Behavior Analysis, 1*, 329-333.

MacMath, S., Wallace, J., & Chi, X. (2009). *Problem-based learning in mathematics: A tool for developing students' conceptual knowledge*. What Works? Research into Practice. Research Monograph #22. Ontario Ministry of Education. Retrieved May 4, 2012 from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/whatWorks.html

Maccini, P., & Gagnon, J. (2005). *Math graphic organizers for students with disabilities*. Washington, DC: American Institutes for Research.

Marzano, R.J., Pickering, D.J., & Pollock, J.E. (2001). *Classroom instruction that works: Research-based strategies for increasing student achievement*. Alexandria, VA: Association for Supervision and Curriculum Development (ASCD).

Matsumura, L.C., Slater, S.C., Junker, B., Peterson, M., Boston, M., Steele, M., et al. (2006). *Measuring reading comprehension and mathematics instruction in urban middle schools: A pilot study of the instructional quality assessment*. (CSE Technical Report 691). Los Angeles: University of California-Los Angeles. National Center for Research on Evaluation. Retrieved February 20, 2012 from www.cse.ucla.edu/products/reportsR681.pdf.

Mayer, R. E. (2001, 2009). *Multimedia learning*. Cambridge: Cambridge University Press.

Mayer, R. E. (2005). Principles for managing essential processing in multimedia learning. In R.E. Mayer (Ed.), *The Cambridge Handbook of Multimedia Learning* (pp. 169-182). New York: Cambridge University Press.

Mayes, R., Chase, P.N., & Walker, V.L. (2008). Supplemental practice and diagnostic assessment in an Applied College Algebra Course. *Journal of College Reading and Learning, 38*(2), 7-31.

Means, B., Toyama, Y., Murphy, R., Bakia, M., & Jones, K. (2009). *Evaluation of evidence-based practices in online learning: A meta-analysis and review of online learning studies*. Washington, DC: U.S. Department of Education, Office of Planning, Evaluation, and Policy Development, Policy and Program Studies Service. Retrieved February 28, 2012 from http://www2.ed.gov/rschstat/eval/tech/evidence-based-practices/finalreport.pdf.

Mellard, D.F., & Johnson, E.S. (2008). *RTI: A practitioner's guide to implementing response to intervention*. Thousand Oaks, CA: Corwin Press.

Mevarech, Z.R., & Kramarski, B. (1997). IMPROVE: A multidimensional method for teaching mathematics in heterogenous classrooms. *American Educational Research Journal, 34*, 365-394.

Mid-Continent Research for Education and Learning (McREL). (2010). *What we know about mathematics teaching and learning, third edition*. Bloomington, IN: Solution Tree Press.

Moskal, B. (2000). Understanding student responses to open-ended tasks. *Mathematics Teaching in the Middle School*, 5(8), 500-505.

National Council of Teachers of Mathematics. (2000, 2009). *Principals and standards for school mathematics*. Retrieved February 11, 2009, from the National Council of Teachers of Mathematics web site: http://standards.nctm.org/document/chapter1/index.htm

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education. Retrieved January 30, 3012 from http://www2.ed.gov/about/bdscomm/list/mathpanel/index.html.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Mathematics Learning Study Committee. Washington, DC: The National Academies Press. Retrieved February 21, 2012 from http://www.nap.edu/openbook.php?record_id=9822&page=1.

Newmann, F. M. (Ed.). (1992). *Student engagement and achievement in American secondary schools*. New York: Teachers College Press.

North Central Regional Educational Laboratory. (2003). *A meta-analysis of the effectiveness of teaching and learning with technology on student outcomes*. Naperville, IL: North Central Regional Educational Laboratory.

O'Neil, H.F., & Brown, R.S. (1998). Differential effects of question formats in math assessment on metacognition and affect. *Applied Measurement in Education, 11*(4), 331-351.

Park, S.Y. (2005). Student engagement and classroom variables in improving mathematics achievement. *Asia Pacific Education Review, 6*(1), 87-97.

Partnership for 21st Century Skills. (2009). *P21 Framework definitions*. Washington, DC: Author. Retrieved May 4, 2012 from http://www.p21.org/overview/skills-framework

Pugalee, D.K. (2004). A comparison of verbal and written descriptions of students' problem-solving processes. *Educational Studies in Mathematics, 55*, 27-47.

Pugalee, D.K. (2005). *Writing to develop mathematical understanding*. Norwood, MA: Christopher-Gordon.

Putnam, R. (2003). Commentary on four elementary math curricula. In S. Senk & D. Thompson (Eds.), *Standards-oriented school mathematics curricula: What does it say about student outcomes?* (pp. 161–180). Mahwah, NJ: Erlbaum.

Reed, D.S. (2009). Is there an expectations gap? Educational federalism and the demographic distribution of proficiency cut scores. *American Educational Research Journal*, 46(3), 718-742.

Reed, M.K. (1995). Making mathematical connections in middle school. Columbus, OH: ERIC Clearinghouse for Science Mathematics and Environmental Education.

Reys, R. E., Suydam, M. N., Lindquist, M. M., & Smith, N. L. (1998). *Helping Children Learn Mathematics*. Needham Heights, MA: Allyn & Bacon.

