SAXON™ HOMESCHOOL

Algebra 1 (4th Edition), Geometry (1st Edition), Algebra 2 (4th Edition) Sampler

LESSON 49

Writing Equations in Slope-Intercept Form

Warm Up

1. (35) **Vocabulary** Which ordered pair could be the coordinates of the y-intercept of a graph?
A $(3, 0)$ **B** $(0, 4)$ **C** $(1, 2)$ **D** $(-1, -6)$

Solve for y.

2. (29) $x + 3y - 4 = 0$
3. (29) $3x - 2y - 7 = 0$

Find the y-value given the x-value.

4. (30) $y = 2x + 10;\ x = 4$
5. (30) $y = x^2;\ x = -9$

New Concepts

The slope-intercept form of a linear equation can be used to graph lines quickly because it gives information about the characteristics of the graph of the equation.

Math Language

The ***y*-intercept** is the y-value where a graph intersects the y-axis, and is usually represented by the variable b. The coordinate is $(0, b)$.

Slope-Intercept Form of an Equation
The **slope-intercept form** is $y = mx + b$, where the value of m is the slope of the line and the value of b is the y-intercept.

Example 1 Determining the Slope and y-Intercept of a Line

Determine the slope and the y-intercept of each equation.

a. $y = 3x - 4$

SOLUTION The equation is already written in slope-intercept form. The slope (m) is the coefficient of x, and the y-intercept is the constant value in the equation. Write the equation so that the operation with the constant is addition.

$y = 3x + (-4)$

slope: 3 y-intercept: -4

b. $2x + 3y - 9 = 0$

SOLUTION Isolate the variable y to write the equation in slope-intercept form.

$$2x + 3y - 9 = 0$$
$$\underline{-2x} \qquad \underline{-2x} \quad \text{Subtraction Property of Equality}$$
$$3y - 9 = -2x$$
$$\underline{+9} \qquad \underline{+9} \quad \text{Addition Property of Equality}$$
$$\frac{3y}{3} = \frac{-2x + 9}{3} \quad \text{Division Property of Equality}$$
$$y = -\frac{2}{3}x + 3$$

slope: $-\frac{2}{3}$ y-intercept: 3

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The **solution of an equation in two variables** is an ordered pair or set of ordered pairs that satisfies the equation. Solutions of equations in two variables can be represented in a table of values or as a graph on a coordinate plane.

Example 2 Graphing an Equation of a Line in Slope-Intercept Form

Graph each line using the equation that is in slope-intercept form.

a. $y = \frac{2}{3}x$

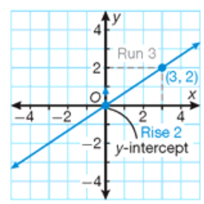
SOLUTION

Identify the slope and y-intercept from the equation. There is no constant, so the value of b is 0.

slope: $\frac{2}{3}$ y-intercept: 0

Graph the y-intercept on the coordinate plane at point (0, 0).

Use the value of the slope to plot another point on the line. The slope is $\frac{2}{3}$, so this means a rise of 2 over a run of 3. Starting at the y-intercept, (0, 0), move 2 units up and 3 units to the right. A second point on the line is (3, 2).



Draw a line through the two points.

b. $2x + y + 5 = 0$

SOLUTION

Write the equation in slope-intercept form.

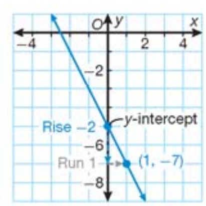
$y = -2x - 5$

Identify the slope and y-intercept from the equation.

slope: -2 y-intercept: -5

Graph the y-intercept on the coordinate plane at point $(0, -5)$.

Use the value of the slope to plot another point on the line. The slope is -2, or $-\frac{2}{1}$, so this means a negative rise of 2 over a run of 1. Starting at the y-intercept, $(0, -5)$, move 2 units down and 1 unit to the right. A second point on the line is $(1, -7)$.



Draw a line through the two points.

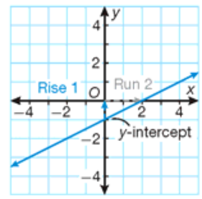
Hint

Slope is often written as $\frac{\text{rise}}{\text{run}}$ or "rise over run." When the value of m is a whole number, think about it in rational form. For example, $6 = \frac{6}{1}$.

You can use a graph to write an equation of the line in slope-intercept form.

Example 3 Writing the Equation of a Line from a Graph

a. Write the equation of the graphed line in slope-intercept form.



SOLUTION

Identify the y-intercept from the graph by identifying the y-value where the line crosses the y-axis.

y-intercept: -1

Identify the slope by determining how to move from one point to another.

rise $= 1$

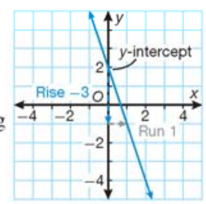
run $= 2$

slope: $\frac{1}{2}$

Write the equation in slope-intercept form.

$y = \frac{1}{2}x - 1$

b. Write the equation of the graphed line in slope-intercept form.



SOLUTION

Identify the y-intercept from the graph by identifying the y-value where the line crosses the y-axis.

y-intercept: 2

Identify the slope by determining how to move from one point to another.

rise $= -3$

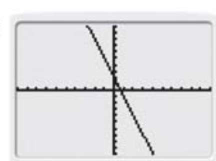
run $= 1$

slope: -3

Write the equation in slope-intercept form.

$y = -3x + 2$

Check Verify the solution by graphing the equation on a graphing calculator and comparing it to the original graph.



Math Reasoning

Generalize What is the slope of a horizontal line? What is the slope-intercept form of the equation of a horizontal line?

When working with a real-world application of linear equations, it is important to correctly identify all of the values. The y-intercept is often a starting value. The slope of a line is the rate of change, so the value that is the rate of change in the problem will be substituted for m.

Example 4 Calculating Rental Rates

Monica is helping prepare a budget for her family vacation. The family has decided to rent canoes for a day on the lake. The rental for a canoe is a \$25 flat fee plus \$10 per hour. Write an equation in slope-intercept form to represent this situation and then graph it.

SOLUTION

Define the variables.

Let y represent the total rental cost and x represent the number of hours of canoe rental.

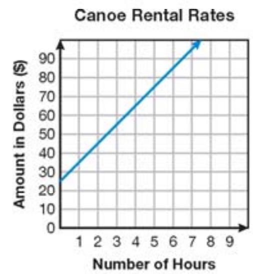
Identify the slope and y-intercept from the information in the problem.

The slope is 10 because it is the rate of change based on the number of hours the canoe is rented. The y-intercept is 25, the flat fee or cost, of renting the canoe for 0 hours.

Write the equation in slope-intercept form.

$$y = 10x + 25$$

Graph the equation of the line.



Math Reasoning

Analyze Why is the graph of the line shown only in the first quadrant?

Caution

Even though a linear equation is often used to represent a real-world problem, pay attention to restrictions on the domain and range.

Lesson Practice

Determine the slope and the y-intercept of the equation. *(Ex 1)*

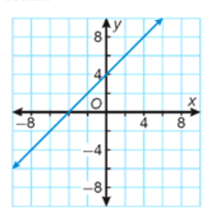
a. $y = 0.7x - 4.9$　　**b.** $-9x + 3y = 12$

Graph each line using the equation that is in slope-intercept form. *(Ex 2)*

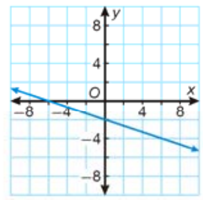
c. $y = \frac{3}{5}x$　　**d.** $x - 4y - 20 = 0$

Write the equation of the graphed line in slope-intercept form. *(Ex 3)*

e. Write the equation of the graphed line in slope-intercept form.



f. Write the equation of the graphed line in slope-intercept form



g. (Ex 4) Monica's family will be renting a car on their vacation. The car rents for an initial fee of \$50 plus \$0.50 per mile. Write a linear equation in slope-intercept form to represent this situation and then graph it.

Practice Distributed and Integrated

Expand each expression by using the Distributive Property.

1. (15) $x^2y^3(3xy - 5y)$

2. (15) $-2x^3y^3(4x^2y - 3xy)$

Evaluate each expression for the given values.

3. (16) x^2y^3z if $x = 3$, $y = -2$, and $z = 4$

4. (16) $-x^2 - y^3$ if $x = -3$ and $y = -2$

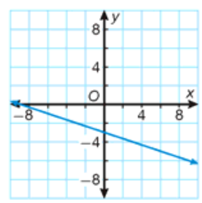
5. (35) Find the x-intercept of the line $3x + 2y - 10 = 0$.

*6. (49) Identify the slope and y-intercept of the line $2x - 5y - 6 = 0$.

*7. (49) **Multiple Choice** What is the equation of the graphed line?

A $y = -\frac{1}{3}x + 3$ B $y = -\frac{1}{3}x - 3$

C $y = -3x + 3$ D $y = -3x - 3$



*8. (49) **Multi-Step** The directions on a box of frozen biscuits state to cook one biscuit for 90 seconds on high and to add 15 seconds of cooking time for each additional biscuit.

a. Write a linear equation in slope-intercept form to represent the situation. Identify what the variables y and x represent.

b. **Analyze** What is the y-intercept of the graph? Does it have any meaning? Why or why not?

*9. (49) **Measurement Conversion** The formula for converting from Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$, where C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit. Identify the slope and the y-intercept of the equation and then graph it.

*10. (49) Pet Care The table shows a feeding chart on the bag of a certain brand of cat food.

Maximum Weight of Cat	Amount of Food Per Day
6 lb	$\frac{3}{4}$ cup
9 lb	$1\frac{1}{4}$ cups
12 lb	$1\frac{3}{4}$ cups

a. Write an equation in slope-intercept form.

b. **Write** The three points in the table show that this is a linear equation. Do you think this relationship would be a linear equation? Explain.

11. (48) Find the mean, median, and mode of the set of data below.
number of pockets in 8 pairs of pants: 4, 2, 0, 4, 5, 4, 2, 3

12. (48) **Error Analysis** Two students studied the table of data. Student A says that the median lowest temperature per month for San Diego, California, is 38.5°F. Student B says that the median is 52.5°F. Which student is correct? Explain the error.

Lowest Recorded Temperature in San Diego (in °F)			
July 2006	68	Jan 2007	35
Aug 2006	63	Feb 2007	45
Sep 2006	61	Mar 2007	45
Oct 2006	55	Apr 2007	50
Nov 2006	42	May 2007	55
Dec 2006	42	Jun 2007	58

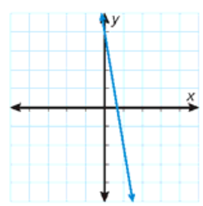
*13. (48) **Probability** Hari surveyed the first 15 students who came to class about the number of pets they owned. According to the data he collected, is the next person he surveys likely to have more than 4 pets? Explain.

2, 1, 3, 1, 1, 0, 2, 2, 3, 7, 0, 4, 2, 1, 1

14. (43) Simplify $\frac{x^2 - 12x}{2x^2 - x}$.

15. (42) Find 22% of 80 using a proportion.

16. (41) **Estimate** Approximate the slope of the line shown in the graph.

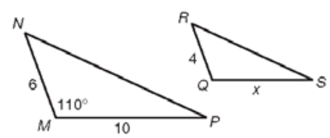


17. (28) Travel Quick Cab charges a \$5 fee for a ride, plus \$0.15 for every block. Speedy Cab charges \$7 for a ride, but only \$0.05 every block. For what number of blocks is the cost of travel the same?

18. (33) A deck of cards contains 15 cards, five of each number 1, 2, and 3. What are the odds of getting a 1? What are the odds against getting a 3?

19. (34) Write a recursive formula for the arithmetic sequence with $a_1 = -3$ and common difference $d = 9$. Then find the first four terms of the sequence.

20. (36) **Multi-Step** Use the similar triangles shown.



a. What angle corresponds with $\angle M$?

b. Find $m\angle Q$.

c. What is the scale factor of the triangles?

d. Use the scale factor to find the value of x.

21. (48) **Tourism** The number of visitors to national parks in the United States each year for the years 1990 to 2005 (in millions) are as follows: 258.7, 267.8, 274.7, 273.1, 268.6, 269.6, 265.8, 275.3, 286.7, 287.1, 285.9, 279.9, 277.3, 266.1, 276.9, 273.5. Find the range of the data.

***22.** (48) **Geometry** Ferdinand found the areas of seven triangles. This was his data set: 7 cm², 8 cm², 12 cm², 6 cm², 7 cm², 10 cm², 6 cm². If the triangle at right has the same area as the mean value in the data set, what is the height (h) of the triangle?

23. (47) **Error Analysis** A box of chocolates sells for \$12.50. The price of the chocolates decreased by 15% from last year. Student A and Student B tried to find the price from last year. Which student is correct? Explain the error.

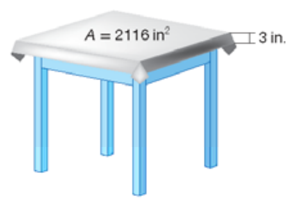
Student A	Student B
$0.15n = \$12.50$	$0.85n = \$12.50$
$\frac{0.15n}{0.15} = \frac{\$12.50}{0.15}$	$\frac{0.85n}{0.85} = \frac{\$12.50}{0.85}$
$n = \$83.33$	$n = \$14.71$

***24.** (46) **Multi-Step** A square table has an area of 2116 square inches. A tablecloth hangs 3 inches over each end of the table.

a. What is the length of one side of the table?

b. What is the area of the tablecloth?

c. What is the area of the part of the tablecloth that hangs over the table?



***25.** (46) **Analyze** Copy and complete the statement with $\sqrt[4]{\sqrt{256}}$ ___?___ $\sqrt{\sqrt[4]{256}}$ >, <, or =.

26. (47) Find the percent of increase or decrease to the nearest percent from an original price of \$20 to a new price of \$23.

27. (45) Write an inequality for the following sentence: The sum of twice a number and $\frac{1}{3}$ is less than $1\frac{2}{3}$.

***28.** (44) **Remote Sensing** Satellites perform sophisticated data collection, but over time their orbits decay, affecting their measurements. In 1980 the NOAA-06 satellite was holding an orbit of 826 km. By 1996 the orbit had decayed to 804 km. What was the average rate of orbital decay for NOAA-06?

29. (Inv 4) **Analyze** Determine if the following statement uses inductive or deductive reasoning. Explain your choice.

"If Sharon made at least one goal at each of her last soccer games, then she will make at least one goal at her next soccer game.