Rittle-Johnson, B., Siegler, R.S., & Alibali, M.W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, 93(2), 346-362.

Roh, K.H. (2003). *Problem-based learning in mathematics*. ERIC Digest. ERIC Clearinghouse for Science Mathematics and Environmental Education. Retrieved May 4, 2012 from http://www.ericdigests.org/2004-3/math.html

Rosenshine, B., & Meister, C. (1992). The use of scaffolds for teaching higher-level cognitive strategies. *Educational Leadership 49*(7), 26-33.

Russek, B. (1998). Writing to learn mathematics. *The WAC Journal, 9*, 36-45.

Russo, M., Hecht, D., Burghardt, M.D., Hacker, M., & Saxman, L. (2011). Development of a multidisciplinary middle school mathematics infusion model. *Middle Grades Research Journal, 6* (2), 113-128.

Schneider, M. (2007). *National Assessment of Educational Progress: Mapping 2005 state proficiency standards onto the NAEP scales*. Washington, DC: IES National Center for Education Statistics.

Scruggs, T.E., Mastropieri, M.A., & Sullivan, G.S. (1994). Promoting relational thinking skills: Elaborative interrogation for mildly handicapped students. *Exceptional Children, 60*, 450-457.

Shannon, G.S., & Bylsma, P. (2003). *Nine characteristics of high-performing schools: A research-based resource for school leadership teams to assists with the School Improvement Process*. Olympia, Washington: Office of the School Superintendent of Public Instruction.

Smith, E.S., & Johnson, L.A. (2011). Response to intervention in middle school: A case story. *Middle School Journal*, 42(3), 24-32.

Stecker, P.M., Fuchs, L.S., & Fuchs, D. (2005). Using curriculum-based measurement to improve student achievement: Review of research. *Psychology in the Schools*, 42, 795-820.

Stetson, R., Stetson, E., & Anderson, K. A. (2007). Differentiated instruction, from teachers' experiences. *The School Administrator, 8* (64), online. Retrieved February 21, 2012 from http://www.aasa.org/SchoolAdministratorArticle.aspx?id=6528.

Sullivan, G.S., Mastropieri, M.A., & Scruggs, T.E. (1995). Reasoning and remembering: Coaching thinking with students with learning disabilities. *Journal of Special Education, 29*, 310-322.

Surbeck, E. (1994). A glimpse through the door: Journal writing with preservice teachers. *Childhood Education, 70*(4), 232-235.

Taylor, J. A., & McDonald, C. (2007). Writing in groups as a tool for non-routine problem solving in first year university mathematics. *International Journal of Mathematical Education in Science and Technology, 38*(5), 639-655.

Teh, G. P. L., & Fraser, J. B. (1995). Gender differences in achievement and attitudes among students using computer-assisted instruction. *International Journal of Instructional Media, 22*(2), 111-120.

Tomlinson, C.A. (1997). Meeting the needs of gifted learners in the regular classroom: Vision or delusion? *Tempo*, 17(1), 1, 10-12.

Tomlinson, C.A. (1999). *The differentiated classroom: Responding to the needs of all learners*. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C.A., (2000). Reconcilable differences: Standards-based teaching and differentiation. *Educational Leadership*, 58, 6-13.

Tomlinson, C. A., (2001). *How to differentiate instruction in mixed-ability classrooms*. (2nd Ed.) Alexandria, VA: ASCD.

Tomlinson, C.A., (2005). Traveling the road to differentiation in staff development. *Journal of Staff Development*, 26, 8-12.

Tomlinson, C. A., & Allan, S. D. (2000). *Leadership for differentiating schools and classrooms*. Alexandria, VA: ASCD.

Urquhart, V. (2009). *Using writing in mathematics to deepen student learning*. Denver, CO: McREL. Retrieved May 8, 2012 from www.mcrel.org.

Valencia, S. W. (2007). Inquiry-oriented assessment. In Paratore, J. R., & McCormack, R. L. (Eds.), *Classroom literacy assessment: Making sense of what students know and do* (pp. 3-20). New York: Guilford.

Vygotsky, L.S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University.

Weglinsky, H. (1998). *Does it compute? The relationship between educational technology and student achievement in mathematics*. Princeton, NJ: Educational Testing Service Policy Information Center. Retrieved February 28, 2012 from http://www.ets.org/research/policy_research_reports/pic-technology.

Weiss, I., Kramarski, B., & Talis, S. (2006). Effects of multimedia environments on kindergarten children's mathematical achievements and style of learning. *Educational Media International, 43*(1), 3-17.

White, B. C., & Frederiksen, J. R. (1998). Inquiry, modeling, and metacognition: Making science accessible to all students. *Cognition and Instruction*. 16(1), 3-117.

Williams, K. M. (2003). Writing about the problem-solving process to improve problem-solving performance. *Mathematics Teacher, 96*(3), 185-187.

Williams, L. (2008). Tiering and scaffolding: Two strategies for providing access to important mathematics. *Teaching Children Mathematics, 14*(6), 324-330.

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