30. (40) **Multi-Step** A leather footstool in the shape of a cube is being made for a living room. The length of one side of the footstool is $2x$ inches.

a. What is the area of one side of the footstool?

b. What is the surface area of leather needed to make the footstool?

LESSON 59

Solving Systems of Linear Equations by Substitution

Warm Up

1. (55) **Vocabulary** A (n) ________ to a system of linear equations is an ordered pair or set of ordered pairs that satisfies all the equations in the system.

2. (55) Is $(-1, 3)$ a solution to the system below?

$3x + 2y = 3$

$x - 3y = -10$

Solve each equation.

3. (23) $3x + 7 = 5x - 28$

4. (26) $5x + 12 = 3x + 36$

New Concepts

To be a solution to a system of equations, an ordered pair must satisfy both equations. One method for finding solutions to systems of equations is to use the substitution method.

Steps for Solving by Substitution
1. Rearrange one of the equations so that it is of the form $y = mx + b$, or $x = my + b$, if necessary.
2. Substitute the equivalent expression for the variable from the first step into the second equation of the system. The result is an equation with one unknown.
3. Solve the resulting equation from the second step for the variable.
4. Substitute the value of the variable from the third step into one of the original equations to find the value of the other unknown.
5. Write the values of the unknowns as an ordered pair.

Example 1 Using Substitution

Solve the system of equations by substitution.

$y = 2x - 5$

$y = 5x + 7$

SOLUTION

$y = 5x + 7$	Write the second equation.
$2x - 5 = 5x + 7$	Substitute $2x - 5$ for y.
$-5 = 3x + 7$	Subtract $2x$ from both sides.
$-12 = 3x$	Subtract 7 from both sides.
$-4 = x$	Divide both sides by 3.

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The value of x in the solution is -4. The y-value can be found by substituting -4 into either of the original equations.

First Equation	Second Equation
$y = 2x - 5$	$y = 5x + 7$
$y = 2(-4) - 5$	$y = 5(-4) + 7$
$y = -8 - 5$	$y = -20 + 7$
$y = -13$	$y = -13$

The solution to the system is $(-4, -13)$.

Example 2 Using the Distributive Property

Hint

You can substitute for x or y. Choose the equation that is already solved for a variable.

Solve the system of equations by substitution. Check your answer.

$12x - 6y = 12$

$x = -2y + 11$

SOLUTION

$12x - 6y = 12$	Write the first equation.
$12(-2y + 11) - 6y = 12$	Substitute $-2y + 11$ for x.
$-24y + 132 - 6y = 12$	Distribute.
$-30y + 132 = 12$	Combine like terms.
$-30y = -120$	Subtract 132 from both sides.
$y = 4$	Divide both sides by -30.

The value of y in the solution is 4. The x-value can be found by substituting 4 into either of the original equations.

First Equation	Second Equation
$12x - 6y = 12$	$x = -2y + 11$
$12x - 6(4) = 12$	$x = -2(4) + 11$
$12x - 24 = 12$	$x = -8 + 11$
$12x = 36$	$x = 3$
$x = 3$	

The solution to the system is $(3, 4)$.

Check To determine whether the ordered pair satisfies both of the original equations, substitute the x-value and y-value into the original equations.

First Equation	Second Equation
$12x - 6y = 12$	$x = -2y + 11$
$12(3) - 6(4) \stackrel{?}{=} 12$	$3 \stackrel{?}{=} -2(4) + 11$
$36 - 24 \stackrel{?}{=} 12$	$3 \stackrel{?}{=} -8 + 11$
$12 = 12$ ✓	$3 = 3$ ✓

Hint

Use the variable with the coefficient of 1 whenever possible.

So far, every system of equations has had at least one equation of the form $y = mx + b$ or $x = my + b$. If neither equation is in this form, the first step is to rearrange one of the equations.

Example 3 Rearrange Before Substitution

a. Solve the system of equations by substitution. Check your answer.

$$2x + y = -4$$
$$5x - 2y = -1$$

SOLUTION

Because neither equation is in a form that can be used for substitution, one equation will need to be rearranged. The first equation can be rearranged to be in the form $y = mx + b$ easily because the coefficient of y is 1.

$2x + y = -4$	Write the first equation.
$y = -2x - 4$	Subtract 2x from both sides.

Caution

Be sure to distribute the negative sign with the number.

Now substitute $-2x - 4$ for y in the second equation.

$5x - 2y = -1$	Write the second equation.
$5x - 2(-2x - 4) = -1$	Substitute $-2x - 4$ for y.
$5x + 4x + 8 = -1$	Distribute.
$9x + 8 = -1$	Combine like terms.
$9x = -9$	Subtract 8 from both sides.
$x = -1$	Divide both sides by 9.

The value of x in the solution is -1. The y-value can be found by substituting -1 into either of the original equations.

First Equation	Second Equation
$2x + y = -4$	$5x - 2y = -1$
$2(-1) + y = -4$	$5(-1) - 2y = -1$
$-2 + y = -4$	$-5 - 2y = -1$
$y = -2$	$-2y = 4$
	$y = -2$

The solution to the system is $(-1, -2)$.

Hint

When neither variable has a coefficient of 1, choose a variable with a small coefficient or one that easily divides into the other coefficients and constants in the equation.

Check To check that this ordered pair satisfies both of the original equations, substitute the x-value and y-value into the original equations.

First Equation	Second Equation
$2x + y = -4$	$5x - 2y = -1$
$2(-1) + (-2) \stackrel{?}{=} -4$	$5(-1) - 2(-2) \stackrel{?}{=} -1$
$-2 + (-2) \stackrel{?}{=} -4$	$-5 + 4 \stackrel{?}{=} -1$
$-4 = -4$ ✓	$-1 = -1$ ✓

b. Solve the system of equations by substitution. Check your answer.

$$4x + 7y = 43$$
$$2x - 3y = -11$$

SOLUTION

Again, one equation will need to be rearranged so that it can be used for substitution. None of the variables have a coefficient of 1, so solve the second equation for x.

$2x - 3y = -11$	Write the second equation.
$2x = 3y - 11$	Add $3y$ to both sides.
$x = \frac{3}{2}y - \frac{11}{2}$	Divide both sides by 2.

Now substitute $\frac{3}{2}y - \frac{11}{2}$ for the x in the first equation.

$4x + 7y = 43$	Write the first equation.
$4\left(\frac{3}{2}y - \frac{11}{2}\right) + 7y = 43$	Substitute $\frac{3}{2}y - \frac{11}{2}$ for x.
$\frac{12}{2}y - \frac{44}{2} + 7y = 43$	Distribute.
$6y - 22 + 7y = 43$	Simplify.
$13y - 22 = 43$	Combine like terms.
$13y = 65$	Add 22 to both sides.
$y = 5$	Divide both sides by 13.

Math Reasoning

Write When substituting the value of one variable to find the value of the other, only one equation needs to be used. So, why must both equations be used to check the answer?

The value of y in the solution is 5. Substitute 5 into either of the original equations to find x.

First Equation	Second Equation
$4x + 7y = 43$	$2x - 3y = -11$
$4x + 7(5) = 43$	$2x - 3(5) = -11$
$4x + 35 = 43$	$2x - 15 = -11$
$4x = 8$	$2x = 4$
$x = 2$	$x = 2$

The solution to the system is (2, 5).

Check To determine whether the ordered pair satisfies both of the original equations, substitute the x-value and the y-value into the original equation.

First Equation	Second Equation
$4x + 7y = 43$	$2x - 3y = -11$
$4(2) + 7(5) \stackrel{?}{=} 43$	$2(2) - 3(5) \stackrel{?}{=} -11$
$8 + 35 \stackrel{?}{=} 43$	$4 - 15 \stackrel{?}{=} -11$
$43 = 43$ ✓	$-11 = -11$ ✓

> **Hint**
>
> To determine how to define the variables, look at the question. The question will indicate what needs to be found; that is, what is unknown. Variables represent the unknown.

Example 4 Application: Play Tickets

A school play charged adults \$8 and students \$5 for tickets. There were 75 people who attended the play. The box office collected \$444. How many adults and how many students attended the play? Use substitution to solve.

SOLUTION

First define the variables.

Let x = number of adults.

Let y = number of students.

Translate the situation into a system of equations.

$x + y = 75$	Adults plus students equals the total number of people.
$8x + 5y = 444$	The total amount of money collected can be found by multiplying the cost of the ticket by the number of people buying that kind of ticket.

Now use substitution to solve the system.

$y = -x + 75$	Solve the first equation for y.
$8x + 5(-x + 75) = 444$	Substitute $-x + 75$ for y in the second equation.
$8x - 5x + 375 = 444$	Distribute.
$3x + 375 = 444$	Combine like terms.
$3x = 69$	Subtract 375 from both sides.
$x = 23$	Divide both sides by 3.
$23 + y = 75$	Substitute $x = 23$ into the first equation.
$y = 52$	Subtract 23 from both sides.

The solution is (23, 52). There were 23 adults and 52 students at the play.

Check

First Equation	Second Equation
$x + y = 75$	$8x + 5y = 444$
$23 + 52 \stackrel{?}{=} 75$	$8(23) + 5(52) \stackrel{?}{=} 444$
$75 = 75$ ✓	$184 + 260 \stackrel{?}{=} 444$
	$444 = 444$ ✓

Lesson Practice

Solve each system of equations by substitution. Check your answer.

a. (Ex 1) $y = 4x - 3$
$y = 3x - 5$

b. (Ex 2) $x = 3y - 11$
$5x + 2y = -4$

c. (Ex 3) $4x + 3y = 2$
$2x + y = 6$

d. (Ex 3) $4x + 3y = 19$
$7x - 6y = -23$

e. (Ex 4) **Finance** The cost of 5 books and 10 pencils is \$36. The cost of 2 books and 40 pencils is \$18. How much do books cost and how much do pencils cost?

Practice Distributed and Integrated

Solve.

1. (28) $-[-(-k)] - (-2)(-2 + k) = -k - (4k + 3)$

2. (26) $\frac{1}{3} + 5\frac{1}{3}k + 3\frac{2}{9} = 0$

3. (13) Simplify $\sqrt{9} + \sqrt{16} - \sqrt{225}$.

4. (1) Give an example of a rational number that is not an integer.

5. (13) True or False: The number $\sqrt{49}$ is a rational number.

***6.** (59) Solve by substitution: $y = 3x - 5$
$y = -2x + 15$.

***7.** (59) Solve by substitution: $y = -8x + 21$
$y = -3x + 6$.

***8.** (59) **Write** How do you know if a point is a solution to a system of equations?

***9.** (59) **Multiple Choice** Which ordered pair is a solution to the system of equations?

$$4x + 9y = 75$$
$$8x + 6y = 66$$

A (3, 7)
B (0, 11)
C (10, 4)
D (12, −5)

***10.** (59) The sum of two numbers is 64. Their difference is 14. Find each of the numbers.

11. (58) **Error Analysis** Students were asked to find the product of $2b^2(b^3 + 4)$. Which student is correct? Explain the error.

Student A	Student B
$2b^2(b^3 + 4)$	$2b^2(b^3 + 4)$
$2b^2(b^3) + 2b^2(4)$	$2b^2(b^3) + 2b^2(4)$
$2b^5 + 8b^2$	$2b^6 + 8b^2$

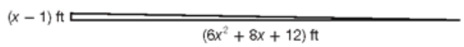
*12. (58) **Geometry** The length of a rectangular pool is four times the width. A four-foot-wide deck surrounds the pool. Write a polynomial expression for the area of the pool and deck. Use the Distributive Property and write your answer in standard form.

*13. (58) **Multi-Step** Henry has a game that includes a number cube. The side length of the cube is $(5x + 1)$ inches. Find the volume of the number cube.

a. Find the area of the base. Multiply the length times the width.

b. Find the volume by multiplying the product found in part **a** by the height.

*14. (58) **Measurement** Tim has a garden in the shape of a right triangle. The triangle has a base of $6x^2 + 8x + 12$ feet and a height of $(x - 1)$ feet. What is the area of Tim's garden?



15. (58) Find the product of $4x(x^2 + 2x - 9)$.

16. (49) Write an equation of the line in slope-intercept form. The slope is 2; the y-intercept is -1.

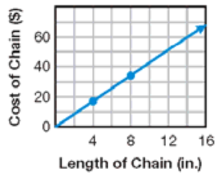
17. (53) What is the degree of $12x^4x^3 + 6xy + 41x^2y^3$?

18. (54) **Baseball** The percentage of games won is used to determine a team's standing in the league. In the American League, the following percentages were recorded:

0.604, 0.540, 0.505, 0.465, 0.380, 0.600, 0.584, 0.505, 0.446, 0.430, 0.580, 0.545, 0.475, 0.451

Make a box-and-whisker plot of these percentages and determine if there is an outlier.

*19. (56) **Estimate** The cost of a chain is directly proportional to its length. Use the graph to estimate the cost of a chain that is 18 inches long.



20. (56) **Multiple Choice** Which point represents the same direct variation as $(3, -9)$?

A $(4, -8)$ **B** $(4, -7)$ **C** $(4, -12)$ **D** $(4, -16)$

21. Find the LCM of $16c^6$ and $24c^3$.
(57)

*22. **Games** Every $20x^3y$ turns, you win \$500. Every $12xy^3c$ turns, you get to roll again. How many turns do you take before you win \$500 and get to roll again on the same turn?
(57)

23. Find the LCM of $300d^2$ and $90d^4$.
(57)

24. **Formulate** The equations $y = 3x - 24$ and $y = 24x + 9$ define two lines. Write a rational expression that represents the ratio of the first line to the second line. Simplify the expression, if possible.
(43)

25. Determine the slope of the line that goes through the points $(-1, 1)$ and $(1, -1)$.
(44)

26. **Biology** Write a counterexample for the following statement: If an animal has wings, then the animal is an insect.
(Inv 4)

27. **Multi-Step** A canister of oatmeal has a height of 7 inches. Its volume is 28π cubic inches.
(46)

a. Write an equation you can use to find the radius of the cylinder.

b. Find the radius of the canister.

28. **Astronomy** The relative gravity on Jupiter is 2.34. This means that the weight of an object on Jupiter is 2.34 times greater than its weight on Earth. Identify the slope and the y-intercept of the equation representing this relationship and then write an equation for the situation in slope-intercept form.
(49)

29. **Verify** Write the converse of the following statement: If a number is an integer, then it is a rational number. Give an example to show that the converse is false.
(Inv 5)

30. **Write** What is an excluded value for a rational expression? Why is it excluded?
(51)

LESSON
100 Solving Quadratic Equations by Graphing

Warm Up

1. (84) **Vocabulary** The U-shaped curve that results from graphing a quadratic function is called a(n) ________.

Evaluate each expression for the given values.

2. (9) $3(x - y)^2 - 4y^2$ for $x = -5$ and $y = -2$
3. (9) $-x^2 - 3xy + y$ for $x = 3$ and $y = -1$

Determine the direction that the parabola opens.

4. (84) $f(x) = 3x^2 + x - 4$
5. (84) $f(x) = -2x^2 + x + 1$

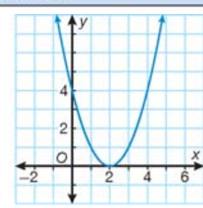
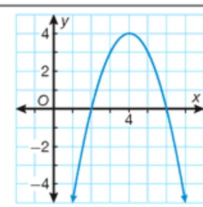
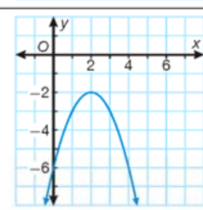
New Concepts

The solution(s) of a quadratic equation, $0 = ax^2 + bx + c$, can be found by graphing the related function, $f(x) = ax^2 + bx + c$. The U-shaped graph of a quadratic function is called a parabola. The solutions of the equation are called roots and can be found by determining the x-intercepts or zeros of the quadratic function. These zeros can be found by graphing the related function to see where the parabola intersects the x-axis.

Math Language

The same function is described by $y = 3x^2 - 5$ and $f(x) = 3x^2 - 5$.

The **function notation** for y is $f(x)$. It is read, "f of x."

Graphical Solutions	
One Real Solution The graph intersects the x-axis at the vertex.	
Two Real Solutions The graph intersects the x-axis at two distinct points.	
No Real Solutions The graph does not intersect the x-axis.	

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Example 1 Solving Quadratic Equations by Graphing

Solve each equation by graphing the related function.

a. $x^2 - 36 = 0$

SOLUTION

Step 1: Find the axis of symmetry.

$x = -\frac{b}{2a}$ Use the formula.

$x = -\frac{0}{2(1)} = 0$ Substitute values for a and b.

The axis of symmetry is $x = 0$.

Step 2: Find the vertex.

$f(x) = x^2 - 36$

$f(0) = (0)^2 - 36$ Evaluate the function for $x = 0$ to find the vertex.

The vertex is $(0, -36)$.

Step 3: Find the y-intercept.

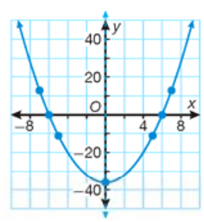
The y-intercept is c, or -36.

Step 4: Find two more points that are not on the axis of symmetry.

$f(5) = 5^2 - 36$ $(5, -11)$

$f(7) = 7^2 - 36$ $(7, 13)$

Step 5: Graph.



Graph the axis of symmetry $x = 0$, the vertex and the y-intercept, both at coordinate $(0, -36)$. Reflect the points $(5, -11)$ and $(7, 13)$ over the axis of symmetry and graph the points $(-5, -11)$ and $(-7, 13)$. Connect the points with a smooth curve.

From the graph, the x-intercepts appear to be 6 and -6.

Check Substitute the values for x in the original equation.

$x^2 - 36 = 0; x = 6$

$(6)^2 - 36 \stackrel{?}{=} 0$

$36 - 36 \stackrel{?}{=} 0$

$0 = 0$ ✓

$x^2 - 36 = 0; x = -6$

$(-6)^2 - 36 \stackrel{?}{=} 0$

$36 - 36 \stackrel{?}{=} 0$

$0 = 0$ ✓

The solutions are 6 and -6.

Hint

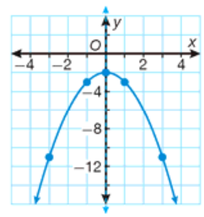
When the coefficient of the x^2-term is positive, the parabola will open upward.

When the coefficient of the x^2-term is negative, the parabola will open downward.

Math Reasoning

Write Why are the x-intercepts substituted into the original equation?

b. $-x^2 - 2 = 0$



SOLUTION

Graph the related function $f(x) = -x^2 - 2$.

axis of symmetry:	$x = 0$
vertex:	$(0, -2)$
y-intercept:	$(0, -2)$
two additional points:	$(1, -3)$ and $(3, -11)$

Reflect these two points across the axis of symmetry and connect them with a smooth curve.

From the graph, it can be seen that there is no x-intercept because the graph does not intersect the x-axis.

There is no real-number solution.

Caution

When a parabola does not cross the x-axis, there is no real-number solution to the quadratic equation.

c. $x^2 + 16 = 8x$

SOLUTION

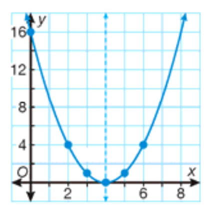
Write the equation in standard form.

$x^2 - 8x + 16 = 0$

Graph the related function $f(x) = x^2 - 8x + 16$.

axis of symmetry:	$x = 4$
vertex:	$(4, 0)$
y-intercept:	$(0, 16)$
two additional points:	$(2, 4)$ and $(3, 1)$

Reflect these two points across the axis of symmetry and connect them with a smooth curve.



From the graph, the x-intercept appears to be 4.

Check Substitute 4 for x in the original equation.

$$x^2 - 8x + 16 = 0;\ x = 4$$
$$(4)^2 - 8(4) + 16 \stackrel{?}{=} 0$$
$$16 - 32 + 16 \stackrel{?}{=} 0$$
$$0 = 0 \ \checkmark$$

The solution is 4.

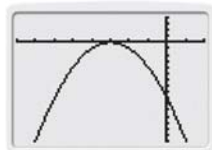
Example 2 Solving Quadratic Equations Using a Graphing Calculator

Solve each equation by graphing the related function on a graphing calculator.

> **Hint**
>
> For help with graphing quadratic functions, see Graphing Calculator Lab 8: Characteristics of Parabolas on p. 583.

a. $-6x - 9 = x^2$

SOLUTION



Write the equation in standard form.

$-x^2 - 6x - 9 = 0$

Graph the related function

$f(x) = -x^2 - 6x - 9.$

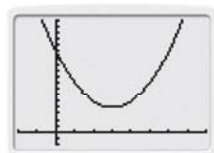
The graph appears to have an x-intercept at -3.

Use the Table function to determine the zeros of this function.

The solution is -3.

b. $-6x = -x^2 - 13$

SOLUTION



Write the equation in standard form.

$x^2 - 6x + 13 = 0.$

Graph the related function $f(x) = x^2 - 6x + 13.$

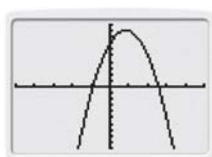
The graph opens upward and does not intersect the x-axis.

There is no solution.

c. $-3x^2 + 5x = -7$

Round to the nearest tenth.

SOLUTION



Write the equation in standard form.

$-3x^2 + 5x + 7 = 0$

Graph the related function $f(x) = -3x^2 + 5x + 7.$

The graph appears to have x-intercepts at 3 and -1.

Use the Zero function to determine the zeros of this function. Round to the nearest tenth.

The solutions are $x = 2.6$ and -0.9.

Hint

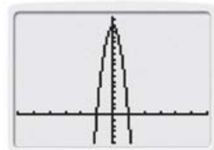
The time t is plotted on the x-axis. The height h is plotted on the y-axis.

Example 3 Application: Physics

Gill drops a baseball from the top of a platform 64 feet off the ground. The height of the baseball is described by the quadratic equation $h = -16t^2 + 64$, where h is the height in feet and t is the time in seconds. Find the time t when the ball hits the ground.

SOLUTION

Graph the related function $h(t) = -16t^2 + 64$ on a graphing calculator.



Height h is zero when the ball hits the ground. Use the Zero function of the graphing calculator to determine the zeros of this function.

There are two zeros for the given parabola: $t = 2$ and $t = -2$. Only values greater than or equal to zero are considered. So, $t = 2$ is the only solution.

The baseball hits the ground in 2 seconds.

Lesson Practice

Solve each equation by graphing the related function. (Ex 1)

a. $3x^2 - 147 = 0$

b. $5x^2 + 6 = 0$

c. $x^2 - 10x + 25 = 0$

Solve each equation by graphing the related function on a graphing calculator.

d. (Ex 2) $x^2 + 64 = 16x$

e. (Ex 2) $x^2 + 4 = 2x$

f. (Ex 2) Round to the nearest tenth: $-7x^2 + 3x = -7$.

g. (Ex 3) Marcus shot an arrow while standing on a platform. The path of its movement formed a parabola given by the quadratic equation $h = -16t^2 + 2t + 17$, where h is the height in feet and t is the time in seconds. Find the time t when the arrow hits the ground. Round to the nearest hundredth.

Practice Distributed and Integrated

Solve.

1. (98) $x(2x - 11) = 0$

2. (99) $\frac{12}{x-6} = \frac{4}{x}$

***3.** (100) **Generalize** Using the path of a ball thrown into the air as an example, describe in mathematical terms each part of the graph the path of the ball creates.

***4.** (100) **Generalize** What does the graph of a quadratic equation look like when there is no solution? one solution? two solutions?

5. (Inv 7) Given that y varies directly with x, identify the constant of variation such that when $x = 15$, $y = 30$.

***6.** (100) **Basketball** Ramero shoots a basketball into the air. The ball's movement forms a parabola given by the quadratic equation $h = -16t^2 + 7t + 7$, where h is the height in feet and t is the time in seconds. Find the maximum height of the path the basketball makes and the time t when the basketball hits the ground. Round to the nearest hundredth.

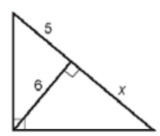
***7.** (100) **Multiple Choice** What is the equation of the axis of symmetry of the parabola defined by $y = \frac{1}{4}(x - 4)^2 + 5$?
A $x = 1$ **B** $x = 4$ **C** $x = 5$ **D** $x = -4$

***8.** (100) Solve $-7x^2 - 10 = 0$ by graphing.

***9.** (99) Solve $\frac{6}{x} = \frac{8}{x + 7}$.

10. (80) A deck of ten cards has 5 red and 5 black cards. Cards are replaced in the deck after each draw. Use an equation to find the probability of drawing a black card twice and rolling a 6 on a number cube.

11. (99) **Geometry** The altitude of the right triangle divides the hypotenuse into segments of lengths x units and 5 units. To find x, solve the equation $\frac{x+5}{6} = \frac{6}{x}$.



***12.** (99) **Multi-Step** Henry starts working a half-hour before Martha. He can complete the job in 4 hours. Martha can complete the same job in 3 hours.

a. Let t represent the total time they work together. In terms of t, how long does Henry work?

b. Use an equation to find how long they work together to complete the job.

c. How long does Henry work?

13. (93) Find the quotient of $\frac{a^2 + 10a - 24}{a - 2}$.

14. (61) Simplify $\sqrt{49y^5}$.

15. (94) **Profit** An entrepreneur makes \$3 profit on each object sold. She would like to make \$270 plus or minus \$30 total. What is the minimum and maximum number of objects she needs to sell?

16. (94) **Data Analysis** A student knows there will be 4 tests that determine her semester grade. She wants her average to be an 85, plus or minus 5 points. What is the minimum and maximum number of points she needs to earn during the semester?

17. (94) Solve the equation $|10x| - 3 = 87$.

18. (95) **Exercise** Tom ran a total of $\frac{7x}{x^2 + 3x - 18}$ miles in August and $\frac{2x + 1}{7x + 42}$ miles in September. How many more miles did he run in August?

19. (96) Graph the function $y = 5x^2 - 10x + 5$.

***20.** (97) **Verify** A boundary line is a vertical line. The inequality contains a $<$ symbol. Which half-plane should be shaded on the graph?

21. (97) **Multiple Choice** Which point does not satisfy the inequality $x + 2y < 5$?
A $(0, 0)$ **B** $(2, 1)$ **C** $(3, -4)$ **D** $(-1, 3)$

***22.** (98) **Ages** A boy is b years old. His father is 23 years older than the boy. The product of their ages is 50. How old is each person?

***23.** (98) **Error Analysis** Two students find the roots of $3x^2 - 6x = 24$. Which student is correct? Explain the error.

Student A	Student B
$3x^2 - 6x = 24$	$3x^2 - 6x = 24$
$3x(x - 2) = 24$	$3x^2 - 6x - 24 = 0$
$3x = 0 \quad x - 2 = 0$	$3(x^2 - 2x - 8) = 0$
$x = 0 \quad x = 2$	$3(x - 4)(x + 2) = 0$
	$x - 4 = 0 \quad x + 2 = 0$
	$x = 4 \quad x = -2$

24. (84) Does the graph of $y + 2x^2 = 12 + x$ open upward or downward?

25. (85) Do the side lengths 18, 80, and 82 form a Pythagorean triple?

26. (87) **Multi-Step** The volume of a prism is $3x^3 + 12x^2 + 9x$. What are the possible dimensions of the prism?

a. Factor out common terms.

b. Factor completely.

c. Find the dimensions.

27. (90) **Travel** The Jackson family drove 480 miles on Saturday and 300 miles on Sunday. Their average rate on Sunday was 10 miles per hour less than their rate was on Saturday. Write a simplified expression that represents their total driving time.

28. (91) **Multi-Step** At the carnival, a man says that he will guess your weight within 5 pounds.

a. You weigh 120 pounds. Write an absolute-value inequality to show the range of acceptable guesses.

b. Solve the inequality to find the actual range of acceptable guesses.

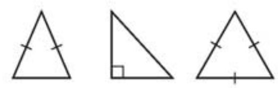
29. (92) **Verify** If the numerator of a rational expression is a polynomial and the denominator of the rational expression is a different polynomial, will factoring the polynomials always provide a way to simplify the expression? Verify your answer by giving an example.

30. (47) If a 9% decrease from the original price resulted in a new price of \$227,500, what was the original price?

LESSON
15 Introduction to Polygons

Warm Up

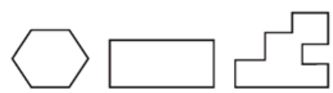
1. **Vocabulary** A and B are the ________ of $\overline{AB}$. (2)
2. How many endpoints does a ray have? (3)
3. Classify each of the triangles in the diagram by both sides and angles. (13)



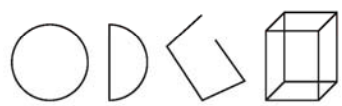
New Concepts

A **polygon** is a closed plane figure formed by three or more segments. Each segment intersects exactly two other segments only at their endpoints. No two segments with a common endpoint are collinear.

The segments that form a polygon are called its **sides**. A **vertex of a polygon** is the intersection of two of its sides.



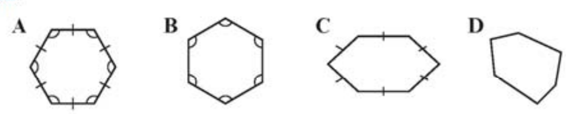
Polygons



Not Polygons

> **Hint**
>
> Equilateral and equiangular polygons have the same traits as equilateral and equiangular triangles, as introduced in lesson 13.

An **equiangular polygon** is a polygon in which all angles are congruent. An **equilateral polygon** is a polygon in which all sides are congruent. If a polygon is both equiangular and equilateral, then it is called a **regular polygon**. If a polygon is not equiangular and equilateral, then it is called an **irregular polygon**.



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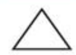
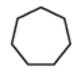
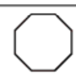
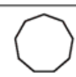
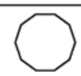
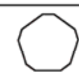
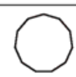
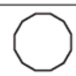
In the diagram, polygons A and B are equiangular. Polygons A and C are equilateral. Since polygon A is both equiangular and equilateral, it is a regular polygon. Polygons B, C and D are all irregular.

Polygons are named by the number of sides they have. The chart below shows some common polygons and their names.

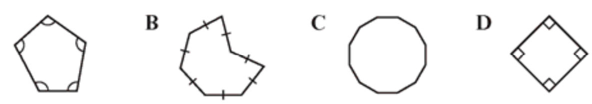
Math Language

An ***n*-gon** is a polygon with *n* sides. Problems may refer to *n*-gons when the number of sides of a polygon is not known, or when a solution is desired for all possible polygons.

Name	Sides	Regular Polygon	Irregular Polygon
Triangle	3		
Quadrilateral	4		
Pentagon	5		
Hexagon	6		
Heptagon	7		
Octagon	8		
Nonagon	9		
Decagon	10		
Hendecagon	11		
Dodecagon	12		

Example 1 Classifying Polygons

Classify each polygon. Determine whether it is equiangular, equilateral, regular, irregular, or more than one of these.



SOLUTION

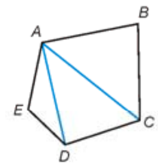
Polygon **A** has 5 sides, so it is a pentagon. It is equiangular but not equilateral, so it is irregular.

Polygon **B** has 7 sides, so it is a heptagon. It is equilateral and irregular.

Polygon **C** is a dodecagon. It is irregular.

Polygon **D** is a quadrilateral. It is equilateral and equiangular, so it is regular.

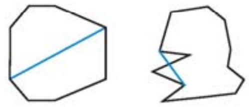
A **diagonal of a polygon** is a segment that connects two nonconsecutive vertices of a polygon. For example, pentagon $ABCDE$ has two diagonals, $\overline{AC}$ and $\overline{AD}$, from vertex A. Three other diagonals could be drawn: $\overline{BD}$, $\overline{BE}$, and $\overline{CE}$.



Math Language

The **exterior of a polygon** is the set of all points outside a polygon. The **interior of a polygon** is the set of all points inside a polygon.

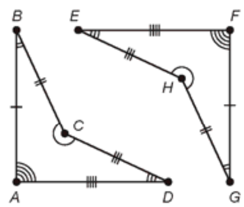
Diagonals can help determine whether a polygon is concave or convex. In a **convex polygon**, every diagonal of the polygon lies inside it, except for the endpoints. In a **concave polygon**, at least one diagonal can be drawn so that part of the diagonal contains points in the exterior of the polygon.



If two polygons have the same size and shape, they are **congruent polygons**.

Example 2 Identifying Polygon Properties

a. Find a diagonal that contains points in the exterior of polygon $ABCD$.



SOLUTION

Diagonal $\overline{BD}$ lies outside polygon $ABCD$, except for its endpoints.

b. Determine whether polygon $EFGH$ is convex or concave. Explain.

SOLUTION

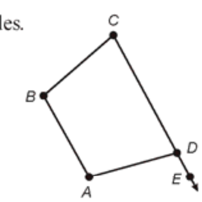
Diagonal $\overline{EG}$ contains points in the exterior of polygon $EFGH$. Therefore, polygon $EFGH$ is concave.

c. Are polygons $ABCD$ and $FGHE$ congruent? Justify your answer.

SOLUTION

Write a congruency statement for all corresponding sides and angles. Angle pairs $\angle A \cong \angle F$, $\angle B \cong \angle G$, $\angle C \cong \angle H$, and $\angle D \cong \angle E$. Sides $\overline{AB} \cong \overline{FG}$, $\overline{BC} \cong \overline{GH}$, $\overline{CD} \cong \overline{HE}$, and $\overline{DA} \cong \overline{EF}$. Therefore, $ABCD \cong FGHE$.

At each vertex of a polygon, there are two special angles. An **interior angle of a polygon** is an angle formed by two sides of a polygon with a common vertex. An **exterior angle of a polygon** is an angle formed by one side of a polygon and the extension of an adjacent side. In the diagram, $\angle CDA$ is an interior angle and $\angle ADE$ is an exterior angle.

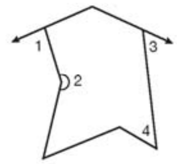


Math Reasoning

Generalize How many interior angles does a convex n-gon have? How many exterior angles does a convex n-gon have?

Example 3 Identifying Interior and Exterior Angles of Polygons

For each numbered angle in the polygon, determine whether it is an interior angle or an exterior angle.

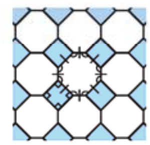


SOLUTION

Angles 2 and 4 are interior. Angles 1 and 3 are exterior.

Example 4 Application: Tile Patterns

This floor tile pattern uses polygonal tiles that fit together exactly.



a. Name the two types of polygons used in the pattern. Are they regular or irregular? Explain.

SOLUTION

Square and octagon; both types are regular, because they have all sides and all angles congruent, respectively.

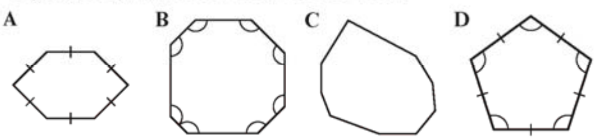
b. Pick any pair of unshaded polygons. Are they congruent? Are they convex or concave? Explain.

SOLUTION

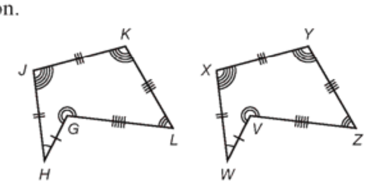
All pairs of unshaded polygons are congruent, because corresponding sides and angles are congruent. Each unshaded polygon is convex, because none of the polygon's diagonals contain points in its exterior.

Lesson Practice

a. *(Ex 1)* Name each polygon. Determine whether it is equiangular, equilateral, regular, irregular, or more than one of these.



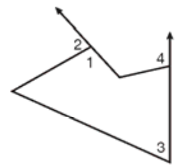
b. *(Ex 2)* Find a diagonal in polygon $GHJKL$ that contains points in the exterior of the polygon.



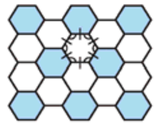
c. *(Ex 2)* Determine whether polygon $VWXYZ$ is convex or concave. Explain.

d. *(Ex 2)* Are polygons $GHJKL$ and $VWXYZ$ congruent? Justify your answer.

e. (Ex 3) For each numbered angle in the polygon, determine whether it is an interior angle or an exterior angle.



f. (Ex 4) Name the type of polygon used in this pattern. Are the polygons regular or irregular? Explain.



g. (Ex 4) Pick any pair of polygons in this pattern. Are they congruent? Are they convex or concave? Explain.

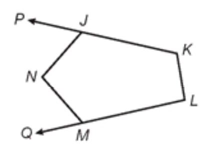
Practice Distributed and Integrated

1. (14) **Write** Explain why the pair of numbers 3 and 3 is a counterexample to this statement. *If the sum of two numbers is even, then both numbers are even.*

2. (5) **Agriculture** A farm is being divided so that each section of land has equal access to the canal running through the property for watering crops. If the road on the opposite side of the property runs parallel to the canal, explain how this can be done.

3. (8) **Wallpaper** A family wants to install wallpaper around the bottom half of a room. If the room has 12-foot tall ceilings and each wall is 14 feet long, calculate the area the wallpaper will cover.

4. (15) In polygon *JKLMN*, name each angle and identify it as an interior or exterior angle.



Write a conditional statement from each sentence.

5. (10) The absolute value of a number is a nonnegative number.

6. (10) A bilingual person speaks two languages.

7. (11) Determine the midpoint M of $\overline{XY}$ connecting $X(0, 2)$ and $Y(6, 1)$.

8. (14) **Model** Find a counterexample to this conjecture.
If two lines intersect, then any third coplanar line intersects both of them.

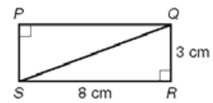
9. (7) Use inductive reasoning to determine the pattern in the following sequence:
2, 3, 5, 9, 17, 33, 65

10. (2) Identify the property that justifies this statement:
$a = b$ *and* $b = c$, *so* $a = c$.

11. (Inv 1) Name all the pairs of angles that are congruent when a transversal cuts a pair of parallel lines.

12. (9) Find the length of the segment connecting (1.3, 4.1) and (2.8, 6.1).

13. (13) **Verify** Rectangle $PQRS$ is divided into two triangles by diagonal $\overline{QS}$.

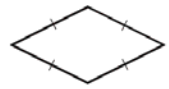


a. Determine the area of rectangle $PQRS$.

b. Given that $\triangle PQS$ and $\triangle QRS$ have equal areas, use your answer to part **a** to determine the area of $\triangle PQS$.

c. Verify that the formula for the area of a triangle gives the same answer as part **b** for the area of $\triangle PQS$.

14. (15) Classify this polygon. Is it equiangular? Is it equilateral? Is it regular?



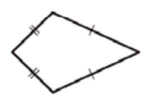
15. (12) **Home Renovation** Tasha is using two wooden rails to construct a pair of stair rails along the walls of a staircase. To make sure that the rails are parallel, she measures the acute angle each rail makes with the vertical edge of the wall at the base of the stairs.

a. What type of angles are these?

b. Tasha measures each angle to be 42°. Explain how she can make sure that the rails are parallel.

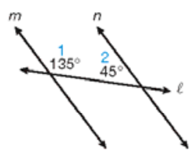
16. (14) **Verify** Confirm that this quadrilateral is a counterexample to the conjecture.

If a quadrilateral has two pairs of congruent sides, then both pairs of opposite sides are parallel.



17. (Inv 1) If two parallel lines are cut by a transversal and one pair of same-side interior angles has angles that measure $(10x + 90)°$ and $(4x + 6)°$, what is the measure of each angle?

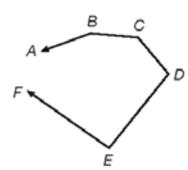
18. (12) **Write** In this figure, the transversal line ℓ intersects lines m and n. Write a paragraph explaining how you know that m and n are parallel.



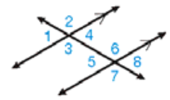
19. (15) This figure shows a polygon with one vertex and two sides missing.

a. Copy the figure and add a point G that makes $ABCDEFG$ concave.

b. Make a second copy of the figure and add a point H that makes $ABCDEFH$ convex.

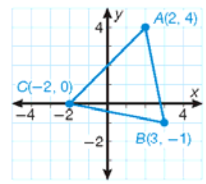


20. (Inv 1) Name every pair of corresponding angles in the diagram.



21. (8) **Building** A builder wants to add a diagonal beam to add support to a structure. The beam needs to extend across a height of 15 feet and a distance of 28 feet. What will the length of the beam be to the nearest hundredth of a foot?

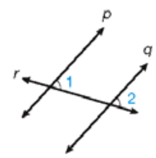
22. (11) Determine the midpoint of each side of $\triangle ABC$.



23. (13) **Algebra** The height of a triangle is 12.7 centimeters and its area is 31.75 square centimeters. Use the Triangle Area Formula to determine its base length.

24. (4) **Multiple Choice** Which statement is always true?
A Two planes intersect in a straight line.
B Two lines are contained in exactly one plane.
C Two lines can intersect at two points.
D Any four points can be contained in exactly one plane.

25. (12) **Multiple Choice** If $\angle 1$ and $\angle 2$ are congruent, which of these should be used to prove that lines p and q are parallel?
A Converse of the Alternate Exterior Angles Theorem
B Converse of the Alternate Interior Angles Theorem
C Converse of the Corresponding Angles Postulate
D Converse of the Same-Side Interior Angles Theorem



26. (7) **Predict** Use inductive reasoning to find the next term in this sequence. Explain the rule for the pattern.

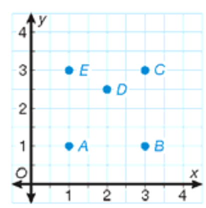
2, 3, 5, 9, 17, …

27. (8) **Multi-Step** Find the perimeter of a square if its area is 289 square centimeters.

28. (Inv. 1) If two parallel lines are intersected by a transversal, what is the sum of the measures of all four interior angles that are formed?

29. (8) Find the perimeter of a regular hexagon with side lengths of 6.8 inches

30. (15) If the points A, B, C, D, and E are connected, is polygon $ABCDE$ convex or concave?



LESSON 54

Representing Solids

Warm Up

1. (49) **Vocabulary** A prism with six square faces is called a ______.
2. (49) Name each of the pictured solids. If the solid is a prism or pyramid, classify it.

3. (49) According to Euler's Formula, if a polyhedron has 7 faces and 10 vertices, how many edges does it have?

New Concepts

In a **perspective drawing**, nonvertical parallel lines appear to meet at a point called a vanishing point. If you look straight down a highway, it appears that the edges of the highway eventually come together at a vanishing point, like point *A* in the diagram. In a perspective drawing, the **horizon** is the horizontal line that contains the vanishing point(s). A drawing with just one vanishing point is called **one-point perspective**.

> **Math Language**
>
> The **vanishing point** is the point in a perspective drawing on the horizon where parallel lines appear to meet.

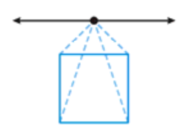
Example 1 Drawing in One-Point Perspective

Draw a rectangular prism in one-point perspective. Use a pencil with an eraser.

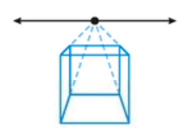
SOLUTION

Step 1 Draw a square and a horizontal line above it representing the horizon. Mark a vanishing point on the horizon.

Step 2 Draw a dashed line from the vanishing point to each of the four corners of the square.



Step 3 Using the dashed lines drawn in Step 2, draw the sides of a smaller square.



Step 4 Connect the two squares and erase the reference lines and the horizon that are located behind the prism. This prism is drawn from a one-point perspective.

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A drawing with two vanishing points is said to have two-point perspective. Look at the following example to see how a drawing can be made from a two-point perspective.

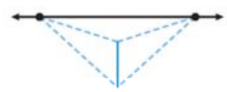
Example 2 Drawing in Two-Point Perspective

Draw a rectangular prism in two-point perspective in which the vanishing points are above the prism.

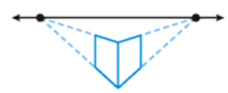
SOLUTION

Step 1 Draw a horizontal line that represents the horizon. Place two vanishing points on the horizon. Draw a vertical line segment below the horizontal line and between the two vanishing points, representing the front edge of the prism.

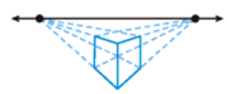
Step 2 Draw dashed lines from each vanishing point to the top and bottom of the vertical line as shown.



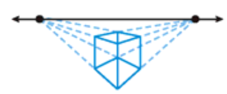
Step 3 Draw vertical segments between the dashed lines from Step 2 as shown and draw segments to connect them to the first segment.



Step 4 Draw dashed perspective lines from the segments drawn in Step 3 to each of the vanishing points as shown.



Step 5 Draw a dashed vertical line between the two intersections of the perspective lines just drawn. Sketch the segments that make the top of the prism.

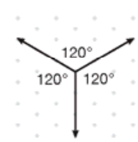


Step 6 Erase the horizon line and the dashed perspective lines. Keep the dashed lines inside the prism that represent the edges that are hidden.

This prism is drawn from a two-point perspective.

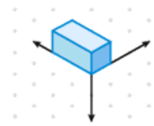
> **Math Reasoning**
>
> **Model** Could you also make a two-point perspective drawing by placing the vanishing points below the original line segment?

An **isometric drawing** is a way of drawing a three-dimensional figure using isometric dot paper, which has equally spaced dots in a repeating triangular pattern. The drawings can be made by using three axes that intersect to form 120° angles, as shown in the diagram.



Example 3 Creating Isometric Drawings

Create an isometric drawing of a rectangular prism.



SOLUTION

Draw the three axes on the isometric dot paper as shown above. Use this vertex as the bottom corner of the prism. Draw the box so that the edges of the prism run parallel to the three axes. Shading the top, front, and side of the prism will add the perception of depth.

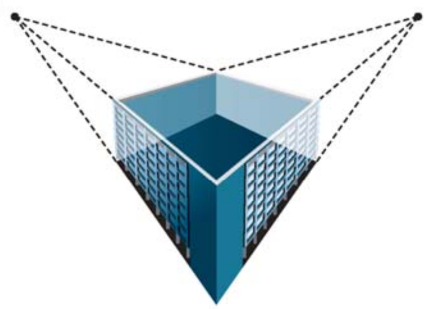
Example 4 Application: Drafting

Hint

In a two-point perspective drawing, it appears that one edge of the solid is the front of the diagram. In a one-point perspective drawing, it appears that a face of the solid is the front.

An architecture firm is planning to construct a rectangular building on a corner lot. The client would like a drawing that shows the building as though someone is looking at it from one edge. Should the drawing be from a one-point or two-point perspective? Make a sketch of what the drawing should look like.

SOLUTION



Since the front of the drawing will be an edge of the building, a two-point perspective drawing is appropriate. The diagram shows a completed view of the building.

Lesson Practice

a. (Ex 1) Draw a rectangular prism in one-point perspective in which the vanishing point is to the left of the square.

b. (Ex 2) Draw a cube in two-point perspective with the vanishing points and horizon below the vertical line.

c. (Ex 3) Make an isometric drawing of a triangular prism.

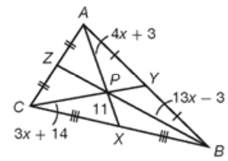
d. (Ex 4) **Drafting** Morgan wants to make a wooden bookshelf with two shelves. The bookshelf will be 1 meter wide, 1 meter deep, and 1.5 meters tall. To decide how much wood to buy, Morgan will draw his plans for the bookshelf. Should the drawing be from a one-point or two-point perspective? Sketch what Morgan's drawing should look like.

Practice Distributed and Integrated

*1. (54) Draw a triangular prism in one-point perspective so that the vanishing point is below the prism.

2. (52) **Write** Explain why the following statement is true.
If a quadrilateral is a square, then it is a rectangle.

3. (32) **Algebra** Find the length of $\overline{ZP}$ in the diagram.



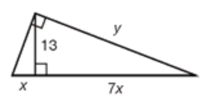
4. (42) What is the shortest distance from (5, 3) to the line $y = -2x + 8$?

*5. (54) **Architecture** An architect is creating different perspective drawings for a new building. The building is a rectangular prism and the client would like a drawing that focuses on the front façade of the building. Should the architect create the drawing using a one-point or two-point perspective? Sketch a sample drawing of the building.

6. (49) A figure has a hexagonal base and triangular lateral faces. Classify the figure.

7. (37) **Multi-Step** Graph the line and find the slope of the line that passes through the points $L(4, 1)$ and $M(3, -1)$. Then find a perpendicular line that passes through point $N(-2, -2)$.

8. (50) Find the value of x and y in the triangle shown to the nearest tenth.



9. (Inv 3) What is the sum of the exterior angles of a convex 134-sided polygon?

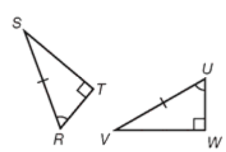
10. (52) Is the following statement always, sometimes, or never true?
A parallelogram is a rectangle.

*11. (54) Trace the figure at right on your paper. Then locate the vanishing point and the horizon line.

12. (30) **Algebra** In $\triangle ABC$, $m\angle ABC = 90°$, $AB = (3x - 7)$, and $m\angle BCA = 60°$, and in $\triangle DEF$, $m\angle DEF = 90°$, $DE = (5x - 17)$, and $m\angle EFD = 60°$. What value of x will make $\triangle ABC \cong \triangle DEF$?

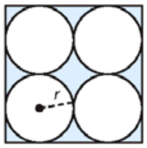
13. (32) The point where three or more lines intersect is the ________________.

*14. (36) Use the Hypotenuse-Angle Congruence Theorem to prove that $\triangle RST \cong \triangle UVW$.

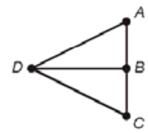


*15. (53) Find the exact length of the hypotenuse of a 45°-45°-90° right triangle with a leg that is 57 feet long.

16. (40) **Formulate** Four congruent circles are cut out of a square as shown. Write an expression for the area of the shaded region in terms of the radius of each circle, r.



17. (51) **Aviation** Four jet aircraft are flying in a triangular formation. Jets A, B, and C form a line perpendicular to the flight heading, while jet B is midway between the other two. Jet D flies directly in front of jet B. If $m\angle ADB = 37°$, what does the vertex angle of the triangular formation measure? Which theorem did you use?



***18.** (48) Use an indirect proof to prove that if two altitudes, $\overline{BX}$ and $\overline{CY}$ of $\triangle ABC$ are congruent, then the triangle must be isosceles.
Given: $\overline{BX} \cong \overline{CY}$, $\overline{BX}$ and $\overline{CY}$ are altitudes.
Prove: $\triangle ABC$ isosceles.

***19.** (53) Find the area, to the nearest hundredth, of a 45°-45°-90° right triangle with a hypotenuse of 17 centimeters.

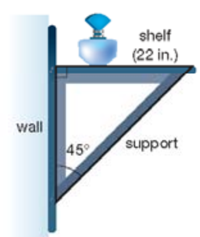
20. (43) **Algebra** If a chord perpendicular to a radius cuts the radius in two pieces that are 7 and 2 inches long, respectively, what are the two possible lengths of the chord to the nearest tenth?

***21.** (54) **Justify** How does a two-point perspective differ when the vanishing points are located close together compared with when they are located further apart? Justify your reasoning with drawings.

22. (50) Find the geometric mean of $\sqrt{2}$ and 5.

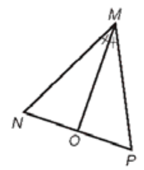
23. (34) **Multiple Choice** If the diagonals of parallelogram $JKLM$ intersect at P, which of the following is true?
A $JP = LP$ **B** $JP = KP$
C $JL = KM$ **D** $JM = KM$

***24.** (53) **Construction** The support of a shelf forms a 45°-45°-90° right triangle, with the shelf and the wall as the legs. Exactly how long is this support?



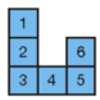
***25.** (44) **Analyze** $\triangle NPQ$ and $\triangle STV$ are similar isosceles triangles. How many of their six sides do you need numerical values for in order find all the other side lengths and the perimeters of both triangles? Explain.

26. (38) Using the diagram on the right, find the length of $\overline{MP}$ if $OP = 5$, $NO = 8$, and $MN = 18$.



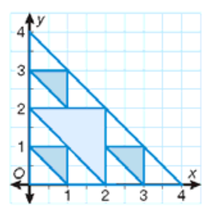
27. (Inv 4) **Cycling** Katya and Sareema start from the same location and bicycle in opposite directions for 2 miles each. Katya turns to her right 90° and continues for another mile. Sareema turns 45° to her left and continues for another mile. At this point, who is closer to the starting point?

28. (Inv 5) **Error Analysis** Darius drew this net of a number cube. Explain his error.



29. (45) **Analyze** Square $RSTU$ has vertices at $R(0, 4)$ and $S(0, 0)$. What are the possible coordinates of T and U?

30. (11) **Design** A white triangle with vertices at (0, 0), (4, 0), and (0, 4) is used to create a logo. A blue triangle is added to the design so that its vertices are the midpoints of the sides of the white triangle. The blue triangle divides the white triangle into three smaller white triangles. Smaller blue triangles are placed in each small white triangle so that their vertices are the midpoints of the sides of the small white triangles.



a. Find the coordinates of the vertices of the large blue triangle.

b. Find the coordinates of the vertices of each small blue triangle.

c. Which of the triangles are congruent, if any? Justify your answer.

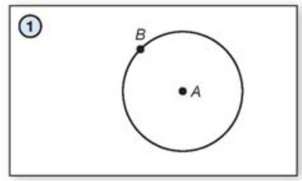
LAB 8

Tangents to a Circle

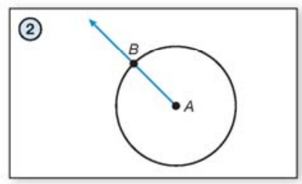
Construction Lab 8 *(Use with Lesson 58)*

Lesson 58 shows you how to identify lines tangent to a circle. This lab demonstrates how to construct lines tangent to a circle through a point on the circle.

1. To construct a tangent line through a given point on the circle, begin with $\odot A$ and a point on the circle, B.

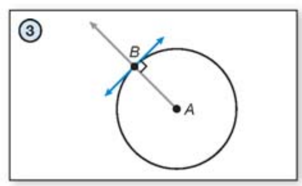


2. Draw $\overrightarrow{AB}$.



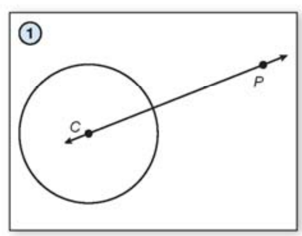
3. Using the method from Construction Lab 2, construct the line perpendicular to $\overrightarrow{AB}$ through B.

   The perpendicular line is tangent to $\odot A$ at B.



This lab demonstrates how to construct lines tangent to a circle through a point not on the circle.

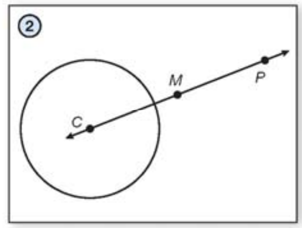
1. To construct two tangent lines through a point external to a circle, draw a circle and label the center C. Choose a point exterior to the circle, and label it P. Draw $\overleftrightarrow{CP}$.



Hint

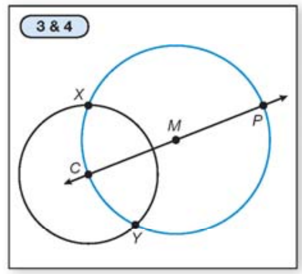
To construct the midpoint, use the method you learned in Construction Lab **3**.

(2.) Construct the midpoint of $\overline{CP}$. Label the midpoint M.

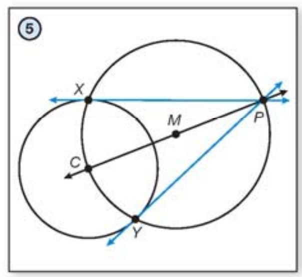


(3.) Draw a circle with a radius, CM centered on M. Notice that P is also on this circle.

(4.) Label the points of intersection of $\odot C$ and $\odot M$ as points X and Y.



(5.) Draw $\overleftrightarrow{XP}$. This line is tangent to $\odot C$ at point X. Draw $\overleftrightarrow{YP}$. Notice that $\overleftrightarrow{YP}$ is tangent to $\odot C$ at point Y.



Lab Practice

Use a compass to draw $\odot D$ and $\odot E$. Draw point A on $\odot D$. Draw points F and G outside, but near to, $\odot D$ and $\odot E$. Perform each construction indicated below.

a. a line tangent to $\odot D$ at point A

b. a line tangent to $\odot D$ from point F

c. a line tangent to $\odot E$ from point F

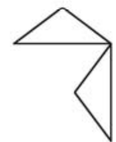
d. a line tangent to $\odot E$ from point G

INVESTIGATION

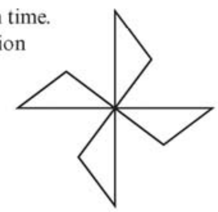
8

Patterns

Finding patterns is a valuable problem-solving skill. In this investigation, you will study patterns made by transforming a figure. The basic figure we will work with is an isosceles triangle like the one shown here. Copy this triangle by connecting the points (0, 0), (8, 0), and (4, 3) on a coordinate plane, and cut the triangle out so you can trace it onto paper.



Trace the cutout triangle onto a blank sheet of paper. First, transform the triangle by rotating it. With the triangle oriented as shown in the diagram above, rotate it 90° counterclockwise around either one of its acute angles. Trace the triangle again in its new position. You should now have a design like the one shown.



Continue to rotate the triangle 90° and trace it each time. Since the triangle will return to its original orientation after four rotations, the final design will look like the one shown.

1. What is the order of the rotational symmetry in the final pattern?

2. Does the final pattern have any lines of symmetry? If so, how many?

Trace your cutout triangle onto a blank sheet of paper. This time you will make a pattern by reflecting the triangle. Draw x- and y-axes and orient the triangle so its base lies on the x-axis and one of its acute angles lies on the origin.

3. Reflect the triangle over the y-axis and draw the resulting pattern.

4. Reflect the pattern from step 3 over the x-axis and draw the resulting pattern.

5. Does the final pattern have rotational symmetry? If so, what is the order of rotational symmetry?

6. Does the final pattern have any lines of symmetry? How many?

Math Reasoning

Model Could you have used different transformations of the triangle to obtain the same image as the one in step 4? Explain how.

Translation symmetry is a type of symmetry describing a figure that can be translated along a vector so that the image coincides with the preimage. A **frieze pattern** is a pattern that has translation symmetry along a line.

Trace your cutout triangle onto a blank sheet of paper. Orient the triangle so its base is parallel to the bottom edge of the paper. Translate the triangle to the right until the vertices of the opposite acute angles lie on the same point, as in the figure shown.

Translate the triangle to the right again. Continue this process until you have 4 triangles in a row. This is a frieze pattern.

7. What other transformation(s) could have been used to create this same pattern?

8. Does the final pattern have rotational symmetry? If so, what is the order of rotational symmetry?

9. Does the final pattern have any lines of symmetry? How many?

Now we will explore some geometric patterns. Draw two points. There is only one segment that can be drawn connecting these two points. What about 3 points? Draw 3 noncollinear points and draw segments connecting them. You find that there are three segments that can be drawn.

10. Draw four non-collinear points. Draw a line segment between each pair of points. How many line segments do you have?

11. **Predict** Based on the pattern you have seen so far, predict how many line segments you can draw connecting 5 noncollinear points. What about 6 noncollinear points? … 7 noncollinear points?

12. **Formulate** Write a rule for the number of line segments, L, between n points, in terms of the number of line segments between $n - 1$ points (denoted L_{n-1}).

The numbers in the series you have just discovered are called triangular numbers. Triangular numbers are numbers that are equal to the sum of the first n whole numbers. The first 8 triangular numbers are: 1, 3, 6, 10, 15, 21, 28, and 36.

Can an algebraic expression be written for the n^{th} triangular number? The n^{th} triangular number is given by the formula below.
$x = 1 + 2 + 3 + 4 + \ldots + n - 1 + n$

To find an expression for the n^{th} triangular number, take this series and add it to itself. Instead of adding the terms in order though, add the first term to the last term, the second term to the second to last term, and so on.

$$\begin{array}{r} x = 1 + 2 + 3 + 4 \ldots + n - 1 + n \\ \underline{+\,x = 1 + 2 + 3 + 4 \ldots + n - 1 + n} \\ 2x = (1 + n) + (2 + n - 1) + (3 + n - 2) + \ldots \end{array}$$

Now notice that in the sum, the expressions in the parenthesis can be simplified. After being simplified, the sum becomes
$2x = (n + 1) + (n + 1) + \ldots$

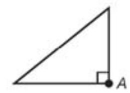
Each expression in parenthesis is the same. Moreover, we know that the series has n terms. So it can be simplified further, resulting in $2x = n(n + 1)$. To solve for x, which is the n^{th} triangular number, divide by 2.

$$x = \frac{n(n + 1)}{2}$$

13. Using the expression above, what is the 50^{th} triangular number? What is the 100^{th} triangular number?

Investigation Practice

a. Using the right triangle given here, sketch the result of rotating the figure 90° counterclockwise about the point A.



b. Continue to rotate the triangle 90° until it coincides with itself, sketching the result of each rotation. What is the order of rotational symmetry in the final figure? Does it have any lines of symmetry?

c. Return to the initial figure and reflect it over the vertical leg of the triangle. Then reflect it over the horizontal leg of the triangle. What kind of polygon is the resulting figure?

d. Does the resulting figure have any lines of symmetry? Does it have rotational symmetry?

e. Square numbers are whole numbers that could be the area of a square. The series begins: 1, 4, 9, 16, 25, ... Write an equation to find the n^{th} square number.

f. What is the 30th square number?

LESSON 58

Completing the Square

Warm Up

1. (11) **Vocabulary** In the trinomial $ax^2 + bx + c$, c is the ________ term.
2. (23) Factor the expression $x^2 + 2x + 1$.
3. (3) True or False. If a number x is a perfect square, then $\sqrt{x}$ is a whole number.

New Concepts

Recall that a perfect square trinomial can be factored into a squared binomial.

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

The Square Root Property can be used in solving quadratic equations in the form of a binomial square.

Square Root Property
If $x^2 = a$, where $a > 0$, then $x = \sqrt{a}$ or $x = -\sqrt{a}$. In general, if $x^2 = a$, then $x = \pm\sqrt{a}$ for any $a > 0$.

Example 1 Solving Quadratic Equations That Are Perfect Squares

a. Solve $x^2 - 10x + 25 = 9$.

SOLUTION

Step 1: Factor the perfect square trinomial.

$$x^2 - 10x + 25 = 9$$
$$x^2 - 2(5)x + 5^2 = 9$$
$$(x - 5)^2 = 9$$

Caution

Do not forget that the Square Root Property yields exactly **two** solutions.

Step 2: Apply the Square Root Property.

$$(x - 5)^2 = 9$$
$$x - 5 = \pm\sqrt{9}$$
$$x - 5 = \pm 3$$

Step 3: Solve for x.

$$\begin{array}{ll} x - 5 = 3 & x - 5 = -3 \\ \underline{+5 \quad +5} & \underline{+5 \quad +5} \\ x = 8 & x = 2 \end{array}$$

$$x = 8 \quad \text{or} \quad x = 2$$

Online Connection
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b. Solve $x^2 + 24x + 144 = 5$.

SOLUTION

Step 1: Factor the perfect square trinomial.

$$x^2 + 24x + 144 = 5$$
$$x^2 + 2(12)x + 12^2 = 5$$
$$(x + 12)^2 = 5$$

Step 2: Apply the Square Root Property.

$$(x + 12)^2 = 5$$
$$x + 12 = \pm\sqrt{5}$$

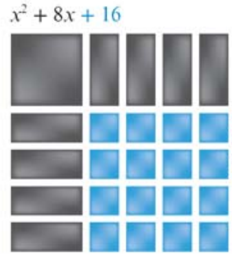
Since 5 is not a perfect square, it can be left as $\sqrt{5}$.

Step 3: Solve for x.

$$\begin{aligned} x + 12 &= \pm\sqrt{5} \\ \underline{-12} &\ \ \underline{-12} \\ x &= -12 \pm \sqrt{5} \end{aligned}$$
$$x = -12 + \sqrt{5} \quad \text{or} \quad x = -12 - \sqrt{5}$$

Some quadratic equations cannot be solved by factoring. In these cases the method of completing the square can be used. To **complete the square,** a term can be added to a quadratic expression of the form $x^2 + bx$ to form a perfect square trinomial.

Exploration Modeling Completing the Square

The model represents the quadratic binomial $x^2 + 8x$. $x^2 + 8x$ 	How many more unit tiles should be added to the model to create a square? $x^2 + 8x + 16$ 
The result is a perfect square trinomial.	$x^2 + 8x + 16 = (x + 4)^2$

Draw a model of $x^2 + 10x$. Use your model to find a perfect square trinomial and its factors.

Completing the Square

Given a quadratic of the form $x^2 + bx$, add to it the square of half the coefficient of x, $\left(\frac{b}{2}\right)^2$, to create a perfect square trinomial.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example 2 Completing the Square

Complete the square and factor the resulting perfect square trinomial $x^2 + 18x$.

SOLUTION Add the square of half the coefficient of x to the binomial. The coefficient of x is 18. Add $\left(\frac{18}{2}\right)^2 = 9^2$ to the binomial to create a perfect square trinomial.

$$x^2 + 18x + 9^2 = x^2 + 18x + 81 = (x + 9)^2$$

When using the method of completing the square to solve a quadratic equation, keep in mind that when you add a constant term to one side of an equation you must also add that constant to the other side of the equation.

Example 3 Solving Quadratic Equations by Completing the Square

Solve $x^2 - 14x - 8 = 0$ by completing the square.

SOLUTION

Step 1: $x^2 - 14x - 8 + 8 = 0 + 8$

$x^2 - 14x = 8$ Isolate the binomial $x^2 - 14x$.

Step 2: Add the square of half the coefficient of x to both sides of the equation.

The coefficient of x is -14; $\left(\frac{-14}{2}\right)^2 = (-7)^2 = 49$.

$x^2 - 14x + 49 = 8 + 49$

$x^2 - 14x + 49 = 57$

Step 3: $x^2 - 14x + 49 = 57$

$x^2 - 2(7)x + 7^2 = 57$ Factor the perfect square trinomial.

$(x - 7)^2 = 57$

Step 4: $x - 7 = \pm\sqrt{57}$ Apply the Square Root Property.

Step 5: $x - 7 + 7 = \pm\sqrt{57} + 7$

$x = 7 \pm \sqrt{57}$ Solve for x.

Math Reasoning

Estimate Without using a calculator, estimate the decimal approximation for the solutions to Example 3.

Example 4 Application: Projectile Motion

During a baseball game a player hits a fly ball. The height $f(t)$, in feet, of the ball at time t, in seconds, can be described by the function $f(t) = -16t^2 + 64t + 4$. Write the equation in vertex form by completing the square. Then determine the maximum height of the ball and the time it takes to reach the maximum height.

SOLUTION

> **Caution**
>
> When completing the square, be sure to keep the equation equivalent to the original equation. Notice $(-16)(4)$ was added to complete the square, so $+64$ was added to keep the equation equivalent to the original equation.

Complete the square for the trinomial expression that describes the height of the ball.

$f(t) = -16t^2 + 64t + 4$

$= (-16t^2 + 64t) + 4$ — Group the binomial $-16t^2 + 64t$

$= -16(t^2 - 4t) + 4$ — Factor out -16 from $(-16t^2 + 6t)$.

$= -16(t^2 - 4t + 4) + 4 + \boxed{64}$ — Complete the square.

$= -16(t - 2)(t - 2) + 68$ — Factor. Then simplify.

$f(t) = -16(t - 2)^2 + 68$ — Write the function in vertex form.

To determine the maximum height of the ball use the vertex form of a function. Recall the vertex form $f(x) = a(x - h)^2 + k$ of a quadratic function. The point (h, k) is the vertex.

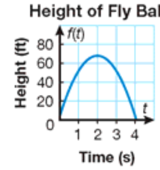
For the function $f(t) = -16(t - (+2))^2 + 68$, the vertex is $(2, 68)$. Therefore, the ball reaches a maximum height of 68 feet 2 seconds after the ball was hit.

Check Use a table to graph the function. Then estimate the vertex of the parabola on the graph.

x	0	1	2	3	4
$f(x)$	4	52	68	52	4

The graph verifies that the vertex occurs at (2, 68)

Height of Fly Ball



Lesson Practice

a. (Ex 1) Solve $x^2 - 2x + 1 = 36$.

b. (Ex 1) Solve $x^2 + 16x + 64 = 6$.

c. (Ex 2) Complete the square and factor the resulting perfect square trinomial

$$x^2 - 20x.$$

d. (Ex 3) Solve $x^2 + 26x - 2 = 0$ by completing the square.

e. (Ex 4) A pebble is thrown into the air from ground level with an initial velocity of 20 feet per second. The height of the pebble h at any given time t can be described by the equation

$$h = -16t^2 + 20t.$$

Find the time at which the height of the pebble is 4 feet. Round your answer to the hundredths place.

Practice Distributed and Integrated

Solve by using either substitution or elimination.

1. (24) $2y - 2x = 8$, $y + x = -2$

2. (24) $y - 2x = 1$, $y = -2$

3. (24) $3x + 2y = 12$, $5x - 4y = 8$

4. (10) Solve the inequality $-3 < d + 2$ and $d + 2 < 12$.

5. (10) Solve and graph the inequality $-4 \leq 4x - 8 < 12$.

Determine the kind of variation, if any, for each equation.

6. (12) $y = \frac{1}{x}$, for $x \neq 0$

7. (12) $z = \frac{1}{xy + a}$, for $x, y \neq 0$ and any value of a

Solve.

***8.** (58) $x^2 - 8x + 16 = 3$

***9.** (58) $x^2 + 26x + 169 = 81$

10. (30) **Physics** An object dropped from a height 4 meters above the ground after x seconds is modeled by the function $y = -4.9x^2 + 4$. If measured in feet, the height is modeled by $y = -16x^2 + 1.2$. Without graphing, tell how the graphs of each function compare to each other.

11. (22) **Error Analysis** Sam listed the following points $\{(2, 3), (1, 6), (5, -3), (1, 2)\}$ when giving an example of a discrete function. Correct his example.

12. (51) **Multiple Choice** What is the result of dividing $x^3 - 7x + 3$ by $x - 2$?

A $x^2 + 2x + 3 - \frac{3}{x - 2}$

B $x^2 + 2x - 3$

C $x^2 + 2x - 3 - \frac{3}{x - 2}$

D $x^2 + 2x$

***13.** (58) **Multi-Step** Use $x^2 + 104x - 97 = 4$.

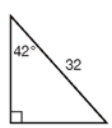
a. What is the first step before you can complete the square?

b. What constant can be added to the binomial to create a perfect square trinomial?

c. What is the factored form of the perfect square trinomial?

d. Solve the equation for x.

14. (46) **Justify** Show that it is possible to solve for x by using either the sine or cosine function.



*15. **Formulate** Formulate an expression for the amount Stu will have in his bank account at the end of a year if he invests a principal of $1,850 at an annual interest rate of 2.4%, compounded monthly.
(57)

*16. **Multiple Choice** Chose the letter that best represents the value of x in the equation $x^2 - 42x + 441 = 2$.
(58)

A $x = -21 \pm \sqrt{2}$ **B** $x = 20, 22$

C $x = 21 \pm \sqrt{2}$ **D** $x = -20, -22$

17. **Geometry** Miriam is going to create a rectangular vegetable garden along the side of her house and is going to enclose it within a fence on three sides. If the total amount of fencing she purchases is 80 feet, write an equation in standard form for finding all possible lengths and widths for the garden.
(26)

18. **Write** Explain how to multiply $(2x + 8)(3x - 6)$ using the FOIL method.
(19)

19. **Formulate** Describe the correlation coefficient r using a compound inequality and absolute value symbols.
(45)

*20. **Pass Codes** A computer generates temporary pass codes to users. Each pass code is exactly 3 letters long and it can have repeating letters. What is the probability of getting a pass code with 3 vowels (A, E, I, O, or U)?
(55)

21. **Deck Building** The area of a triangular section of a deck is $72x^2 - 98$ ft^2. If the base of the section of deck is $12x + 14$ ft, find the height of the section.
(23)

22. **Agriculture** Lee and his brother own a piece of land that has the shape of an equilateral triangle with side length 200 m. They divide the land between them into two equal parts along the altitude of the triangle by putting a fence. What is the length of the fence? What area of land does each of them own? Use a graphing calculator and round your answers to the nearest hundredth.
(52)

*23. **Graphing Calculator** Describe the feasible region for the set of inequalities
(54)

$$x \geq 0,\ y \geq 0,\ y \leq -0.75x + 8.$$

24. **Error Analysis** Find and explain the error in solving the system $\begin{matrix} x - 10y = 10 \\ 2x - 20y = 23 \end{matrix}$ in the step below.
(24)

$$\begin{array}{r} -2(x - 10y = 10) \\ +2x - 20y = 23 \\ \hline \end{array} \qquad \begin{array}{r} -2x + 20y = -20 \\ +2x - 20y = 23 \\ \hline 40y = 3 \end{array}$$

25. **Pendulums** The period T (in seconds) of a simple pendulum as a function of its length l (in meters)is given by the equation $T = 2\pi\sqrt{\frac{l}{9.8}}$. Express the length l as a function of the time t.
(50)

***26.** (57) **Multi-Step** The half-life of Uranium-234 is 245,000 years.

a. Formulate an exponential function representing the amount of a grams Uranium-234 remaining after t years.

b. Use the function to find how much of a 150-gram sample of Uranium-234 will be left after 182,000 years.

27. (53) Let $f(x) = 3x$ and $g(x) = 3x + 2$. Find the composite function $g(f(x))$.

28. (40) **Justify** A student saw his friend write that $-\sqrt{81} = -9$, and he told his friend that it is wrong because a square root cannot yield a negative number. Why is the student wrong and his friend correct?

29. (44) **Generalize** How might you rationalize the denominator for the expression $\frac{1}{\sqrt[3]{5}}$, when there is a *cube* root in the denominator?

***30.** (58) **Projectile Motion** The world record for the World Championship Punkin Chunkin, held each year in the state of Delaware, is held by an air cannon named "2nd Amendment." The height of a pumpkin t seconds after being shot from this machine can be described by the equation $h = -16t^2 + 880t + 70.7$. After how many seconds will a pumpkin shot from this machine land on the ground? Round your answer to the tenths place.

LESSON 69

Simplifying Complex Expressions

Warm Up

1. (62) **Vocabulary** A number that can be written in the form $a + bi$ where a and b are real numbers is a ____________.
2. (19) Multiply $(3x - 5)(4x + 2)$.
3. (40) Simplify $\frac{5}{\sqrt{5}}$.
4. (62) Add $(3 + 2i) + (6 - 8i)$.

New Concepts

Complex numbers $(a + bi)$ can be graphed on a complex plane, where the horizontal axis represents the real part, a, and the vertical axis represents the imaginary part, bi.

Math Language

The **imaginary axis** is sometimes referred to as the ***i*-axis.**

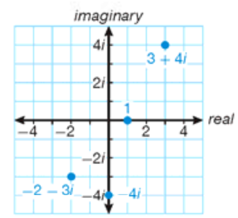
Example 1 Graphing Complex Numbers

Graph each number on the complex plane.

a. $3 + 4i$ **b.** $-2 - 3i$

c. $-4i$ **d.** 1

SOLUTION



a. Move 3 units to the right and 4 units up.

b. Move 2 units to the left and 3 units down.

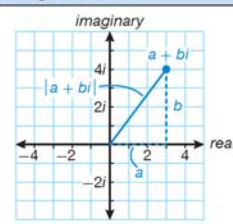
c. Move 0 units right or left and 4 units down.

d. Move 1 unit to the right and 0 units up or down.

The **absolute value of a complex number** is its distance from the origin. Unless the point is on the x- or i-axis, the distance is the length of a line segment that forms the hypotenuse of a right triangle and can be found using the Pythagorean Theorem.

Absolute Value of a Complex Number

$$|a + bi| = \sqrt{a^2 + b^2}$$



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Example 2 Finding the Absolute Value of Complex Numbers

Find the absolute value of each number.

a. $3 + 4i$

SOLUTION $|a + bi| = \sqrt{a^2 + b^2}$

$|3 + 4i| = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$

b. $-2 - 3i$

SOLUTION $|a + bi| = \sqrt{a^2 + b^2}$

$|-2 - 3i| = \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{4 + 9} = \sqrt{13}$

Math Reasoning

Verify Use the formula for the absolute value of a complex number to show that $|-1|$ is 1.

c. $-5i$

SOLUTION $|a + bi| = \sqrt{a^2 + b^2}$

$|-5i| = \sqrt{0^2 + (-5)^2}$

$= \sqrt{25} = 5$

When multiplying complex numbers, remember that $i^2 = -1$.

Example 3 Multiplying Complex Numbers

Multiply. Write answers in the form $a + bi$.

a. $-4i(7 + 3i)$

SOLUTION

$-4i(7 + 3i)$	
$-28i - 12i^2$	Distributive Property
$-28i - 12(-1)$	$i^2 = -1$
$-28i + 12$	Simplify.
$12 - 28i$	Write in standard form.

b. $(5 + 4i)(6 - 2i)$

SOLUTION

$(5 + 4i)(6 - 2i)$	
$30 - 10i + 24i - 8i^2$	Use the FOIL method.
$30 + 14i - 8i^2$	Simplify.
$30 + 14i - 8(-1)$	$i^2 = -1$
$30 + 14i + 8$	Simplify.
$38 + 14i$	Write in standard form.

Study the powers of i below.

Powers of i	
$i^1 = i$	$i^5 = i \cdot i^4 = i(1) = i$
$i^2 = -1$	$i^6 = i \cdot i^5 = i(i) = i^2 = -1$
$i^3 = i \cdot i^2 = i(-1) = -i$	$i^7 = i \cdot i^6 = i(-1) = -i$
$i^4 = i \cdot i^3 = i(-i) = -i^2 = -(-1) = 1$	$i^8 = i \cdot i^7 = i(-i) = -i^2 = -(-1) = 1$

Notice that the four simplified values of i, -1, $-i$, and 1 repeat such that $i = i^5$, $i^2 = i^6$, $i^3 = i^7$, and $i^4 = i^8$. Any power of i can be found by dividing the exponent of i by four and using the remainder.

When the remainder is …	The expression is equivalent to …
0	$i^4 = 1$
1	$i^1 = i$
2	$i^2 = -1$
3	$i^3 = -i$

Example 4 Evaluating Powers of i

Simplify each expression.

a. i^{30}

SOLUTION $30 \div 4 = 7$ R 2, so $i^{30} = i^2 = -1$.

b. $5i^{12}$

SOLUTION $12 \div 4 = 3$ R 0, so $i^{12} = i^4 = 1$.
$5i^{12} = 5(1) = 5$

c. $-2i^{13}$

SOLUTION $13 \div 4 = 3$ R 1, so $i^{13} = i^1 = i$.

$-2i^{13} = -2(i) = -2i$

Caution

A simplified expression does not have a radical in the denominator. Because $i = \sqrt{-1}$, an expression with i in the denominator is not simplified.

To eliminate a radical in a denominator, multiply both the numerator and denominator of the fraction by the conjugate of the denominator. The **complex conjugate** of $a + bi$ is $a - bi$.

Example 5 Dividing Complex Numbers

a. Divide 3 by $6 + i\sqrt{5}$. Write the answer in the form $a + bi$.

SOLUTION $\dfrac{3}{6 + i\sqrt{5}}$

$\dfrac{3}{6 + i\sqrt{5}}\left(\dfrac{6 - i\sqrt{5}}{6 - i\sqrt{5}}\right)$ Multiply the numerator and the denominator by the complex conjugate of the denominator.

$\dfrac{18 - 3i\sqrt{5}}{36 - 5i^2}$ Distribute and simplify.

$\dfrac{18 - 3i\sqrt{5}}{41} = \dfrac{18}{41} - \dfrac{3\sqrt{5}}{41}i$

b. Divide $4 + i$ by $6i$. Write the answer in the form $a + bi$.

SOLUTION $\frac{4+i}{6i}$

$\frac{4+i}{6i}\left(\frac{-6i}{-6i}\right)$ Multiply the numerator and the denominator by the complex conjugate of the denominator.

$\frac{-24i-6i^2}{-36i^2}$ Distribute.

$\frac{-24i-6(-1)}{-36(-1)}$ Simplify.

$\frac{-24i+6}{36} = -\frac{24}{36}i + \frac{6}{36} = -\frac{2}{3}i + \frac{1}{6} = \frac{1}{6} - \frac{2}{3}i$

Math Reasoning

Analyze Why is $-6i$ the conjugate of $6i$?

Lesson Practice

Graph each number on the complex plane. (Ex 1)

a. $1 - 3i$ **b.** $-2 + 4i$ **c.** -3 **d.** $2i$

Find the absolute value of each number. (Ex 2)

e. $7 - 2i$ **f.** $-5 + i$ **g.** $10i$

Multiply. Write answers in the form $a + bi$. (Ex 3)

h. $8i(-9 - 5i)$ **i.** $(2 - 3i)(5 + 9i)$

Simplify each expression. (Ex 4)

j. i^{18} **k.** $-3i^{11}$ **l.** $\frac{1}{2}i^{21}$

m. (Ex 5) Divide $3 + 3i$ by $3i$. Write the answer in the form $a + bi$.

n. (Ex 5) Divide -4 by $2 + i\sqrt{3}$. Write the answer in the form $a + bi$.

Practice Distributed and Integrated

Solve using the quadratic formula.

1. (65) $f(x) = x^2 + 8x - 3$ **2.** (65) $f(x) = 3x^2 - 10x + 4$

3. (55) **Probability** A bag holds 53 yellow marbles, 17 red marbles, and 30 green marbles. What is the probability of not choosing a red marble?

4. (57) A savings account earns interest at an annual rate of 3.8%, compounded quarterly. If the account begins with a principal amount of $2,700, what will its value be after 14 years?

Solve for x.

5. (59) $x^{\frac{1}{7}} = 5$ **6.** (59) $x^{\frac{2}{3}} = 36$

*7. (69) **Multiple Choice** In which quadrant of the complex plane is $5 - i$ located?
A Quadrant I **B** Quadrant II **C** Quadrant III **D** Quadrant IV

8. (61) **Write** Explain how to use synthetic division to determine if $x + 6$ is a factor of $x^3 + 3x^2 - 28x - 60$ and then perform the division.

Simplify.

9. (62) $\frac{1}{4}\sqrt{-256}$.

10. (62) $\frac{1}{2}\sqrt{-2500}$.

11. (56) Given $\theta = -920°$, find the measure of the reference angle.

*12. (69) **Generalize** Show that the product of any two complex conjugates, $a + bi$ and $a - bi$, is a real number.

*13. (67) Evaluate the inverse trigonometric function $\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)$. Give your answer in both radians and degrees.

14. (Inv 4) Why can't the matrix below be used as an encoding matrix for a cipher?

$$\begin{bmatrix} 5 & -2 & 3 \\ 0 & 0 & 7 \\ 0 & 0 & 13 \end{bmatrix}$$

15. (56) **Amusement Parks** You get on a ferris wheel ride, and the ferris wheel turns through 1370°. How much further must the wheel turn before you are at the bottom again?

*16. (68) **Error Analysis** Find and correct the error a student made in finding $P(\text{AM}|\text{Male})$.

$$P(\text{AM}|\text{Male}) = \frac{16}{20} = \frac{4}{5} = 80\%$$

	Male	Female
AM	16	4
PM	8	12

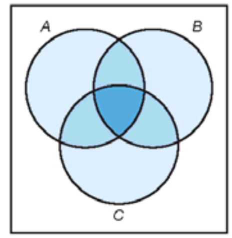
*17. (66) **Graphing Calculator** **a.** Graph $y = x^3 + 6x^2 - 32$ on a graphing calculator. What are the x-intercepts?

b. Find the multiplicity of each root of $x^3 + 6x^2 - 32 = 0$ by factoring.

*18. (68) **Formulate** Give a rule for finding $P(B|A)$. *Hint:* Consider the rule for $P(A \text{ and } B)$ for dependent events.

19. (64) **Multiple Choice** Which equation is equivalent to $10^x = 0.1$?
A $\log_{0.1}10 = x$ **B** $\log_{0.1}x = 10$
C $\log_{10}0.1 = x$ **D** $\log_{10}x = 0.1$

20. (60) **Formulate** Write a formula for $P(A \text{ or } B \text{ or } C)$. Use the diagram to the right to help you.



21. (67) **Multi-Step** Let $f(\theta) = \text{Sin}\,\theta$ and $g(\theta) = \text{Sin}^{-1}\theta$, where θ is in radians.

a. Evaluate $f\left(g\left(\frac{1}{2}\right)\right)$. Show your steps.

b. Evaluate $g\left(f\left(\frac{\pi}{6}\right)\right)$. Show your steps.

c. Based on part a, what is the value of $f(g(x))$? Based on part b, what is the value of $g(f(x))$?

22. (58) **Error Analysis** Examine the work below. Find and explain the error. Then find the correct solution.

$$x^2 - 44x + 484 = 121$$
$$(x - 22)^2 = 121$$
$$x - 22 = 11$$
$$x = 33$$

*23. (65) **Probability** A student graphs $y = 9x^2 - 29x + 22$ and uses a graphing calculator to find the value of the zeros one at a time. What is the probability that the first zero the student finds is negative? Why?

*24. (69) **Multi-Step** Simplify $i^{22} + i^{15}$. Then graph the number on the complex plane.

25. (63) **Planet Rotation** Venus is only slightly smaller than Earth; its radius is approximately 3757 miles. But it rotates much more slowly; it takes 243 Earth days for Venus to make one complete rotation. If an object is fixed on Venus's equator, how far does it travel in 1 hour due to Venus's rotation?

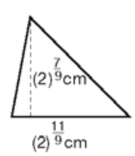
26. (64) **Analyze** Describe the value(s) of b^x when $b = 1$ and when $b \neq 1$.

*27. (69) Find the absolute value of $|3 - 4i|$.

*28. (69) Find the absolute value of $|6 + 2i|$.

29. (62) **Sports** A football player playing in a domed stadium kicks a football straight up into the air. The height of the football after t seconds can be modeled by $-16t^2 + 80t$. The dome has a height of about 288 feet. The solutions of $-16t^2 + 80t = 288$ give the number of seconds it will take for the football to reach the top of the dome. Solve the equation. Will the football reach the top of the dome? If so, when?

30. (59) **Geometry** Find the area of the triangle to the right.



LESSON 95

Factoring Higher-Order Polynomials

Warm Up

1. (35) **Vocabulary** If $f(x) = ax^2 + bx + c$ is divisible by $(x - d)$, then $(x - d)$ is a ________ of $f(x)$.

2. (51) Use synthetic division to test if $6x^3 - 5x^2 - 34x + 40$ is divisible by $(x - 2)$.

3. (51) Use synthetic division to test if $20x^3 - 53x^2 - 122x + 56$ is divisible by $(x - 2)$.

New Concepts

A constant a is a root of polynomial $P(x)$ if $P(a) = 0$.

You can use the Reminder Theorem and the Factor Theorem to test for roots of a polynomial.

Remainder Theorem
The **Remainder Theorem** states:
If the polynomial function $P(x)$ is divided by $x - a$, then the remainder r is $P(a)$.

Math Reasoning

Generalize Suppose a polynomial $P(x)$ has no real roots. What does the Factor Theorem state about such a polynomial?

Factor Theorem
The **Factor Theorem** states:
For any polynomial $P(x)$,$(x - a)$ is a factor of $P(x)$ if and only if $P(a) = 0$.

Use the Factor Theorem to test if a value is a root of a polynomial.

Example 1 Using the Factor Theorem to Test for Roots

a. Determine if $x = 5$ is a root of $P(x)$.

$$P(x) = 2x^6 - 43x^5 + 75x^4 + 1765x^3 - 857x^2 - 22{,}542x - 30{,}240$$

SOLUTION

Use synthetic division to test if $r(5) = 0$. Divide $P(x)$ by $(x - 5)$.

$$\begin{array}{r|rrrrrrr} 5 & 2 & -43 & 75 & 1765 & -857 & -22{,}542 & -30{,}240 \\ & & 10 & -165 & -450 & 6{,}575 & 28{,}590 & 30{,}240 \\ \hline & 2 & -33 & -90 & 1{,}315 & 5{,}718 & 6{,}048 & 0 \end{array}$$

Since $P(5) = r(5) = 0$, 5 is a root of the polynomial.

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b. Determine if $x = -7$ is a root of $P(x)$.

$$P(x) = 9x^8 - 101x^7 + 195x^6 + 622x^5 - 621x^4 - 1565x^3 - 1947x^2 + 2772x + 2940$$

SOLUTION

Use synthetic division to test if $r(-7) = 0$. Divide $P(x)$ by $(x + 7)$.

$$\begin{array}{r|rrrrrrrrr} -7 & 9 & -101 & 195 & 622 & -621 & -1565 & -1947 & 2772 & 2940 \\ & & -63 & 1148 & -9401 & 61{,}453 & -425{,}824 & 2{,}991{,}723 & -20{,}928{,}432 & 146{,}479{,}620 \\ \hline & 9 & -164 & 1343 & -8779 & 60{,}832 & -427{,}389 & 2{,}989{,}776 & -20{,}925{,}660 & 146{,}482{,}560 \end{array}$$

Since $P(-7) = r(-7) \neq 0$, -7 is not a root of the polynomial.

Example 2 Finding Roots of $P(x)$

Find all the rational roots of $P(x) = x^5 + 4x^4 - 10x^2 - x + 6$.

SOLUTION

By the Rational Root Theorem, the possible rational roots are ± 1, ± 2, ± 3, ± 6.

Use synthetic division to test the roots.

$$\begin{array}{r|rrrrrr} 1 & 1 & 4 & 0 & -10 & -1 & 6 \\ & & 1 & 5 & 5 & -5 & -6 \\ \hline & 1 & 5 & 5 & -5 & -6 & 0 \end{array}$$

Since $P(1) = 0$, 1 is a root of the polynomial.

$P(x) = (x - 1)(x^4 + 5x^3 + 5x^2 - 5x - 6)$.

By the Rational Root Theorem, the possible rational roots of the quartic quotient are ± 1, ± 2, ± 3, ± 6. Use synthetic division to test the roots.

$$\begin{array}{r|rrrrr} 1 & 1 & 5 & 5 & -5 & -6 \\ & & 1 & 6 & 11 & 6 \\ \hline & 1 & 6 & 11 & 6 & 0 \end{array}$$

Since $P(1) = 0$, 1 is a root of the polynomial with multiplicity 2.

$P(x) = (x - 1)(x - 1)(x^3 + 6x^2 + 11x + 6)$.

By the Rational Root Theorem, the possible rational roots of the cubic quotient are ± 1, ± 2, ± 3, ± 6. Use synthetic division to test the roots.

$$\begin{array}{r|rrrr} 1 & 1 & 6 & 11 & 6 \\ & & 1 & 7 & 18 \\ \hline & 1 & 7 & 18 & 24 \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

Since $P(-1) = 0$, -1 is a root of the polynomial.

$P(x) = (x - 1)(x - 1)(x + 1)(x^2 + 5x + 6)$

The last quotient is a quadratic that can be factored.

$P(x) = (x - 1)(x - 1)(x + 1)(x + 2)(x + 3)$

The roots are **1**, **−1**, **−2**, and **−3**. The root 1 has a multiplicity 2.

Math Reasoning

Analyze Suppose that a polynomial $P(x)$ has two real roots a and b. Could $P(x)$ be of degree 3?

Example 3 Application: Braking Distance

When a car needs to be brought to an immediate stop, it takes time for someone to step on the brake and time for the car to come to a complete stop. During these times, the car is still moving. The stopping distance is how far the car travels during this time frame. A reasonable mathematical model for the stopping distance d, in feet, based on the car's speed s, in miles per hour, is shown below. Use the Remainder Theorem to evaluate $d(20)$, $d(30)$, and $d(55)$.

$d(s) = 0.05s^2 + 2.2s$

SOLUTION Use synthetic division.

$$\begin{array}{r|rrr} 20 & 0.05 & 2.2 & 0 \\ & & 1 & 64 \\ \hline & 0.05 & 3.2 & 64 \end{array}$$

$$\begin{array}{r|rrr} 30 & 0.05 & 2.2 & 0 \\ & & 1.5 & 111 \\ \hline & 0.05 & 3.7 & 111 \end{array}$$

$$\begin{array}{r|rrr} 55 & 0.05 & 2.2 & 0 \\ & & 2.75 & 272.25 \\ \hline & 0.05 & 4.95 & 272.25 \end{array}$$

The stopping distances are $d(20) = 64$ ft, $d(30) = 111$ ft

$d(55) = 272.25$ ft.

Lesson Practice

Determine if $x = 7$ is a root of $P(x)$.
(Ex 1)

a. $P(x) = 2x^6 - 43x^5 + 75x^4 + 1765x^3 - 857x^2 - 22{,}542x - 30{,}240$

b. $P(x) = 9x^8 - 101x^7 + 195x^6 + 622x^5 - 621x^4 - 1565x^3 - 1947x^2 + 2772x + 2940$

c. (Ex 2) Find all the rational roots of $P(x) = x^2 + 4x^2 + 1x - 6$.

d. (Ex 3) The height of an object thrown at 40 feet per second can be modeled by $h(t) = -16t^2 + 40t$ where h is the height in feet and t is time in seconds. Use the remainder theorem to find the height of the object after 2 seconds.

Practice Distributed and Integrated

Solve.

1. (84) $x - \frac{6}{x} = 1$

2. (84) $\frac{x^2 + x - 6}{x + 1} = 0$

3. (84) $\frac{7x}{3x + 2} = 2$

4. (57) Does the graph of $y = b^x + k$, through the points $(3, -3)$ and $(5, 0)$, model exponential growth or decay?

***5.** (94) Solve $\frac{1}{x^3} \geq 5$. Round to the nearest thousandth.

6. (8) In a certain recipe, the amount of sugar is directly proportional to the amount of flour. If 3 cups of sugar are used with 8 cups of flour, how many cups of sugar are used with 12 cups of flour?

***7.** (95) **Error Analysis** Two students were evaluating $P(-4)$ for the polynomial $P(x)$, but they got different results. Which student made the mistake?

$$P(x) = x^6 + 4x^5 - x^4 - 6x^3 - 8x^2 + 2x + 8$$

Student A

-4	1	4	-1	-6	-8	2	8
		-4	0	4	8	0	-8
	1	0	-1	-2	0	2	0

$P(-4) = 0$

Student B

4	1	4	-1	-6	-8	2	8
		4	32	124	472	1856	7432
	1	8	31	118	464	1858	7440

$P(-4) = 7440$

***8.** (94) **Analyze** When solving $\frac{(x-4)(x+6)}{(x+2)(x-3)} \geq 0$, which intervals will use strict inequalities ($<$ or $>$) and why?

***9.** (64) **Multi-Step** As of the 2000 U.S. census, Nevada had the fastest growing population of all the states, and Connecticut had nearly the slowest (47th out of 50). The table below is based on U.S. census statistics.

	Nevada	Connecticut
Population in 2000 (to nearest thousand)	1,998,000	3,406,000
Average annual increase (1990–2000)	5.21%	0.36%

a. Write a function ($y_1 = ab^t$), where y_1 represents Nevada's population t years after 2000.

b. Write a function ($y_2 = ab^t$), where y_2 represents Connecticut's population t years after 2000.

c. Write and solve an equation to predict the year in which Nevada's population will overtake Connecticut's population.

Simplify each of the following.

10. (81) $2 \ln e^x$

11. (81) $x \cdot \ln e^3$

12. (73) **Estimate** A researcher visits a wildlife preserve and marks endangered African elephants. He marks 6 elephants and releases them. On a return visit several months later, he comes across 16 elephants, and 4 of them are marked from the previous visit. Estimate the elephant population in the preserve.

13. (77) **Surveying** A surveyor finds that the lengths between three stakes on the ground are 125 feet, 182 feet, and 211 feet. He connects the stakes with string, forming a triangle. To the nearest tenth, what is the measure of the largest angle in this triangle?

14. (79) Determine the domain and range of the function $f(x) = \begin{cases} -x, & \text{if } x < 0 \\ 4, & \text{if } 0 \leq x < 10. \\ 18, & \text{if } x \geq 10 \end{cases}$

15. (80) **Research** In a study of 413 men, the mean height was 174.1 centimeters and the standard deviation was 7 centimeters. Assuming that the distribution is normal, approximate the z-score of a man whose height is 182 centimeters.

16. (80) A set of values has a mean of 13 and a standard deviation of 0.625. Find the z-score of a value of 10.25.

17. (11) **Justify** Formulate a conjecture (statement) about the degree of a polynomial that is a sum or difference of polynomials compared to the degree of the polynomials that are added or subtracted to get that sum or difference. Give examples to justify your conjecture. (Hint: Name the polynomials P_1, P_2, P_3, etc. to make it easy to refer to them.)

18. (23) **Geometry** The area of a rectangular field is 100 meters. The field is 15 meters longer than it is wide. Find the length and width of the field.

***19.** (95) **Multiple Choice** Which of the following polynomials has $P(5) = 0$?

A $P(x) = 48x^6 + 212x^5 - 1098x^4 - 5298x^3 + 1174x^2 + 22{,}350x + 18{,}900$

B $P(x) = 48x^6 + 644x^5 + 2754x^4 + 2154x^3 - 12{,}974x^2 - 30{,}750x - 18{,}900$

C $P(x) = 48x^6 + 868x^5 + 6282x^4 + 23{,}238x^3 + 46{,}274x^2 + 46{,}950x + 18{,}900$

D $P(x) = 48x^6 - 268x^5 - 818x^4 + 4282x^3 + 6254x^2 - 14{,}790x - 18{,}900$

20. (1) **Multiple Choice** Identify which property or properties of real numbers are being demonstrated.

$$(2 \cdot 9) \cdot 5 = (2 \cdot 5) \cdot 9$$

A Commutative Property of Multiplication

B Associative Property of Multiplication

C Distributive Property

D Both A and B

***21.** (95) Write a possible polynomial $P(x)$ that fits the parameters $P(0) = 0$, $P(2) = 20$, and $P(-4) = 0$.

***22.** (87) Use the properties of logarithms to expand the expression $\ln (8x)^3 + \ln e^{\left(\frac{2x}{3}\right)}$.

23. (13) **Predict** The table below shows the sales in dollars, y, made by a gift shop x years after it opened.

Years	2	5	8
Sales	\$30,000	\$75,000	\$120,000

a. Plot the data on a coordinate grid and draw a line that fits the data.

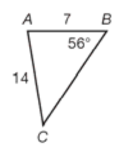
b. Use the line to predict the total sales 15 years after the shop opened.

24. (76) **Error Analysis** A student performs the following steps while finding the roots of $y = (x^2 - 7)(3x^2 + 4) - (x^2 - 7)(8x - 3)$. What is his mistake?

$$(x^2 - 7)(3x^2 + 4) - (x^2 - 7)(8x - 3) = (x^2 - 7)(3x^2 - 8x + 1)$$

***25.** (92) **Travel** A driver on the Pennsylvania turnpike turned on the cruise control at milepost 30, Warrendale, and kept it on until milepost 242, Harrisburg West, exactly 4 hours later. Show how to use an arithmetic sequence to find the speed at which the car traveled during those 4 hours.

26. (71) Solve $\triangle ABC$.



***27.** (91) Write the equation for the circle centered at (3, 8) with a radius of 4.

28. (1) John saves \$125 each month. Use the Distributive Property to mentally calculate the amount of money that he will save in 18 months.

***29.** (90) **Graphing Calculator** Use a graphing calculator to graph the function $y = 3\tan(2x - 2\pi) - 4$. Identify its period, undefined values, and phase shift.

***30.** (83) Write a quadratic equation whose root is -3.

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